The Role of Heterogeneity in Collective Decision Processes

Frank Schweitzer

fschweitzer@ethz.ch

in collaboration with:
L. Behera, H.U. Stark, P. Groeber

Overview

- Decision making
- Voter models
- Spatial heterogeneity
- Agent's heterogeneity
- Heterogeneity in social relations

Decision Making

- decision making: selection among alternatives
  - *basic* process in social and economic systems
- individual perspective of social actor ("*agent*"):
  - decision outcome ⇒ increase private utility
- classical approach: *rational agent*
  - calculation of utility function
  - common knowledge assumption
  - dissemination of information: fast, loss-free, error-free
- problems
  - incomplete (limited) information ⇒ *bounded rationality*
  - how to quantify private utility in social systems? (public votes)
  - ambigious solutions, conflicts ("frustrated system")

Social elements ...
... reduce the risk of making wrong decisions

- *imitation strategies*
  - biology, cultural evolution: adapt to the community
  - economy: copy successful strategies
- "information contagion", herding behavior
  - agents more likely do what others do
  - examples: financial markets, mass panics, fashion, ...

Collective Decisions

- aggregated outcome of many individual decisions
  - most individual implications are averaged out
  - interaction among agents play crucial role
  - system utility (social welfare) $\neq \sum_i U_i^{\mathrm{indep}}$
- *our focus:*
  - prediction of global/system quantities, not of individual decisions
  - role of local/neighborhood effects in collective decisions
  - influence of social elements (herding behavior)

Voter Models

- simple model of opinion formation
- population of agents: $i = 1, ..., N$
- each agent i: "spatial" position i, "opinion" $\theta_i(t) \Rightarrow \{0, 1\}$
- "decision": to keep or change opinion $\theta_i(t)$

$$\theta_i(t+1) = \begin{cases} \theta_i(t) & \text{keep} \\ 1 - \theta_i(t) & \text{change} \end{cases}$$

- rate to change opinion depends on other agents
  - neighbors (networks, spatial models)
  - randomly choosen agents ($\rightarrow$ mean-field)

Simplified version

- procedure (in a static network, grid)
  - randomly choose agent i
  - randomly choose one of its direct neighbors j ($c_{ij} = 1$)
  - assign opinion $\theta_j \longrightarrow \theta_i$, i.e. "keep": $\theta_j = \theta_i$, "change": $\theta_j \neq \theta_i$
- problem: "social" interpretation
  - "voters" don't vote
  - no "intertia" (role of agent's own opinion?)
  - binary interactions
  - random sequential update

"Aggregated" version

- rate to change opinion depends on *frequency*

$$w(1-\theta_i|\theta_i) = \kappa(f)\ f_i^{1-\theta_i}$$

  - $0 \leq f_i^{1-\theta_i} \leq 1$: frequency of agents with *opposite* opinions in "neighborhood" of agent i
  - $\kappa(f)$: nonlinear response to frequency of other opinions
- crucial for social, population biology applications
  - frequency considers agent i and n nearest neighbors ($\longrightarrow$ bias)
  - update in *generations* (parallel update)

Linear Voter Model

$$w(1-\theta_i|\theta_i) = c\, f_i^{1-\theta_i}$$

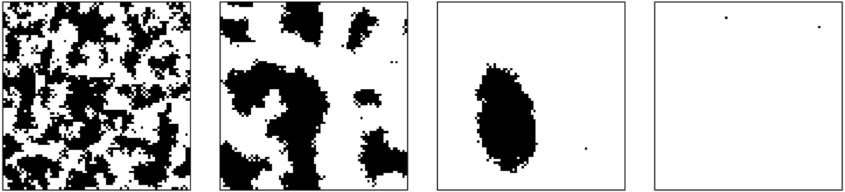
- important properties (nice!)
  - asymptotically only *one* opinion ⇒ *consensus*
  - two "absorbing´´ states: 0, 1
  - probability to reach a given attractor equals initial frequency $f(0)$
- "drawbacks" (depending on perspective)
  - very limited social/biological interpretation
    what about *coexistence* of opinions?
  - "only" interesting features:
    time to reach consensus (TTC)
    intermediate dynamics: domain sizes, interface density, ...

Stochastic computer simulations

- initially: $f(0) = 0.5$, random distribution

Online Simulation

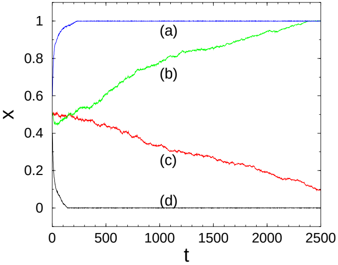
- results:
  - coordination of decisions on medium time scales
  - asymptotically: "no opposition" ($\rightarrow$ equilibrium)

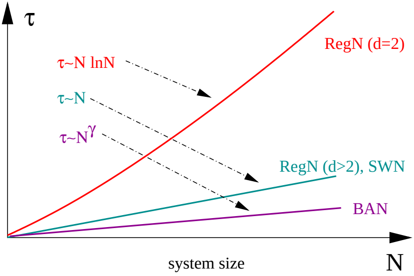


$t = 10^1, 10^2, 10^3, 10^4$

Spatial Heterogeneity

- agents do *not* interact *randomly* ($\longrightarrow$ mean-field)
  - dependence on local neighborhood $\Rightarrow$ formation of domains
- heterogeneity: varying local conditions ($f_i^{1-\theta_i}$)
  - different opinions of neighbors
  - different number of neighbors ($\longrightarrow$ network topology)
- how does this affect *time to reach consensus* τ?

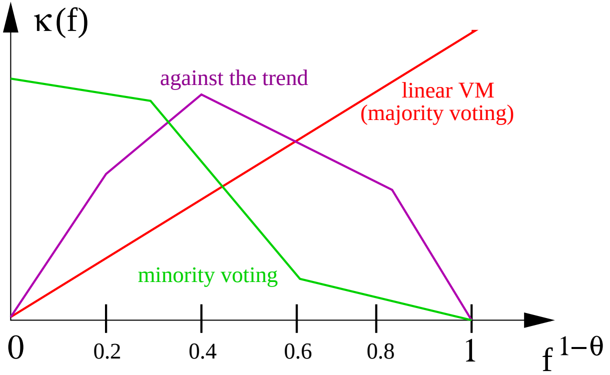




- Voter model dynamics in complex networks: Role of dimensionality, disorder, and degree distribution – K. Suchecki, V. M. Eguiluz, and M. San Miguel, Phys. Rev. E 72, 036132 (2005)
- Effect of network topology on the ordering dynamics of voter models – C. Castellano, AIP Conf. Proc. 779, 114 (2005)
- Conservation laws for the voter model in complex networks – K. Suchecki, V. M. Eguiluz and M. San Miguel, Europhysics Letters 69 (2), pp. 228-234 (2005)
- Incomplete ordering of the voter model on small-world networks – C. Castellano1, 2, D. Vilone1 and A. Vespignani, Europhysics Letters 63 (1), pp. 153-158 (2003)

Nonlinear Voter Models

- vary influence of heterogeneity: $\Rightarrow$ *nonlinear response:* $\kappa(f)\, f_i^{1-\theta_i}$



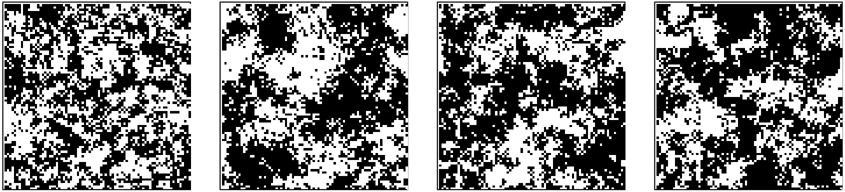
Coexistence?

1. adjust $\kappa(f)$
   - coexistence, but no spatial coordination

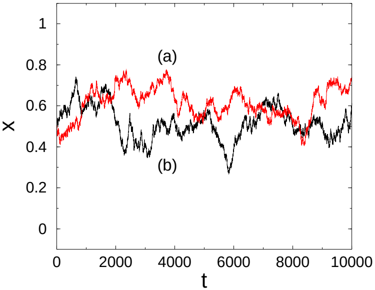
Online simulation 1

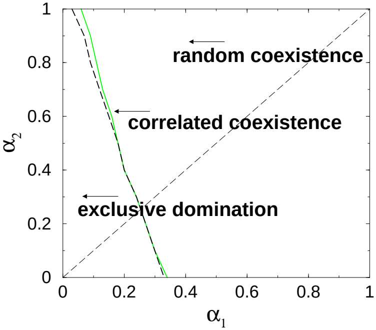
Coexistence?

1. adjust $\kappa(f)$ — Online simulation 1
   - coexistence, but no spatial coordination
2. destabilize absorbing states — Online simulation 2
   - small pertubation for $f^{1-\theta} = 1$ ($\rightarrow \varepsilon = 10^{-4}$)
   - coordination of decisions on long time scales
   - asymptotically: coexistence, but non-equilibrium



$\varepsilon = 10^{-4}$ $t = 10^1,\ 10^2,\ 10^3,\ 10^4$



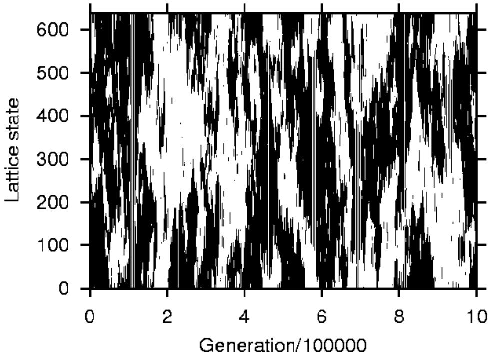


(a) $\varepsilon = 10^{-4}$, $\alpha_1 = 0.2$, $\alpha_2 = 0.4$ (linear VM)

(b) $\varepsilon = 10^{-4}$, $\alpha_1 = 0.25$, $\alpha_2 = 0.25$

Phase diagram for coexistence

1d CA:



- long-term nonstationarity
- only *temporal* domination of one opinion

Agent's Heterogeneity

- $\nu_i(\tau_i)$: reluctance of agent i to change opinion θ_i
  - persistence time τ_i (opinion was *not* changed) $\Rightarrow$ "history"
  - reflects local experience with agents in neighborhood (memory effects)

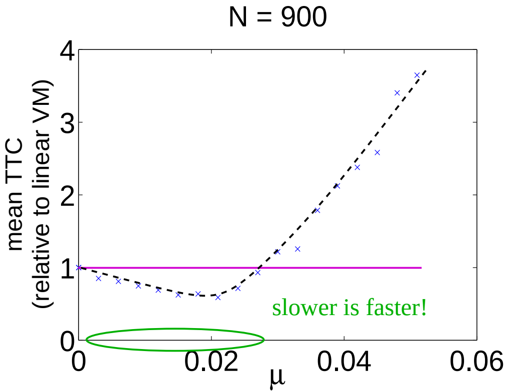
$$\frac{d\nu}{d\tau} = \mu\,\nu(1-\nu) \quad \Rightarrow \quad v_i = \frac{1}{1+e^{-\mu\tau_i}}$$

- decision dynamics:

$$w(\theta_i'|\theta_i) = [1-\nu_i(\tau_i)]\; f_i^{\theta_i'}$$

  - $\mu > 0$: slowing down of opinion dynamics
- consensus vs. coexistence of opinions ??
  - decision between 3 opinions: $\{-1, 0, +1\}$ Simulation Video

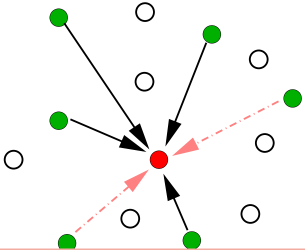
Time to reach consensus



- *heterogeneity* of agents important:
  - local groups of "confident" agents convince an indifferent neighborhood

Heterogeneity in Social Relations

- continuous opinions: $\theta_i(t) \in [0, ..., 1]$ (social behavior)
- different social relations of agent i:
  - *ingroup*: friends $\Rightarrow$ try to reach *consensus* (attraction)
  - *outgroup*: foes $\Rightarrow$ try to depart (repulsion)
  - neutral $\Rightarrow$ no relation
  - $t = 0$: probabilities $p_{\rm in}$, $p_{\rm out}$ (decrease with distance)
    $\Rightarrow$ adjacency matrix: $\mathcal{C}_{ij}$



K_i: size of ingroup
L_i: size of outgroup

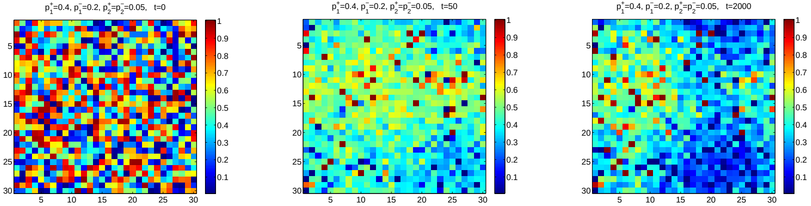
- agent's decision: adopt opinion θ_i which maximizes private utility

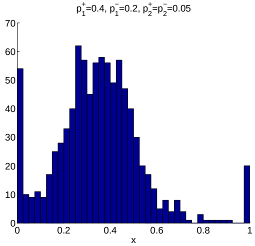
$$U(\theta_i^{t+1}) = -\alpha \times (\theta_i^{t+1} - \theta_i^t)^2 + \\ +(1-\alpha) \times \left[-\sum_{k \in I(i)} (\theta_i^{t+1} - \theta_k^t)^2 + \sum_{l \in O(i)} (\theta_i^{t+1} - \theta_l^t)^2 \right]$$

  - α: weights between importance of own opinion θ_i and opinions of "others"

Simulation Video

Results of computer simulations with $N = 900$ agents





heterogeneous social behavior

- coexistence
- spatial concentration
- stationarity (slow dynamics)

Conclusions

collective decisions ⇔ aggregated individual decisions??

1. nonlinear voter model
   - consensus:
     - ⋆ time scale?, symmetry of outcomes?
   - coexistence:
     - ⋆ non/stationarity? spatial correlations?
   - role of memory effects
     - ⋆ heterogeneity may enhance consensus

2. heterogeneity in social relations
   - various opinions
   - influence of social structure (in/outgroup)
   - agent's utility
   - result: variety of coexisting "norms" with tendency of domain formation

- KISS *(Keep It Simple, Stupid)* principle
  - details: not as much as possible, only as much as necessary
  - systematic understanding: role of parameters, feedbacks ...
  - abstract modeling level: elucidates dynamic key features