Mechanisms of Systemic Risk –

Contagion, Reinforcement, Redistribution

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in collaboration with:

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J. Lorenz, S. Battiston, F. Schweitzer: Systemic Risk in a Unifying Framework for Cascading Processes on Networks, *European Physical Journal B* (2009, forthcoming), http://arxiv.org/abs/0907.5325

Motivation

- **systemic risk**
  - system: comprised of many interacting agents
  - risk that whole system **fails**

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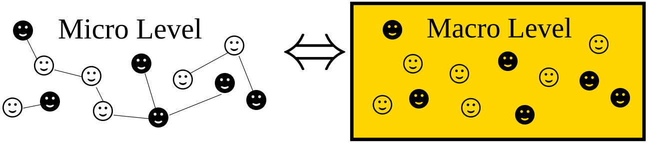
- **systemic risk**
  - system: comprised of many interacting agents
  - risk that whole system **fails**
- **examples**
  - *financial sector* (banks, companies)
  - *epidemics* (humans: SARS, plaque, animals: bird flu)
  - *power grids* (blackout due to overload)
  - *material science* (bundles of fibers)
- **common features**
  - failure of few agents is amplified ⇒ system failure
  - individual agent dynamics: fragility, threshold for failure
  - interaction: network topology

Aim: develop a common framework for systemic risk

- **cover examples from different areas**
  - what do they have in common?, what makes them unique?
- **highlight critical conditions**
  - role of heterogeneity?, leads diversification to larger systemic risk?
- **allow prediction and prevention**
  - how does the fraction of failed nodes evolve over time?
  - Can we counterbalance failure propagation?

Theory of Complex Systems

- system comprised of a *large* number of *strongly* interacting (similar) subsystems (entities, processes, or '*agents*')
  - examples: brain, insect societies (ants, bees, termites), ...
- **complex network:** agents ⇒ *nodes*, interactions ⇒ *links*



- **challenge:** The micro-macro link
  - How are the properties of the elements and their interactions ("microscopic" level) related to the dynamics and the properties of the whole system ("macroscopic" level)?

Micro Dynamics: Individual Agent

- **node** i with interaction matrix $\mathbf{A}$
  - *state* $s_i(t) \in \{0, 1\}$: 'healthy', 'failed' $\Rightarrow \mathbf{s}(t) = s_1(t), ..., s_i(t), ..., s_n(t)$
  - *fragility* $\phi_i(t) > 0$: susceptibility to fail, may depend on other nodes
  - (individual) *threshold* θ_i for failure
- key variable: *net fragility*:

$$z_i(t) = \phi_i(t, \mathbf{s}, \mathbf{A}) - \theta_i$$

- *deterministic dynamics*

$$s_i(t+1) = \Theta[z_i(t)]$$

  - $s_i = 1$ if $z_i(t) \geq 0$; $s_i = 0$ if $z_i(t) < 0$

Macro Dynamics: System Level

- **global fraction of failed nodes** $\Rightarrow$ *prediction*

$$X(t) = \frac{1}{n}\sum_{i=1}^{n} s_i(t)$$

- **dynamics**
  - assumption: probability distribution $p(z)$, $(z_i = \phi_i - \theta_i)$

$$X(t+1) = \int_0^{\infty} p_{z(t)}(z)dz = 1 - \int_{-\infty}^{0} p_{z(t)}(z)dz$$

  - cascading process: failures modify net fragility of other nodes

$$p_{z(t+1)} = \mathcal{F}(p_{z(t)})$$

- **systemic risk:** $X(t \to \infty) = X^{\star} \to 1$
  - iterate $X(t)$ dependent on $\phi(0)$, $\theta(0)$

Models with constant load

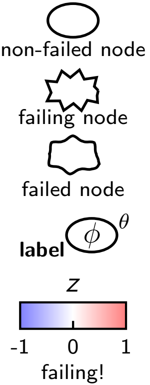
- **assumptions:**
  - 'load' of nodes is constant (equals one)
  - changes in fragility ϕ_i do not depend on ϕ_j
- **(i) 'inward' variant**: increase of fragility depends on *in-degree*

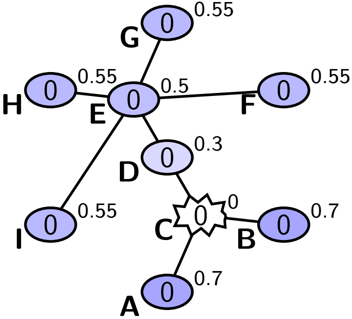
$$\phi_i(t) = \frac{1}{k_i^{\mathrm{in}}} \sum_{j \in \mathrm{nb}_{\mathrm{in}}(i,A)} s_j(t)$$

- **examples:**
  - model of social activation (Granovetter, 1978)
  - model of bankrupcy cascades (Battiston *et. al*, 2009): firms characterized by robustness $\rho_i \Rightarrow \phi_i$, $\theta_i = \rho_i^0/a$

$$\rho_i(t+1) = \rho_i^0 - \frac{a}{k_i^{\mathrm{in}}} \sum_{j \in \mathrm{nb}_{\mathrm{in}}(i,A)} s_i(t)$$

Example: Inward variant - node C fails





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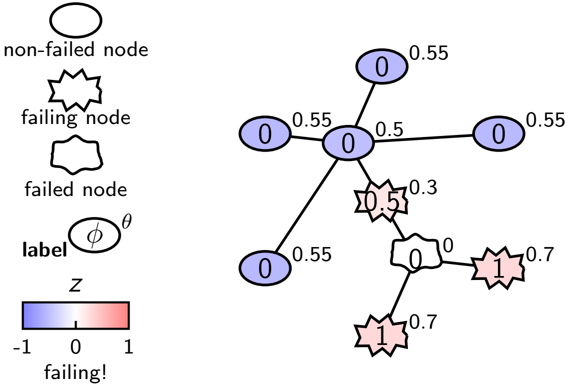

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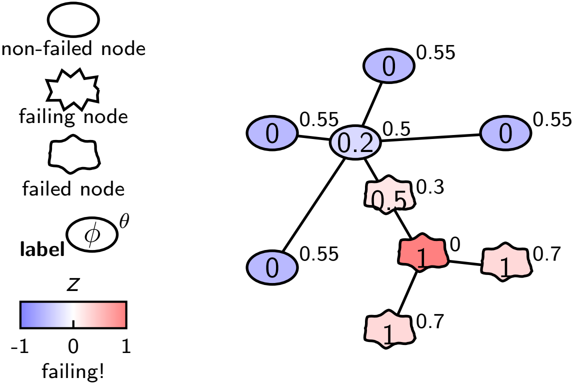
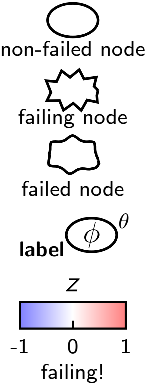
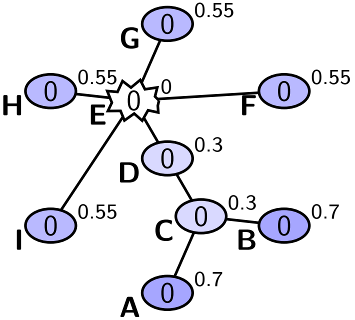


- low degree node ⇒ high vulnerability to fail
  - failure causes little damage, cascade stops after 2 steps ⇒ no 'systemic risk'

Example: Inward variant - node E fails





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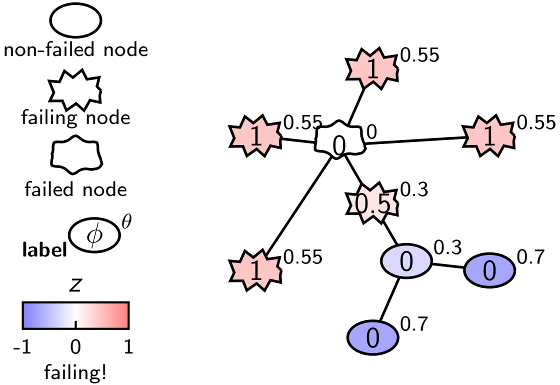

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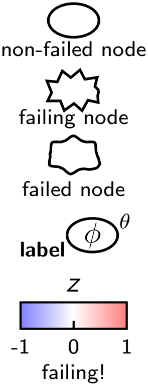

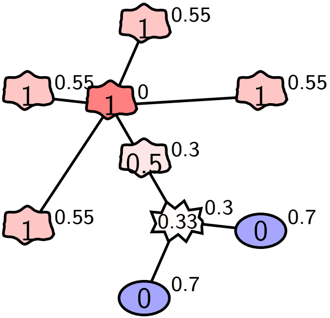

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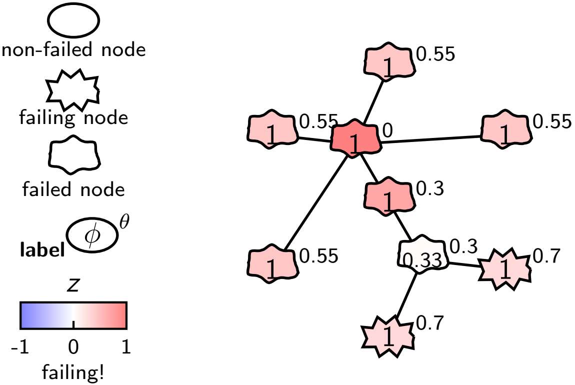

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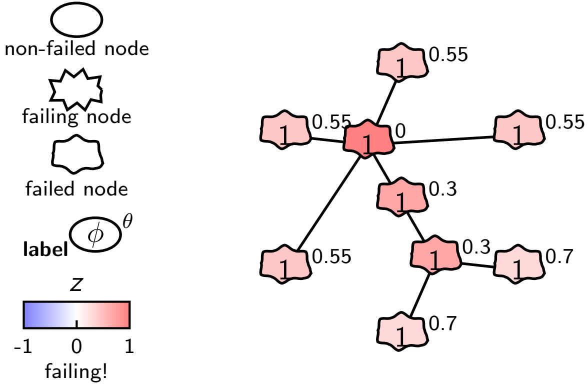


- high degree node ⇒ low vulnerability to fail
  - failure causes big damage (to low degree nodes), cascade involves all nodes ⇒ 'systemic risk'

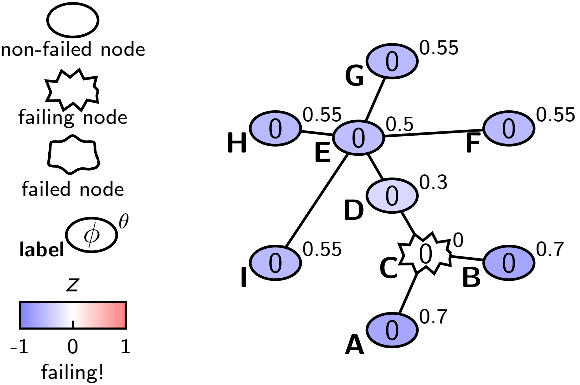
Models with constant load

- **(ii) 'outward variant':** increase of fragility depends on *out-degree*
  - load of failing node (i.e. 1) is shared equally among neighbors

$$\phi_i(t) = \sum_{j \in \mathrm{nb}_{\mathrm{in}}(i,A)} \frac{s_j(t)}{k_j^{\mathrm{out}}}$$

- *undirected, regular networks:*
  - inward and outward variant equivalent
- *heterogeneous degree:*
  - failing high-degree nodes cause *less* damage then low-degree nodes
- high-degree node:
  - high vulnerability if connected to low-degree nodes (dissortative networks)

Example: Outward variant - node C fails



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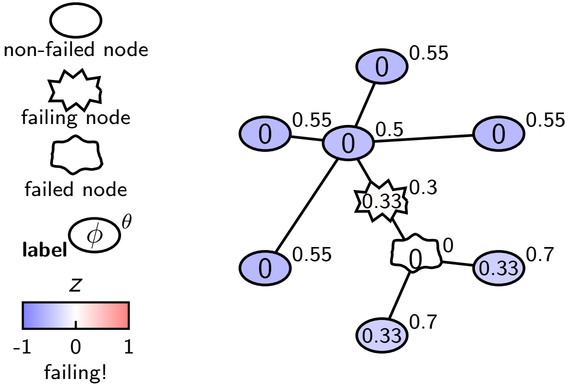

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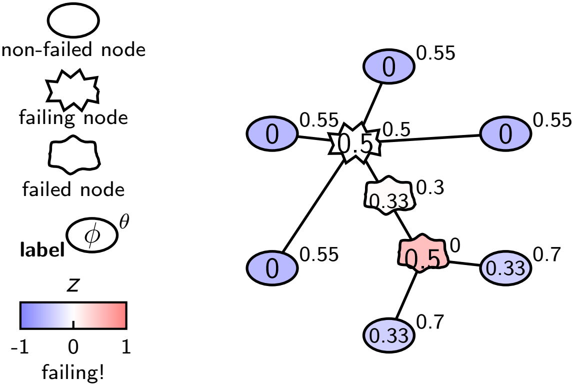
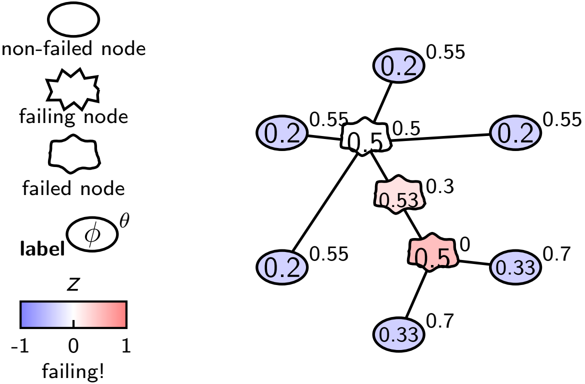


Example: Outward variant - node C fails



- low degree node causes more damage than in 'inward' variant
  - 'systemic risk' strongly depends on initial position, distributions

Models with load redistribution

- **assumptions:**
  - 'load' is represented by fragility ϕ_i
  - failed nodes distribute total fragility
  - changes in fragility ϕ_i *do* depend on ϕ_j
- **examples:**
  - (FBM) fiber bundle model (Kun *et. al*, 2000)
  - cascading models in power grids (Kinney *et. al*, 2005)
- **variants:**
  - LLSC: total load is conserved (FBM), local load is shared if nodes fail, links remain active ⇒ broad redistribution
  - LLSS: local load shedding: if nodes fail, links break ⇒ fragmented network
- *does 'globalization' increases systemic risk?*
  - network allows to redistribute load (risk), but also to receive load (risk) from far distant nodes

Load redistribution

- **LLSC:** network remains active

$$\phi_i(t) = \phi_i^0 + \sum_{j\in \mathrm{reach}_{\mathrm{in}}^{0\to 1}(i,A,s)} \frac{\phi_j^0}{\#\mathrm{reach}_{\mathrm{out}}^{1\to 0}(j,s,A)}$$

  - $\mathrm{reach}_{\mathrm{out}}^{1\to 0}(i,s,A)$: healthy nodes reachable through only failed nodes
  - $\mathrm{reach}_{\mathrm{in}}^{0\to 1}(i,A,s)$: nodes that can reach i through only failed nodes
- **LLSS:** network can be fragmented

$$\phi_i(t) = \begin{cases} \phi_i(t-1) + \sum_{j\in \mathrm{fail}_{\mathrm{in}}(i)} \frac{\phi_j(t-1)}{\#\mathrm{sus}_{\mathrm{out}}(j)} & \text{if } s_i(t) = 0 \\ 0 & \text{otherwise} \end{cases}$$

  - $\mathrm{fail}_{\mathrm{in}}(i)$: set of in-neighbors of i which failed at $t-1$
  - $\mathrm{sus}_{\mathrm{out}}(j)$: set of out-neighbors of j which remain alive after $t-1$
- **twofold reinforcement:** $\mathrm{fail}_{\mathrm{in}}(i)$ *increases*, $\mathrm{sus}_{\mathrm{out}}(j)$ *decreases*
- increase of 'systemic risk' depends on network topology, intitial position of failing nodes, distributions of fragility

Models with overload redistribution

- **assumptions:**
  - failing nodes only distribute *overload* $\Rightarrow$ net fragility
  - nodes still carry load (no complete dropout)
- **example:** economic networks of liabilities
  - fragility: total liability minus expected payments
  - threshold: operating cash flow
- **two variants:** LLSC, LLSS
  - replace $\phi_i \rightarrow (\phi_i - \theta_i)$
- **result:**
  - much smaller cascades (compared to ii)
  - high initial overload needed to trigger cascades

Macroscopic reformulation

- **aim:** compare different model classes $\longrightarrow$ set $p_{z(0)}$
- **assumptions:** fully connected network
  - independent distributions of θ, ϕ, approximate $p(\phi) \longrightarrow \delta_{\langle\phi(t)\rangle}$

$$p_{z(t)} = \delta_{\langle\phi(t)\rangle} * p_{-\theta} \longrightarrow p_{\langle\phi(t)\rangle-\theta}$$

- **macroscopic dynamics**

$$\begin{aligned} X(t+1) &= \int_0^\infty p_{\langle\phi(t)\rangle-\theta}(z)dz = P_\theta(\langle\phi(t)\rangle) \\ P_\theta(x) &= \int_{-\infty}^x p_\theta(\theta)d\theta \end{aligned}$$

- **procedure:** express $\langle\phi(t)\rangle$ in terms of $X(t) \Rightarrow$ recursive equation

- **(i) constant load:**

$$\langle\phi(t)\rangle = X(t)$$

- **(ii) load redistribution:**

$$\langle\phi(t)\rangle = \frac{\phi^0}{1 - X(t)}$$

  - $\mathrm{reach}_{\mathrm{in}}^{0\to 1}(i, A, s) = n\,X(t)$, $\#\mathrm{reach}_{\mathrm{out}}^{1\to 0}(j, s, A) = n(1 - X(t))$

- **(iii) overload redistribution:**

$$\langle\phi(t)\rangle = \frac{-\langle\theta\rangle_{X(t)}\,X(t)}{1 - X(t)}$$

  - $\langle\theta\rangle_{X(t)}$: normalized first moment of θ below X-quantile of p_θ

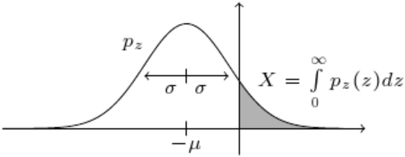
- **recursive dynamics with fix point $X^\star$**

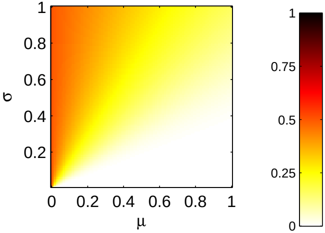
$$X(t+1) = P_\theta(\langle\phi(t)\rangle)$$

J. Lorenz, S. Battiston, F. Schweitzer: Systemic Risk in a Unifying Framework for Cascading Processes on Networks, *European Physical Journal B* (2009, forthcoming), http://arxiv.org/abs/0907.5325

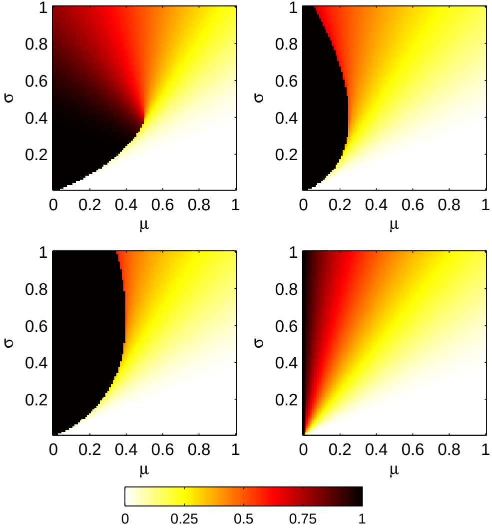
Comparison of Macrodynamics

- initial conditions normally distributed: $z(0) \sim \mathcal{N}(-\mu, \sigma)$
  - cases (i), (iii): $\theta \sim \mathcal{N}(\mu, \sigma)$, case (ii): $\theta \sim \mathcal{N}(\mu + \phi^0, \sigma)$
  - σ: measure of *initial heterogeneity* in θ across nodes
- initial failure: $X(0) = \Phi_{\mu,\sigma}(0)$
  - cumulative normal distribution function





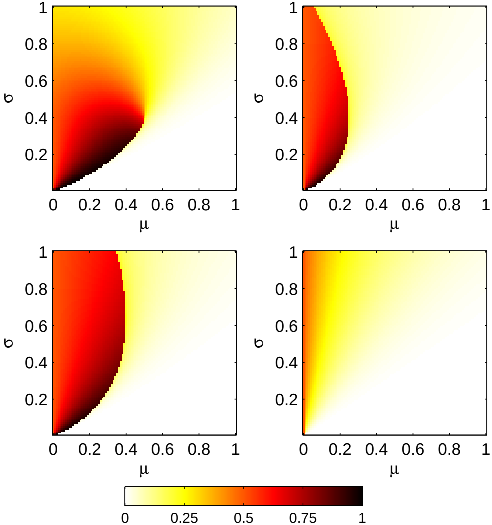
Final fraction of failed nodes $X^{\star}$



- **First-order phase transition:** small variations in initial conditions lead to complete failure
- non-monotonous behavior for case (ii): intermediate σ most dangerous

Top left: class (i) constant load. Top right: class (ii) load redistribution with initial load $\phi^0 = 0.25$. Bottom left: class (ii) with $\phi^0 = 0.4$. Bottom right: class (iii) overload redistribution.

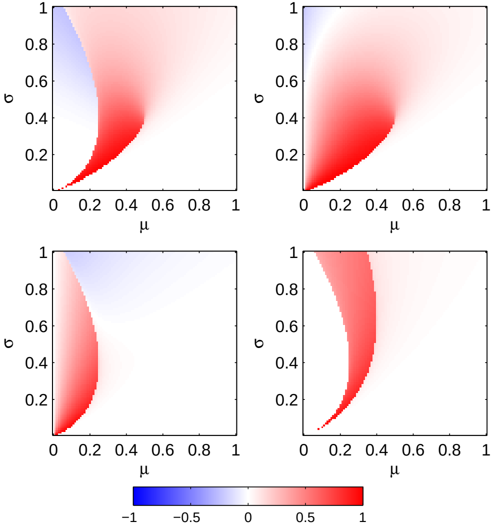
Net fraction of failed nodes $X^{\star} - X(0)$



Systemic risk resulting from *cascades* only

Top left: class (i) constant load. Top right: class (ii) load redistribution with initial load $\phi^0 = 0.25$. Bottom left: class (ii) with $\phi^0 = 0.4$. Bottom right: class (iii) overload redistribution.

Differences of $X^\star$ between classes



- case (i): larger failures for small load than case (ii)
- small μ, large σ: less failure for case (i)
- no model class leads to smaller risk in general

Top Left: $X^*_{(\mathrm{i})} - X^*_{(\mathrm{ii})}$. Top Right: $X^*_{(\mathrm{i})} - X^*_{(\mathrm{iii})}$. Bottom Left: $X^*_{(\mathrm{ii})\phi^0=0.25} - X^*_{(\mathrm{iii})}$. Bottom Right: $X^*_{(\mathrm{ii})\phi^0=0.4} - X^*_{(\mathrm{ii})\phi^0=0.25}$.

Stochastic contagion models

- deterministic dynamics: $s_i(t+1) = \Theta[\phi_i(\mathbf{s},\mathbf{A}) - \theta_i]$
- stochastic dynamics: failure/recovery with some prob. $p(z_i)$

$$s_i(t+1) = \begin{cases} 1 \text{ with } & p_i(1,t+1|1,t;z_i) \quad \text{if } s_i(t)=1 \\ 1 \text{ with } & p_i(1,t+1|0,t;z_i) \quad \text{if } s_i(t)=0 \\ 0 \text{ with } & p_i(0,t+1|0,t;z_i') \quad \text{if } s_i(t)=0 \\ 0 \text{ with } & p_i(0,t+1|1,t;z_i') \quad \text{if } s_i(t)=1 \end{cases}$$

- assumption: recovery transition at different $z_i'(t) = \phi_i - \theta_i'$
- dynamics: Chapman-Kolmogorov equation

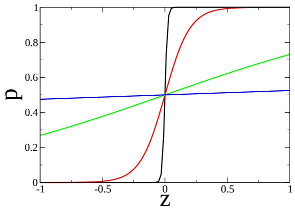
$$p_i(1,t+1) - p_i(1,t) = -p(0|1,z_i')\, p_i(1,t) + p(1|0,z_i)\, [1 - p_i(1,t)]$$

Transition probabilities

- detailed balance condition
$$\frac{p_i(1)}{1-p_i(1)} = \frac{p(1|0;z_i')}{p(0|1;z_i)}$$
- assumption for stationary distribution: logit function
$$p_i(1;\beta,\beta';z_i,z_i') = \frac{\exp(\beta z_i)}{\exp(\beta z_i)+\exp(-\beta' z_i')}$$

transition probabilities

$$p(1|0;z_i) = \gamma \frac{\exp(\beta z_i)}{\exp(\beta z_i)+\exp(-\beta' z_i')}$$
$$p(0|1;z_i') = \gamma' \frac{\exp(-\beta' z_i')}{\exp(\beta z_i)+\exp(-\beta' z_i')}$$



Mean-field approximation

- **global fraction of failed nodes**

$$\langle X(t)\rangle = \frac{1}{n}\sum_i p_i(1, z, t)$$

- **dynamics**

$$\begin{aligned} X(t+1) - X(t) &= (1 - X(t)) \int_{\mathbb{R}} p_z(z(t))\, p(1|0; z(t))\, dz \\ &\quad - X(t) \int_{\mathbb{R}} p_z(z'(t))\, p(0|1; z')\, dz'. \end{aligned}$$

- **deterministic limit:** $p(1|0; z) = \Theta(z)$; $p(0|1; z) = \Theta(-z)$

$$X(t+1) = \int_0^{\infty} p_z(z(t)) dz$$

- stochastic model with **homogeneous threshold** $z_i = z$

$$X(t+1) - X(t) = (1 - X(t))\, p(1|0; z) - X(t)\, p(0|1; z)$$

Example: Linear Voter Model

- **contagion:** driving process in *epidemics*, *social herding*
  - node i 'adopts' state of neighboring nodes j with some probability
  - competition between two absorbing states: system failure/no failure
- transition depends on local frequency, reverse transition possible

$$p_i(1|0) = f_i; \quad p_i(0|1) = 1 - f_i$$

- general framework: LVM recovered by choosing:

$$p(1|0, z_i) = \frac{\gamma}{2}\left[1 + \beta z_i\right] \; ; \quad p(0|1, z_i) = \frac{\gamma'}{2}\left[1 - \beta' z_i'\right]$$
$$\gamma = 1 \; ; \; \beta = 2 \; ; \; \theta = \frac{1}{2} \; \Rightarrow \; \phi_i = f_i$$

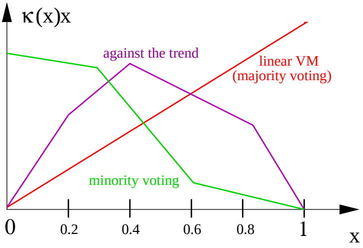
- macroscopic dynamics: mean-field approximation

$$f_i(t) \rightarrow X(t) \; \Rightarrow \; X(t+1) - X(t) = 0$$

  - formation of global state, $\{0\}$, or $\{1\}$
  - but $\langle X \rangle = X(t = 0)$, i.e. probability for systemic risk depends on initial condition

Example: Nonlinear Voter Model

$$p_i(1|0) = f_i(t)\, F_1(f_i(t)); \quad p_i(0|1) = (1 - f_i(t))F_2(f_i(t))$$

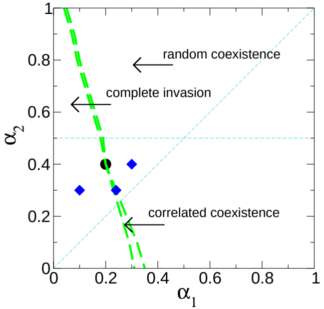


- global dynamics depends on nonlinearity → $F_1(X)$, $F_2(X)$

$$X(t+1) - X(t) = X(t)(1 - X(t))\Big[F_1(X) - F_2(X)\Big]$$

  - **linear VM**: $F_1 = F_2 = 1$
  - nonlinear VM: small non-linearities → global failure or coexistence

Nonlinearity and Systemic Risk



- nonlinear response $F_1(X), F_2(X) \rightarrow \alpha_1, \alpha_2$: different global dynamics
- even for positive frequency dependence $X^{\star} < 1$ possible
- even for 'against the trend' $X^{\star} \rightarrow 1$ (system failure) possible

- **heterogeneity of individual dynamics**: $\kappa \rightarrow \kappa_i(t)$
  - **reluctance** to adjust indiv. state may even speed up global failure

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H.U. Stark, C. Tessone, F. Schweitzer, *PRL* **101** (2008) 018701;
ACS - Advances in Complex Systems **11**/4 (2008) 87-116
F. Schweitzer, L. Behera, European Physical Journal B **67** (2009) 301-318

Example: Epidemic Spreading

- infection of healthy node: $p(1|0, z_i) = \nu\, k_i\, q$;
  - ν: infection rate, q: prob. neighbor is infected, k_i: node degree
- spontaneous recovery of infected node: $p(0|1) = \delta$
- general framework: LVM recovered by choosing:

$$p(1|0, z_i) = \frac{\gamma}{2}\left[1 + \beta z_i\right] \; ; \quad p(0|1, z_i) = \frac{\gamma'}{2}\left[1 - \beta' z_i'\right]$$

$$\gamma = 1 \; ; \; \beta = 2 \; ; \; \theta = \frac{1}{2} \; \Rightarrow \; \phi_i = \nu\, k_i\, q \; ; \; \gamma' = 2\delta \; ; \; \beta' = 0$$

- mean-field approximation: $f_i \sim q \sim X$, $k_i = k$

$$X(t+1) - X(t) = \nu\, k\, X(t)(1 - X(t)) - \delta X(t)$$

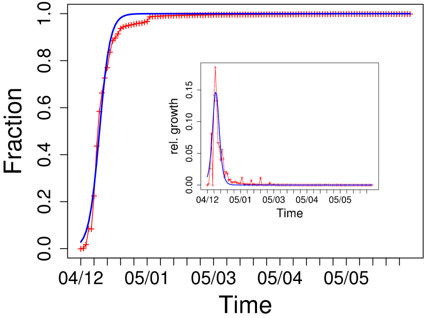
  - $\nu < \nu_c = \delta/k \Rightarrow X^* = 0$; $\quad \nu \geq \nu_c \Rightarrow X^* > 0$ (unique fix point)
- **SI model**: no recovery $\delta = 0 \Rightarrow X^\star = 1$

$$X(t+1) - X(t) = \nu\, k X(t)(1 - X(t)) \; ; \quad X(t) = \frac{1}{1 + e^{(t-\mu)/\tau}}$$

  - global dynamics: logistic growth

Example: Epidemics of Donations

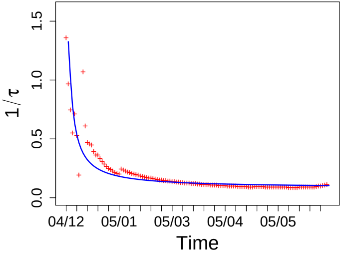
- *data:* donations after tsunami desaster (Dec 2004)
  - 01-06/2005: $N_{\text{tot}} = 1,556,626$, $A_{\text{tot}} = 126,879,803$ EUR



- Fraction of the total number of donations (inset: relative growth of amount of donations)
  - Fit: $\mu = 8.05 \pm 0.07$, $1/c = \tau = 1.98 \pm 0.06$

F. Schweitzer, R. Mach: The Epidemics of Donations: Logistic Growth and Power Laws, in: PLoS ONE vol. 3, no.1 (2008) e1458

Influence of the media



F.S., R. Mach, PLoS ONE (2008)

- slowing-down of mean-field interaction

$$1/\tau = \left[\alpha + (\beta/t) + (\gamma/t)^2\right]$$

- $c = 1/\tau$: number of successful interactions per time interval
  - early stage: people were more enthusiastic to donate money
  - later stage: became more indifferent
- decrease of $1/\tau$ in time $\Rightarrow$ lack of public interest

Summary of stochastic contagion models

- fit into general framework $\Rightarrow \gamma$, β, θ; ϕ
- VM's belong to *class (i): constant load*
  - but homogeneous threshold and stochatic failure
- SI, SIS model belong to class (i) model
  - but $\phi_i \sim k_i\, f_i$, number of connections important
- asymmetric transitions, hysteresis effects are possible

Credit networks with heterogeneous degree

- idea: firms/banks fail if 'debt' is larger than 'cash'
  - *directed credit network*: firms have extended credit to neighboring firms (debtors), i.e. 'cash' of firm i depends on paid debts of firm r
  - if firm r defaults, this increases the fragility of firm i

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- node i with in-degree k_i (neighboring nodes)
  - fragility: $\phi_i(t) \sim x_i(t)$, local fraction of failed nodes $x_i(t) = j(t)/k$
  - probability of independent failure follows binomial distribution:

$$\mathcal{B}(j,k) = \binom{k}{j} p^j (1-p)^{k-j}$$

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- what happens, when node r with total debt a fails?
  - transfers a load of a/k to its neighours $\Rightarrow$ increase of fragility

$$\phi_i(t) = \phi^0 + aj(t-1)/k \quad \text{if } s_i(t) = 0$$

- **global dynamics** (mean-field limit)
  - assumptions: $p = X(t)$, degree distribution $g(k)$, $\theta_i = \theta$

$$X(t+1) = \sum_k g(k) \sum_{j=0}^{k} \mathcal{B}(j,k,X(t))\ \mathrm{Pr}\left(\phi + \frac{j\,a}{k} > \theta\right)$$

- for narrow distribution $g(k) \longrightarrow k$

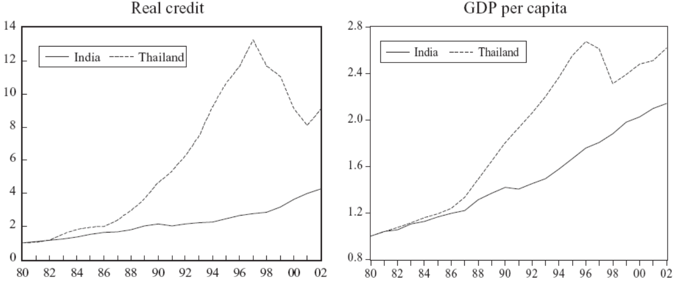
$$X(t+1) = \sum_{j=0}^{k} \mathcal{B}(j,k,X(t))\ \mathrm{Pr}\left(\phi + \frac{ja}{k} > \theta\right)$$

$\Rightarrow$ **prediction of avalanche of failure** for given t

Battiston, Stefano, Delli Gatti, Domenico, Gallegati, Mauro, Greenwald, Bruce, Stiglitz, Joseph E.: Credit chains and bankruptcy propagation in production networks, in: Journal of Economic Dynamics and Control, vol. 31, no. 6 (2007), pp. 2061-2084

Systemic risk in financial systems - good or bad?

- Costs of banking crisis (wave of bank defaults) are high for economy – measured in output loss of GDP*
- taking systemic risk can enhance overall growth despite of occasional severe crisis†



*Hoggarth, G.; Reis, R. & Saporta, V. *Costs of banking system instability: Some empirical evidence* Journal of Banking and Finance 2002

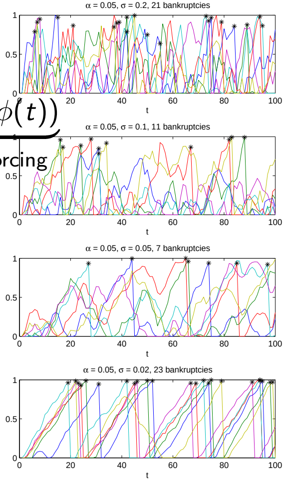
†Ranciere, R.; Tornell, A. & Westermann, F. *Systemic Crises and Growth* Quarterly Journal of Economics, 2008

Trend Reinforcement Model

- Fragility of n firms evolves as

$$\underbrace{\phi(t+1)=\phi(t)}_{\text{fragility}} + \underbrace{\sigma\xi(t)}_{\text{stochastic shocks}} + \underbrace{\alpha\ \mathrm{sign}(\Delta\phi(t))}_{\text{trend reinforcing}}$$

- trend reinforcing $\nearrow \rightsquigarrow \nearrow\nearrow, \searrow \rightsquigarrow \searrow\searrow$
- reducing volatility σ
  - decreases stochastic shocks $\rightarrow$ less bankruptcies, BUT
  - reduces possibility to break bad trends $\rightarrow$ more bankrupcies!
- Conclusion: We are safest with intermediate volatility



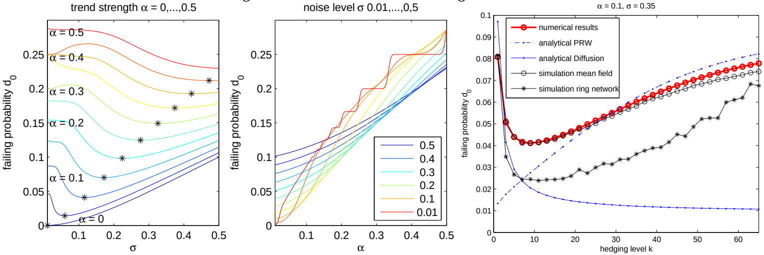
† Lorenz, Jan, Battiston, Stefano: Systemic risk in a network fragility model analyzed with probability density evolution of persistent random walks , Networks and Heterogeneous Media, vol. 3, no. 2, June (2008), pp. 185-200

Local optimum explained by stochastic process

- Scaling of displacement for Gaussian Random Walk (GRW) and Persistent Random Walk (PRW)

$$\phi(t+1) = \phi(t) + \underbrace{\sigma\xi(t)}_{\text{diffusive scaling}} + \underbrace{\alpha\text{trend}}_{\text{ballistic} \rightarrow \text{diffusive}}$$

- GRW dominates for $\frac{\alpha}{\sigma} \rightarrow 0$, PRW for $\frac{\alpha}{\sigma} \rightarrow \infty$



[†] Lorenz, Jan, Battiston, Stefano: Systemic risk in a network fragility model analyzed with probability density evolution of persistent random walks , Networks and Heterogeneous Media, vol. 3, no. 2, June (2008), pp. 185-200

Conclusions

- **general framework for systemic risk**
  - **microlevel:** interplay between fragility (ϕ_i) and threshold (θ_i)
  - **macrolevel:** fraction of failed nodes, $X(t) \Rightarrow$ *prediction*
- **different model classes** with unique behavior
  - (i) constant load, (ii) load redistribution, (iii) overload redistribution
  - phase transition: small changes lead to big impact in systemic risk
  - systemic risk increases for *medium heterogeneity*
- **mechanisms of systemic risk**
  - **contagion:** donations, voter model, social activation,
  - **load redistribution:** additional reinforcement
  - **trend reinforcement:** bankrupcies can increase
- **role of stochasticity**
  - optimal volatility to break bad trends