Evolution of Economic Networks

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in collaboration with:

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Economic Networks

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- ... one example (from Amadeus database)

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- some types of economic networks:
  - ownership networks between firms (investments)
  - networks of board members of firms (decisions)
  - R& D networks (transfer of knowledge)
  - supply networks (transfer of goods)
  - credit networks (transfer of risk)

Download Tools for Network Vizualization (M. Geipel)

from our homepage `www.sg.ethz.ch/research/graphlayout`

Economic Networks

- consist of nodes (⇒ firms) and links (⇒ interactions)

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- *Physics perspective:* focus on the links
  - topological features, degree distribution, path length
  - centrality, modularity, clustering, cliquishness, ...
  - dynamics (preferential attachment)

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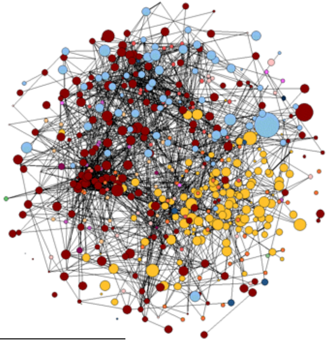
- *Economics perspective:* focus on links *and* nodes
  - eigendynamics of nodes (e.g. growth, R&D, entry/exit)
  - eigendynamics of links (adaptation, creation/removal)
  - different time scales of link and node dynamics
  - quality of links (unidirectional, weight, costs)
  - feedback of links on the node dynamics
  - utility maximization vs. boundedly rational behavior

Modeling: Innovation Networks

- complex technologies ⇒ firms must rely on knowledge transfers
  - Recent studies: relationship/performance of existing R&D networks
  - *emergence* of R&D networks, not just on existing networks



Picture from NEMO: Network Models, Governance and R&D collaboration networks, `www.nemo-net.eu`

Firm-based Network Evolution

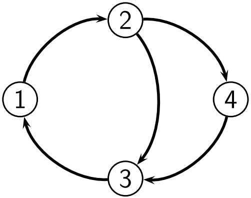
- economic network: nodes (⇒ firms) and links (⇒ interactions)
- *economics perspective:* eigendynamics of links *and* nodes
- quality of links (unidirectional, weight, *costs*)
- *local rules* of network update (*agent driven*)
  - assume a change of link structure
  - evaluate x_i over time period T ($\rightarrow$ equilibrium)
  - perform change if "beneficial" (different cases)
- **questions:**
  - what network structures do emerge (for given costs)?
    - fully connected?, cycles?, decomposed?, equilibrium?
  - do networks fulfill *global* optimality criteria?
    - beneficial for agent $i \neq$ beneficial for network
  - can optimal network be reached?
    - role of path dependence, multiple equilibria

Model Outline: Node Dynamics

- set of firms: $i = 1, ..., N$, $x_i(t)$: knowledge stock
- knowledge growth: $dx_i/dt = \mathcal{F}_i(?) \Rightarrow \mathcal{F}_i = f(x_j, x_k) + ...$
  - interaction (e.g. R&D collaborations) $\Rightarrow$ adjacency matrix $\mathbf{A}$
  - $B_i(\mathbf{A}, \mathbf{x})$: benefits (knowledge spillovers)
  - $C_i(\mathbf{A}, \mathbf{x})$: costs (innovation rent dissipation)
  - $D_i(x_i)$: decay (knowledge obsolescence)
  - $S_i(x_i)$: production ("in-house" R&D capability)

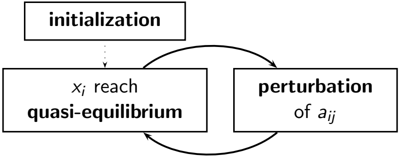
$$\frac{dx_i}{dt} = -D_i(x_i) + S_i(x_i, t) + B_i(\mathbf{A}, \mathbf{x}) - C_i(\mathbf{A}, \mathbf{x})$$

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$



Model Outline: Network Dynamics

- assumption:
  separation of time scales of node and network dynamics



- "pertubation": changes in the link structure
  - *physics:* *global rules* (least fit removal, "extremal dynamics")
  - *economics:* *local rules:* utility driven dynamics

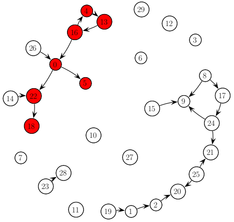
Baseline case: linear benefit and null costs

- node dynamics

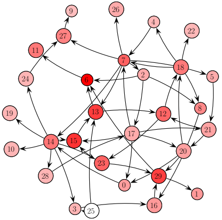
$$\frac{dx_i}{dt} = -dx_i + \sum_{i=1}^{n} a_{ji} x_j$$

- network dynamics: initial *random* network, remov least fit node, replacement with random links

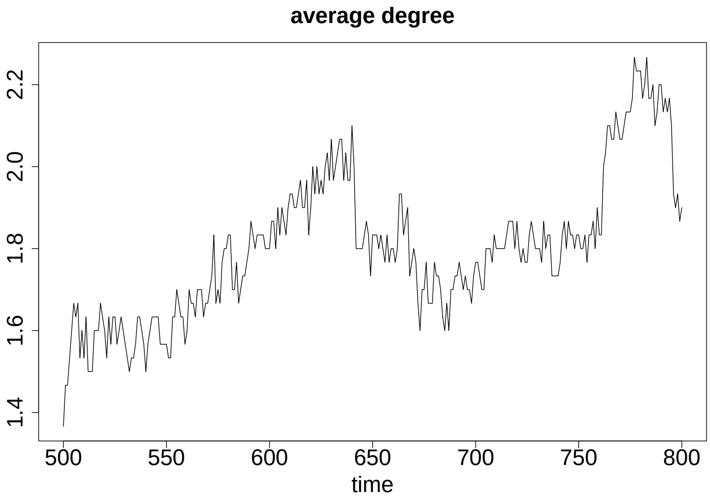
t=0



t=5.000



- emergence of a core of *cooperative* firms, and a *parasitic* pheriphery
- considerable crashes and recovery



- considerable crashes and recovery

Knowledge Growth with Linear Costs

- (no dissipation), no costs, linear benefits ⇒ gross return

$$\dot{x}_i = \sum_{j=1}^{n} a_{ij} x_j \quad \Rightarrow \quad R_i(t) = \frac{\dot{x}_i(t)}{x_i(t)} \sim \lambda_{\mathrm{PF}}(G_i)$$

  - for $t \to \infty$ growth is dominated by largest real eigenvalue $\lambda_{\mathrm{PF}}(G_i)$:
- linear costs c, number of links k_i ⇒ net returns from innovation

$$r_i(t) = R_i(t) - ck_i \quad \Rightarrow \quad r_i^* = \lim_{t\to\infty} r_i(t) = \lambda_{\mathrm{PF}}(G_i) - ck_i$$

- performance measure: π
  - network G is called *efficient*, if it maximizes social welfare Π
  - Π: sum of the individual asymptotic net returns

⇒ task for the social planner

$$\max_{\{G\}} \Pi(G)\ ; \quad \Pi = \sum_{i=1}^{n} r_i^* = \sum_{i=1}^{n} \lambda_{\mathrm{PF}}(G_i) - 2mc$$

Results for static networks

- propositions
  - for each given n, c, there exists *only one* efficient network
  - $c > n$: efficient network is the empty graph
  - $c < n$: efficient network is connected (only one component)
  - $c < 1/2$: the efficient network is the fully connected graph
  - $1/2 < c < n$: the efficient network contains a mixture of hubs ($H_{n,k}$) or cliques ($G_{n,k}$) (dependent on n), if $c \to n$, the $H_{n,k}$ converges to a *star*
- conclusions
  - if costs of collaboration increase, the network becomes sparse
    critical thresholds for c, n involved
  - an efficient network is guarenteed to exist
    analytical results about the structure of the network

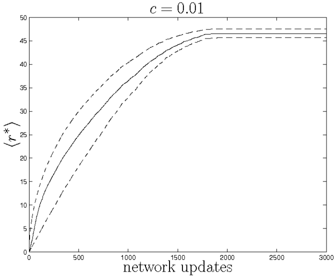
M. König, S. Battiston, M. Napoletano, F. Schweitzer: Efficiency and Stability of Dynamic Innovation Networks (Mimeo, 2007)

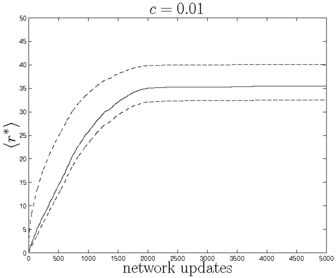
Network Dynamics

1. **initialization**: empty graph
2. **quasi-equilibrium**: fast knowledge growth
   A fixed → net returns **r** reach balanced growth $\lim_{t\to\infty} \mathbf{r}(t) = \mathbf{r}^*$
3. **perturbation** of network
   - pair of agents, i and j, is selected at random
   - link $ij \notin E(G)$ is created if

   (II) link increases r_i^*, r_j^* (*incremental improvement*)
   (BR) link ij is the *best response* of both agents i and j (among *all* possible links with other agents k)
4. **stop** if network is stable, otherwise go to **2**
   - (II) bilaterally stable networks, (BR) Nash networks

Simulations: Average Asymptotic Net Returns



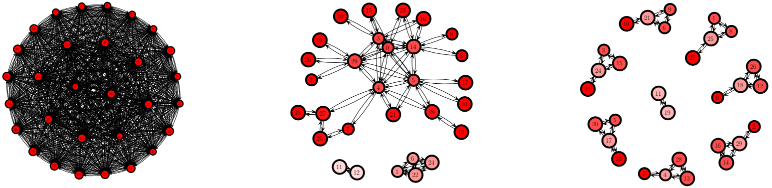


Best Response (BR) Incremental Improvement (II)

- average $\langle r_i^* \rangle$ from single run with $n = 50$ agents, $c = 0.01$
- above critical c, the II strategy outperforms BR w.r.t. $\langle r_i^* \rangle$

Simulations: Growing Networks

- intial setting: empty graph ⇒ final setting: equilibrium network
- $0 < c < 0.5$: fully connected graph is efficient network



$c = 0.01$ $c = 0.2$ $c = 0.5$

- equilibrium networks more sparse and clustered with increasing c
- inefficient equilibrium networks are reached
  - for given cost, *multiple equilibria* exist
  - equilibrium network is *path dependent* (stochastic influences)

Further results

- propositions
  - critical $c < 1/2$ for which fully connected network is not reachable
  - connectivity for BR and II strategies
  - degree heterogeneity (maximum variance)
- summary of analytical results for efficiency vs stability

Parameters	Graph	Efficiency	Stability
$c = 0$	K_n	efficient	stable
$c \geq n$	empty graph	efficient	stable
$c > 1$	empty graph	not efficient	stable
$c < \frac{1}{2}(1 - 2n + \sqrt{9 - 4n + 4n^2}) < \frac{1}{2}$, connected graph G	K_n	efficient	stable
$\frac{1}{2}(1 - 2n + \sqrt{9 - 4n + 4n^2}) \leq c < \frac{1}{2}$, connected graph G	K_n	efficient	not stable
$c > \frac{1}{2}$, connected graph G	$\{H_{n,k}, G_{n,k}\}$	efficient	stable under rewiring
$c \geq 0$, connected graph G	$\forall c\ \exists n^*(c) < n$ s.t. K_{1,n^*-1}	not efficient	stable
$c \geq 0$, graph G	$\forall c\ \exists n^*(c) < n$ s.t. K_{n^*}	not efficient	stable

Knowledge Growth with Quadratic Costs

- node dynamics

$$\frac{dx_i}{dt} = -dx_i + b\sum_{i=1}^{n} a_{ji}x_j - c\sum_{i=1}^{n} a_{ij}x_i^2$$

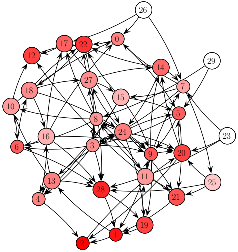
- network dynamics:
  - pair of agents, *i* and *j*, is selected at random
  - different agent strategies
    - ⋆ unilateral link deletion/creation ⇒ *indirect reciprocity*
    - ⋆ bilateral link deletion/creation ⇒ *direct reciprocity*
  - decision are bounded rational
    - ⋆ locally bounded (no complete information on the system)
    - ⋆ temporarily bounded (finite time horizon)

M. König, S. Battiston, F. Schweitzer: Modeling Evolving Innovation Networks, in: Innovation Networks – New Approaches in Modeling and Analyzing (Eds. A. Pyka, A. Scharnhorst), Springer (2007)

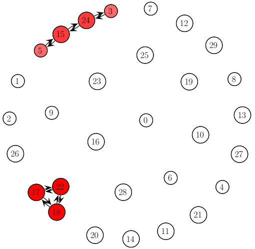
Case 1: indirect Reciprocity, linear benefit, squared costs

- agent i with time horizon T:
  - random unilateral link creation, optimal unilateral link deletion
  - accepted, if knowledge stock x_i increased

t=0



t=2.000

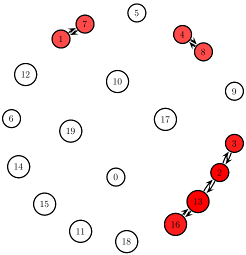


- initial links break down in favour of few bilateral cooperations
- free-riders get isolated

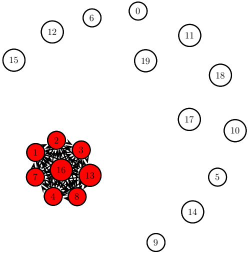
Case 2: direct reciprocity, linear benefit, squared costs

- agents i and j with time horizon T:
  - random bilateral link creation, optimal bilateral link deletion
  - accepted, if both knowledge stocks x_i and x_j increased

t=0



t=1.000



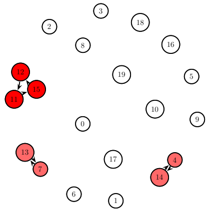
- initially connected agents evolve towards fully connected network
- initially isolated agents have nothing to contribute

Case 3: ind. reciprocity, weighted linear benefit, squared costs

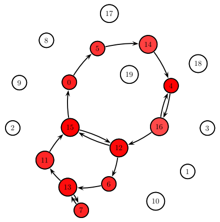
$$\frac{dx_i}{dt} = -dx_i + b\sum_{i=1}^{n} a_{ji}x_j + b_{\text{ext}}\sum_{i=1}^{n} w_{ji}x_j - c\sum_{i=1}^{n} a_{ij}x_i^2$$

- *externalities:* higher weights to
  - links providing shorter paths (Jackson, Watts 2002)
  - links contributing to cycles ⇒ feedback on technology

t=500



t=500



- cyclic externalities support emergence of indirect reciprocity

Conclusions

- important aspect of economic networks: link *and* node dynamics
- example: innovation networks (*modeling*)
  - based on dynamic multi-agent model
  - linear/nonlinear cost functions ⇒ limits for connected networks
  - multiple equilibria: many stable, but inefficient equilibrium networks
  - different agent strategies for link creation/removal
    - best response, mutual benefit/direct reciprocity, indirect reciprocity
  - *both* analytical results *and* computer simulations

Read more?

- M. D. König, S. Battiston, M. Napoletano, F. Schweitzer: On Algebraic Graph Theory and the Dynamics of Innovation Networks, *Networks and Heterogeneous Media* (2007, forthcoming), http://arxiv.org/abs/0712.2752
- M. D. König, S. Battiston, F. Schweitzer: Modeling Evolving Innovation Networks, in: *Innovation Networks - New Approaches in Modeling and Analyzing* (Eds. A. Pyka, A. Scharnhorst), Heidelberg: Springer (2008, forthcoming), http://arxiv.org/abs/0712.2779
- M. D. König, S. Battiston, M. Napoletano, F. Schweitzer: Efficiency and Stability of Dynamic Innovation Networks, *Journal of Economic Dynamics and Control* (2008, submitted)

Open Positions for PostDocs and PhD Students

- transdisciplinary projects:
  - combining physics, economics, computer science, social sciences
  - dealing with agent-based modeling, network interaction, data analysis, analytical results
- offer:
  - excellent working conditions in a lively team, innovative research topics
  - close collaboration with leading teams all over the world
  - competitive salary, working in No. 1 ranked continental European university
- interested???
  - please talk to Frank Schweitzer