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# Network Evolution Based on Centrality

Michael D. König, Claudio J. Tessone

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## Network Evolution Based on Centrality

Michael D. König, Claudio J. Tessone

Chair of Systems Design, ETH Zurich, Kreuzplatz 5, 8032 Zurich, Switzerland

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### Abstract

We study the evolution of networks when the creation and decay of links are based on the position of nodes in the network measured by their centrality. We show that the same network dynamics arises under various centrality measures, and solve analytically the network evolution. During the complete evolution, the network is characterized by nestedness: the neighborhood of a node is contained in the neighborhood of the nodes with larger degree. We find a sharp transition between hierarchical and homogeneous networks, depending on the rate of link decay. We also show that this evolution mechanism may lead to double power-law degree distributions, with interrelated exponents.

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The underlying mechanisms of link formation governing the evolution of a network ultimately determine its emergent properties at the aggregate level [1, 8]. In particular, there exists numerous empirical evidences that the network evolution can be driven by *centrality*, where nodes with higher centrality are more likely to form or receive links [10, 19]. The notion of centrality was recognized to play a fundamental role in the most despair fields, ranging from dynamical systems [23], synchronization [20], biology [14], and economics [10, 19]. In spite of its importance, a formal understanding of how networks evolve when the formation of links depends on the centrality of the nodes involved is still missing.

Depending on the context, several measures of centrality have been introduced to quantify the central position of a node in a network: *degree*, *eigenvector*, *betweenness*, *closeness*, *PageRank* and *Bonacich centrality* are the most prominent ones [6, 26]. Due to this variety, few attempts have been made so far to elucidate the common features underlying the emergent properties of networks evolving by centrality [13].

Going from the microlevel of link formation to the macroscopic level, some real world networks exhibit a high degree of clustering while, coincidentally, their degree distributions show power-law tails. Taken together, these two characteristics indicate a hierarchical organization in the network [22]. In social and economic [17], as well as biological systems [4], it has been found that the hierarchical organization of networks can further be characterized by *nestedness* [4, 25]: the neighborhood of a node is contained in the neighborhood of the nodes with higher degrees. In these examples, the degree of nestedness (defined as the fraction of links belonging to the nested structure) was shown to be above 93% [25].

In this Letter, we study a model of network evolution where links are created or removed based on the centrality of the nodes incident to the links. We show that the network evolution is independent of the centrality measure used. Thus, for the first time, this model provides a general framework to study the evolution of networks under various measures of centrality. In this model, there exist stationary networks which are highly hierarchical when the rate of link creation is low. Moreover, the networks are *nested* during the complete evolution. As we show, both, a hierarchical organization as well as network nestedness can be the outcomes of a centrality based network formation process. Finally, we show that in this framework, double power-law degree distributions [5, 7] can be stationary solutions, and that each power-law exponent unequivocally determines the other.

**Model.** We consider a network composed of  $N$  nodes, initially connected by an arbitrary network. Each node has a centrality associated to it. Note that it is not specified *which* centrality measure this refers to. There is a single parameter  $\alpha \in [0, 1]$  that determines the relative weight between edge addition and deletion. At a rate –that is equal for all nodes– a node is randomly selected and modifies its neighborhood: with probability  $\alpha$ , it creates a link to the node with the highest centrality it is not already connected to. Otherwise, with probability  $1 - \alpha$ , a link of the selected node decays. If this happens then the node removes the link to the neighbor with the lowest centrality. If the node is connected to all the other nodes in the network (resp. it is isolated), and it has to create (resp. remove) a link, nothing happens. It is simple to show that the network formation process is ergodic, and thus starting from any initial network yields the same asymptotic results. Thus, and without any loss of generality, in the following we consider an empty network as initial condition.

Alternatively, we could assume that a node has only local information of the network [9] and creates a link to another one in its second-order neighborhood with the highest centrality. It turns out that this leads to the same network evolution process as the one described above. This makes sense in situations where centrality is known *ex ante*, for example, when centrality is a measure of performance in inter-organizational networks [18], or it indicates the fitness of biological species [14].

Different measures of centrality can be used in the above network formation process, depending on the context. For example, consider a population of biological species in a catalytic network [14]. Let species  $i = 1, \dots, N$  have a fitness value  $y_i \geq 0$  that evolves according to the dynamics  $\dot{y}_i = \sum_{j=1}^N a_{ij}y_j - \phi y_i$ ,  $\phi \geq 0$ , where  $a_{ij} \in \{0, 1\}$  is the  $ij$ -th element of the symmetric adjacency matrix  $\mathbf{A}$ . In terms of relative fitness  $x_i = y_i / \sum_{j=1}^N y_j$ , we get  $\dot{x}_i = \sum_{j=1}^N a_{ij}x_j - x_i \sum_{k,j=1}^N a_{kj}x_j$ . This dynamics have a fixed point given by the eigenvector  $\mathbf{v} \geq 0$  corresponding to the largest real (Perron-Frobenius) eigenvalue  $\lambda_{\text{PF}}$  of  $\mathbf{A}$ . Hence, in this model, fitness is directly given by the eigenvector centrality. A second example comes from a socio-economic context. Consider a set

of agents whose payoffs are interdependent in a network. The agents choose a contribution level  $x_i \geq 0$  and receive a payoff given by  $\pi_i = x_i - x_i^2/2 + \lambda \sum_{j=1}^N a_{ij}x_ix_j$  with  $\lambda < 1/\lambda_{\text{PF}}$  [3]. Then the unique Nash equilibrium is given by their Bonacich centrality [26]. The dynamics introduced above correspond to a game in which agents form links that maximize their equilibrium payoffs in each period [15]. Further examples include degree centrality, closeness centrality [13], betweenness centrality [26], PageRank [6] and *random walk centrality* [21]. Finally, links created (removed) are the ones which increase the most (decrease the least) the largest eigenvalue  $\lambda_{\text{PF}}$ . These links were shown to modify the most the dynamical properties of the system [24].

At every time step, this dynamics yield a network whose adjacency matrix  $\mathbf{A}$  is *stepwise*: the nodes can be ordered by their degree, such that the zero/one entries in the adjacency matrix are separated by a monotonic step-function  $h(x)$  (see Fig. 1, right), where  $x = 1 - r$ , and  $r$  is the degree rank of a node. Networks with stepwise adjacency matrix are *threshold networks* [11, 16]. In such a network, if two nodes  $i$  and  $j$  have degrees such that  $d_i < d_j$ , then their neighborhoods satisfy  $N_i \subset N_j$ . Thus, these networks are characterized by nestedness. Moreover, the nodes can be partitioned into a *dominating set* and *independent sets*. In the dominating set  $S$ , every node not in  $S$  is linked to at least one member of  $S$ . Conversely, an independent set is one in which no two nodes are adjacent (see Fig. 1, left).

We now prove by induction that the adjacency matrix representing the state of the network at every time step is stepwise for the case of eigenvector centrality. First, at time  $t = 0$ , the first link added generates a (trivial) stepwise matrix. Next, let us assume that this is true at time  $t \geq 0$ . Consider the creation of a link  $ij$ . Then  $v_i = 1/\lambda_{\text{PF}} \sum_{k=1}^n a_{ik}v_k = 1/\lambda_{\text{PF}} \sum_{k \in N_i} v_k$ . Thus, the larger is the degree of a node  $i$ , the higher is its eigenvector component  $v_i$ . In this way, the eigenvector centrality of the nodes is ranked in the same way as their degree. Therefore, for the model studied, a node has to establish a link to a node with the highest degree it is still not connected to. This preserves the stepwise property of  $\mathbf{A}$  (see Fig. 1, right). Similarly, for the removal of links, the node with the lowest degree among the neighbors is the least central one, and removing a link to it preserves the stepwise property of  $\mathbf{A}$ .

The nested neighborhood structure allows us to use similar arguments for other centrality measures. Consider two nodes  $i$  and  $j$  in a nested graph with  $d_i > d_j$ . All walks starting at node  $j$  are contained in the set of all walks starting from node  $i$  (after exchanging the starting node  $j$  with  $i$ ). This implies that  $i$  has a higher centrality than  $j$  for any centrality measure that is based on walks or paths in the network. Hence, a proof by induction shows that the ranking of nodes by degree is equivalent to the ranking by centrality for this family of centrality measures. In general, this dynamics leads to a self-reinforcement of the nested structure.

Given the symmetry of the adjacency matrix  $\mathbf{A}$ , in order to solve the dynamic evolution of the network, it is enough to solve the dynamics for the nodes belonging to the independent sets (see Fig. 1). Let us denote by  $n(d, t)$  the number of nodes in the independent sets with degree  $d$  at

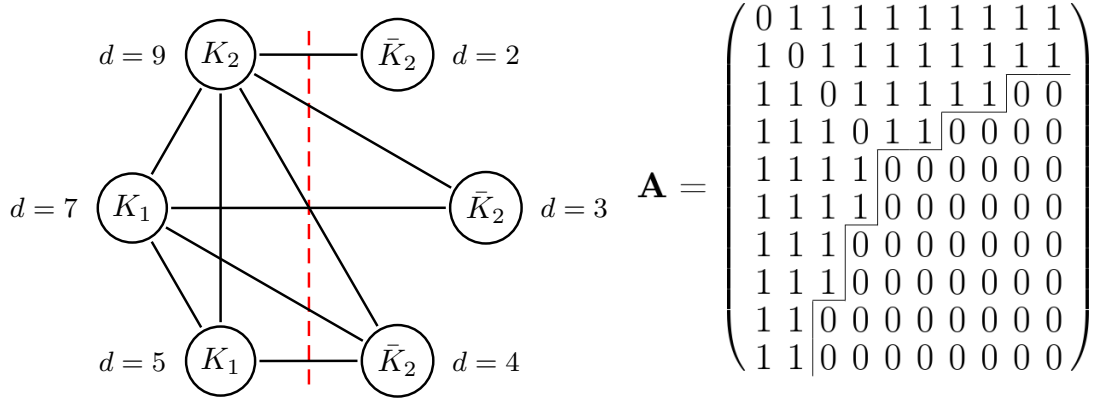


Figure 1: Representation of a nested network (left) and the associated stepwise adjacency matrix (right) with  $N = 10$  nodes. A nested network can be partitioned into subsets of nodes with the same degree (each subset is represented by circle, next to which the degree  $d$  of the nodes in the subset is indicated). A line connecting two subsets indicates that there exists a link between each node in one set to all nodes in the other set. The union of the sets represented by the circles to the left of the dashed line induce a dominating set, while to the right the circles indicate independent sets. In the matrix  $\mathbf{A}$  to the right, the zero-entries are separated from the one-entries by a step-function.

time  $t$ . The dynamic evolution of these populations can be written as a rate equation,

$$\begin{aligned} \partial_t n(d, t) = & \omega[d + 1 \rightarrow d] n(d + 1, t) + \\ & \omega[d - 1 \rightarrow d] n(d - 1, t) - \\ & (\omega[d \rightarrow d - 1] + \omega[d \rightarrow d + 1]) n(d, t), \end{aligned} \quad (1)$$

where the transition rates are simply  $\omega[d \rightarrow d + 1] = \alpha/N$ ,  $\omega[d \rightarrow d - 1] = (1 - \alpha)/N$ . In Eq. (1) we have neglected the contributions of the nodes in the corresponding dominating set (which are selected with probability  $\mathcal{O}(N^{-1})$ ) to the dynamics of the nodes in the independent sets. The dynamics studied is restricted to the profile separating (non-)existing edges, and is thus related to surface-growth models, such as those of polynuclear growth; then, it can also be linked to the one-dimensional Ising model with Kawasaki dynamics [12].

The dynamic evolution of the network can be written in terms of its degree distribution  $P(k; t) = n(d, t)/N$ , where  $k = d/N$  denotes the normalized degree. For a finite population, the minimum increment possible in degree is  $\delta k = 1/N$ . At leading order in  $\delta k$ , the dynamic

evolution corresponding to Eq. (1) is given by

$$\begin{aligned}\partial_t P(k; t) &= (1 - 2\alpha) \partial_k P(k; t) \\ &\quad + \delta k \partial_{kk}^2 P(k; t) + \mathcal{O}(\delta k^2), \\ \partial_t P(0; t) &= (1 - 2\alpha) (\delta k + \partial_k P(0; t)) \\ &\quad - \alpha \delta k P(0; t) + \mathcal{O}(\delta k^2),\end{aligned}\tag{2}$$

with the additional boundary condition  $P(1, t) = \mathcal{O}(\delta k^2)$  and an initial condition  $P(k, 0) = \delta(k)$ .

When the terms of order  $\delta k$  can be neglected, Eq. (2) becomes a usual drift equation whose stationary solution is either a complete network for  $\alpha > 1/2$  (when the link decay is low), or empty for  $\alpha < 1/2$ . If  $\alpha$  is small (and the link decay is high), the network rapidly converges to a hierarchical structure, where only a few nodes immediately become central, and they remain in this central position during the network evolution. In this case it is the competition driven dynamics for centrality which leads to the spontaneous emergence of hubs [2].

There exists a first order phase transition in the network density that gives rise to nontrivial effects around the critical point  $\alpha = 1/2$ . If  $|1 - 2\alpha|/\delta k \sim \mathcal{O}(1)$ , then the diffusion term in Eq. (2) is not negligible anymore. Time scales must be rescaled to  $\tau \equiv t \delta k$ , and we get the Fokker-Planck equations

$$\partial_\tau P(k; \tau) = (1 - 2\alpha) \partial_k P(k; \tau) + \partial_{kk}^2 P(k; \tau)\tag{3}$$

$$\partial_\tau P(0; \tau) = \frac{(1 - 2\alpha)}{\delta k} \partial_k P(0; \tau).\tag{4}$$

This prescription allows to relate the width of the transition from sparse to dense networks: it must be that  $|1 - 2\alpha| \sim \mathcal{O}(1)$ , or conversely,  $\Delta\alpha \sim N^{-1}$ .

We now study the stationary solutions for all values of  $\alpha \in [0, 1]$ . First, notice that the network obtained for a value of  $\alpha > 1/2$  is the complement of the network obtained for  $1 - \alpha < 1/2$ . Thus, in the following we consider only values of  $\alpha \leq 1/2$ . The step-function  $h(x)$  can be decomposed in a part  $h_u(x)$  below the diagonal and a part  $h_l(x)$  above the diagonal of  $\mathbf{A}$  (see Fig. 1, right panel). The point  $x^*$  is implicitly defined by  $h_u(x^*) = h_l(x^*)$ , where the step-function  $h(x)$  intersects with the diagonal. Let  $P(k)$  denote the stationary degree distribution. We have that  $h_u(x) = \int_0^{1-x} P(k) dk$ . From the stationary solution of Eq. (2) we find  $h_u(x) = \mathcal{N} e^{-2(1-2\alpha)x}$ , with  $\mathcal{N} = 2(1 - 2\alpha)/(1 - e^{-2(1-2\alpha)N})$ .

This result for the functional form of the step-function is valid for the elements below the diagonal, i.e. for the nodes with low degree. We now turn our attention to the high degree, central nodes. From the symmetry of the adjacency matrix, one finds that  $h_l(x)$  for these nodes satisfies  $x = \mathcal{N} e^{-2(1-2\alpha)h_l(x)}$ . Thus, inverting this expression we get  $h_l(x) = (\ln(\mathcal{N}) - \ln(x))/(2(1 - 2\alpha))$ .

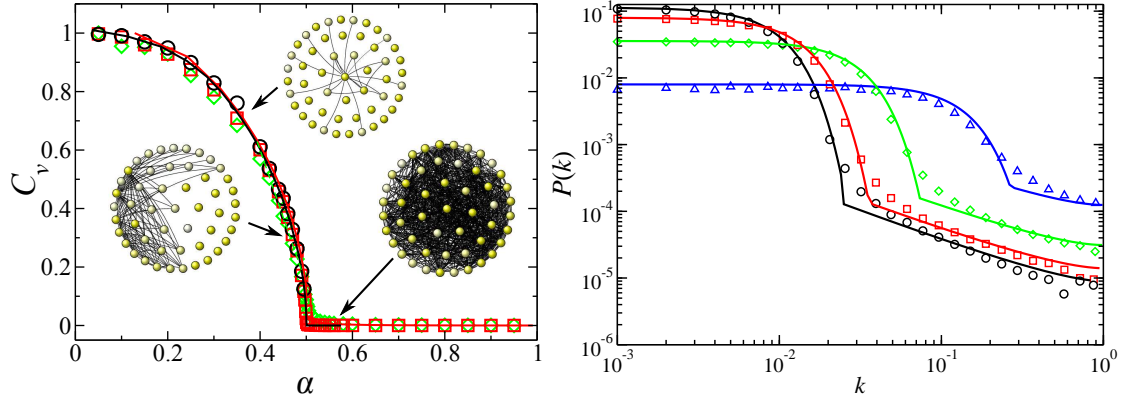


Figure 2: (Left) Eigenvector centralization  $C_v$  in stationary networks as a function of the link formation probability  $\alpha$  for different system sizes  $N = 100$  ( $\circ$ ),  $N = 1000$  ( $\square$ ) and  $N = 5000$  ( $\diamond$ ). Results of numerical simulations are superimposed with lines representing the analytical prediction. (Right) Degree distributions of stationary networks for different values of  $\alpha = 0.45$  ( $\circ$ ),  $0.48$  ( $\square$ ),  $0.49$  ( $\diamond$ ),  $0.495$  ( $\triangle$ ) and system size  $N = 5000$ . The figure reveals that the leading part of the distribution is exponential, while a logarithmic binning shows a power-law tail with exponent  $-1$ .

Conversely, the degree distribution is given by  $P(k) = -h'(1 - k)$ , from which the following stationary degree distribution is found

$$P(k) = \begin{cases} \mathcal{N}e^{-2(1-2\alpha)k}, & \text{if } k < 1 - x^*, \\ \frac{1}{2(1-2\alpha)}k^{-1}, & \text{if } k > 1 - x^*. \end{cases} \quad (5)$$

In particular, for  $\alpha = 1/2$ , it results in a uniform distribution  $P(k) = 1/N$ . Degree distributions for different values of  $\alpha$  in the stationary state can be seen in Fig. 2.

In these nested structures, the adjacency matrix is completely determined by the corresponding degree distribution from Eq. (5) or, conversely, from the profile function  $h(x)$ . Thus, it is possible to compute any network statistic of interest when the degree distribution is known. In doing so, one can show that the stationary networks emerging in the link formation process are characterized by short path length, high clustering, negative degree-clustering correlations and dissortativity. The emerging networks also show a clear core-periphery structure, which can be measured by their centralization, defined as the normalized sum of differences between the centrality of the nodes and the maximum centrality in the network [26]. Fig. 2 shows the eigenvector centralization  $C_v$ . It can be seen that there exists a transition at  $\alpha = 1/2$  from highly centralized to highly decentralized networks. This means that for low arrival rates of linking opportunities



$\alpha$  (and a strong link decay) the stationary network is strongly centralized, while for high arrival rates of linking opportunities, stationary networks are dense and largely homogeneous.

The symmetry condition for the step-function  $h(x)$  implies an important result when part of the degree distribution (for example around the head, i.e.  $k \rightarrow 0$ ) shows a power-law decay: The tail of the distribution (i.e.  $k \rightarrow \infty$ ) also follows a power-law distribution, but with a different exponent. To see this, let us assume that the head of the distribution has the functional dependence  $P(k) = \beta k^{-\eta}$ . If  $\eta > 0$ , this implies that the step-function  $h_l(x)$  for low degree nodes is given by  $h_l(x) = \beta k^{-\eta-1}/1 - \eta$ . By inverting this function, we get  $x = [\beta/(1 - \eta)]h_u(x)^{1/(-\eta+1)}$ ; and the distribution in the tail yields  $P_u(k) = [\beta/(1 - \eta)]^{1/(\eta+1)}(1 + \eta)^{-1}/k^{\eta_u}$ , where  $\eta_u = \eta/(\eta - 1)$ . In the limit  $\eta \rightarrow \infty$ , (there is an exponential distribution for the head), it implies  $\eta_u \rightarrow 1$ , i.e. we recover the previous result of Eq. (5). The power-law distribution in the head and in the tail have the same exponent when  $\eta = 2$ .

So far we have assumed that all nodes are selected at the same rate, regardless of their position in the network. Depending on the context, this assumption may not apply. In order to overcome this limitation, we assume that nodes are selected at a rate which depends on their position in the network. Note that the rate at which nodes are selected affects only the frequency but not the way in which they create or remove links. Therefore, the nestedness of the network is preserved. Moreover, in these nested structures, the nodes with the same degree are indistinguishable, as only their degree rank in the network is important. We therefore assume that the node selection rate  $F$  is a function of the degree of the node. As a simple example, we set  $F(k) = k^\eta + A$ , where  $A > 0$  denotes the idiosyncratic activity of every node, and  $\eta > 0$  a parameter governing nonlinearly the preferential selection of nodes with higher degree. Using similar arguments as in the derivation of Eq. (2), we can write the evolution of the degree distribution as follows,

$$\begin{aligned} \partial_t P(k; t) &= \frac{(1 - 2\alpha)\eta}{\mathcal{N}} k^{\eta-1} P(k; t) \\ &+ \frac{(1 - 2\alpha)}{2\mathcal{N}} [k^\eta + A] \partial_k P(k; t) \delta k + \mathcal{O}(\delta k^2). \end{aligned}$$

In the continuous limit, the stationary solution is given by  $P(k) = D/(A + k^\eta)$ , where  $D$  is a normalization constant such that  $\int_0^1 P(k) dk = 1$ . The solution reduces to the exponential one when  $\eta \rightarrow \infty$  and  $A \ll N$ . In the general case, the degree distribution exhibits two different power-law behaviors and an inflection point. These two power-laws have the functional form  $P(k) \sim k^{-\eta}$  for the head of the distribution, and consequently  $P(k) \sim k^{-\eta/(\eta-1)}$  for the tail.

To summarize, we have introduced a network formation process in which link creation and removal is based on the position of the nodes in the network measured by their centrality. Interestingly, the network evolution is independent of the exact measure of centrality, and does not require global information of the complete network structure. Also, extending the model to allow for heterogeneous activity levels of nodes keeps this property unaltered, although the

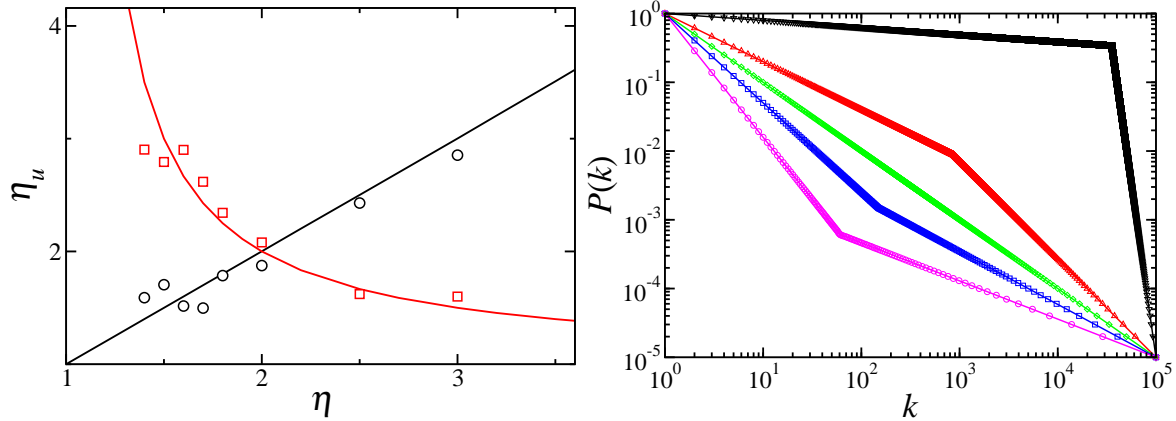


Figure 3: (Right) Degree distribution for different exponents in the head of the distribution  $\eta = 1.2$  ( $\nabla$ ),  $1.5$  ( $\triangle$ ),  $2$  ( $\diamond$ ),  $2.5$  ( $\square$ ),  $3$  ( $\circ$ ). If a nested network exhibits a power-law in the head (tail) of the degree distribution then the distribution will also exhibit a power-law behavior in the tail (head), with an exponent that can be completely determined by the head (tail). (Left) Power-law exponents for the tail of the degree distribution, i.e.  $k \rightarrow \infty$  ( $\square$ ), and the head of the distribution, i.e.  $k \rightarrow 0$  ( $\circ$ ), as a function of the power-law exponent of the head. The symbols correspond to networks of  $N = 10^5$  nodes, and the lines represent the numerical simulations.

degree distribution is modified, and a restricted set of double power-law degree distributions is found.

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## References

- [1] Réka Albert and Albert-László Barabási. Statistical mechanics of complex networks. *Rev. Mod. Phys.*, 74(1):47–97, Jan 2002.
- [2] M. Anghel, Z. Toroczkai, K.E. Bassler, and G. Korniss. Competition-Driven Network Dynamics: Emergence of a Scale-Free Leadership Structure and Collective Efficiency. *Phys. Rev. Lett.*, 92(5):58701, 2004.
- [3] Coralio Ballester, Antoni Calvó-Armengol, and Yves Zenou. Who’s who in networks. wanted: The key player. *Econometrica*, 74(5):1403–1417, 2006.

- [4] U. Bastolla, M.A. Fortuna, A. Pascual-García, A. Ferrera, B. Luque, and J. Bascompte. The architecture of mutualistic networks minimizes competition and increases biodiversity. *Nature*, 458(7241):1018–1020, 2009.
- [5] M. Boss, H. Elsinger, M. Summer, and S. Thurner. Network topology of the interbank market. *Quantitative Finance*, 4(6):677–684, 2004.
- [6] S. Brin and L. Page. The anatomy of a large-scale hypertextual Web search engine. *Computer networks and ISDN systems*, 30(1-7):107–117, 1998.
- [7] G. Csányi and B. Szendrői. Structure of a large social network. *Phys. Rev. E*, 69(3):36131, 2004.
- [8] S. N. Dorogovtsev, A. V. Goltsev, and J. F. F. Mendes. Critical phenomena in complex networks. *Rev. Mod. Phys.*, 80(4):1275, 2008.
- [9] N.E. Friedkin. Horizons of observability and limits of informal control in organizations. *Social Forces*, 62(1):54–77, 1983.
- [10] R. Gulati and M. Gargiulo. Where Do Interorganizational Networks Come From? *American Journal of Sociology*, 104(5):1398–1438, 1999.
- [11] A. Hagberg, P.J. Swart, and D.A. Schult. Designing threshold networks with given structural and dynamical properties. *Phys. Rev. E*, 74(5):56116, 2006.
- [12] H. Hinrichsen. Nonequilibrium Critical Phenomena and Phase Transitions into Absorbing States. *Adv. Phys.*, 49:815–958, 2000.
- [13] Petter Holme and Gourab Ghoshal. Dynamics of networking agents competing for high centrality and low degree. *Phys. Rev. Lett.*, 96:098701, 2006.
- [14] S. Jain and S. Krishna. Autocatalytic sets and the growth of complexity in an evolutionary model. *Phys. Rev. Lett.*, 81(25):5684–5687, 1998.
- [15] D. König, Michael, J. Tessone, Claudio, and Yves Zenou. A dynamic model of network formation with strategic interactions. CEPR Discussion Paper no. 7521, 2009.
- [16] N.V.R. Mahadev and U.N. Peled. *Threshold Graphs and Related Topics*. North Holland, 1995.
- [17] R.M. May, S.A. Levin, and G. Sugihara. Ecology for bankers. *Nature*, 451(21):893–895, 2008.

- [18] Ajay Mehra, Martin Kilduff, and Daniel J. Brass. The social networks of high and low self-monitors: Implications for workplace performance. *Administrative Science Quarterly*, 46(1):121–146, 2001.
- [19] A. Nerkar and S. Paruchuri. Evolution of R&D Capabilities: The Role of Knowledge Networks Within a Firm. *Management Science*, 51(5):771, 2005.
- [20] Takashi Nishikawa, Adilson E. Motter, Ying-Cheng Lai, and Frank C. Hoppensteadt. Heterogeneity in oscillator networks: Are smaller worlds easier to synchronize? *Phys. Rev. Lett.*, 91(1):014101, Jul 2003.
- [21] Jae Dong Noh and Heiko Rieger. Random walks on complex networks. *Phys. Rev. Lett.*, 92(11):118701, Mar 2004.
- [22] E. Ravasz and A.L. Barabási. Hierarchical organization in complex networks. *Phys. Rev. E*, 67(2):26112, 2003.
- [23] Juan G. Restrepo, Edward Ott, and Brian R. Hunt. Onset of synchronization in large networks of coupled oscillators. *Phys. Rev. E*, 71(3):036151, 2005.
- [24] Juan G. Restrepo, Edward Ott, and Brian R. Hunt. Characterizing the dynamical importance of network nodes and links. *Phys. Rev. Lett.*, 97(9):094102, 2006.
- [25] S. Saavedra, F. Reed-Tsochas, and B. Uzzi. A simple model of bipartite cooperation for ecological and organizational networks. *Nature*, 457(7228):463–466, 2008.
- [26] Stanley Wasserman and Katherine Faust. *Social Network Analysis: Methods and Applications*. Cambridge University Press, 1994.