Collective Dynamics of Companies

A Complex Systems Perspective

Part 1: Models of Company Growth

Frank Schweitzer

fschweitzer@ethz.ch

Is Economics the Next Physical Science?

An emerging body of work by physicists addressing questions of economic organization and function suggests new approaches to economics and a broadening of the scope of physics.

J. Doyne Farmer, Martin Shubik, and Eric Smith

In the past decade or so, physicists have begun to do academic research in economics. Perhaps a hundred people are now actively involved in an emerging field often called econophysics, and two new journals and frequent conferences are devoted to the field. At least ten books have been

phy the thro the theo orig Fisl Gibl scie ties cal-physics methods to

The range of topics t cists spans many differ particularly well repre Pimbley, PHYSICS TODA

Physics Today, September 2005, pp. 37-42

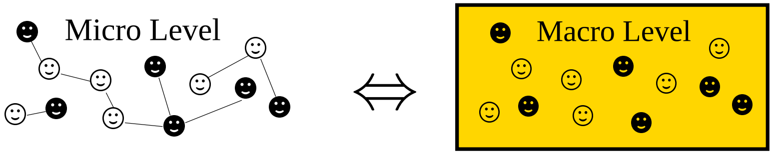
Some historical notes

- involvement of physicists in economics/social sciences
  - Daniel Bernoulli: "utility" (1738)
  - Pierre-Simon Laplace: statistics of dead (1812)
  - Adolphe Quetelet (1796-1874) ("body mass index")
    - introduced the term "social physics" (1835)
- economist Vilfredo Pareto: "scaling laws" $y \sim x^{-\alpha}$ (1897)

...

- "econophysics"
  - coined by H.E. Stanley (1995) at Workshop in Kolcata, India
  - today: several hundred physicists involved (banks, insurance, ...)
  - driving force: high-frequency data of transactions $\Rightarrow$ giant laboratory
- more recent: "sociophysics" (2000)
  - universality in social systems
  - simple opinion dynamics
- **big criticism**: impact in economics, social science?

Complex Systems



- How are the properties of the elements and their interactions ("microscopic" level) related to the dynamics and the properties of the whole system ("macroscopic" level)?
- approach: agent-based models
  - agent: "particle" with "intermediate" internal complexity
  - collective phenomena in multi-agent systems

Two ways to influence complex systems:

Top−Down

hierarchical planning

centralized control

selforganization

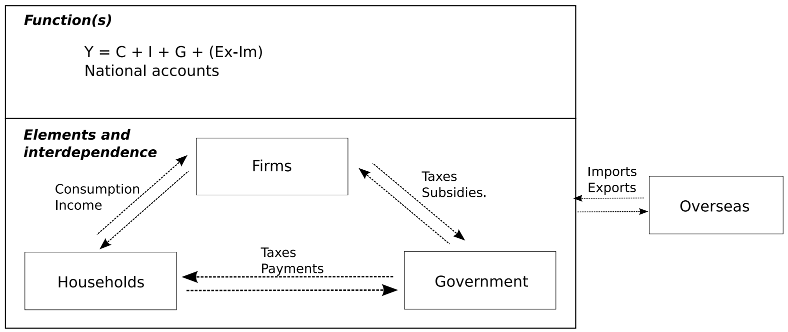
decentralized problem solving

Bottom−Up

- *bottom up:* change *interactions*
  - examples: incentives, communication, learning, ...
- *top down:* design *boundary conditions*
  - examples: taxes, laws, ...

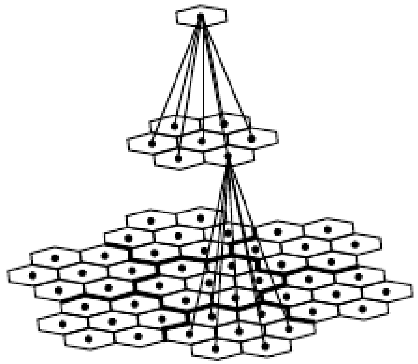
Systems Dynamics Perspective: Top-Down

The system of an open economy with state activity



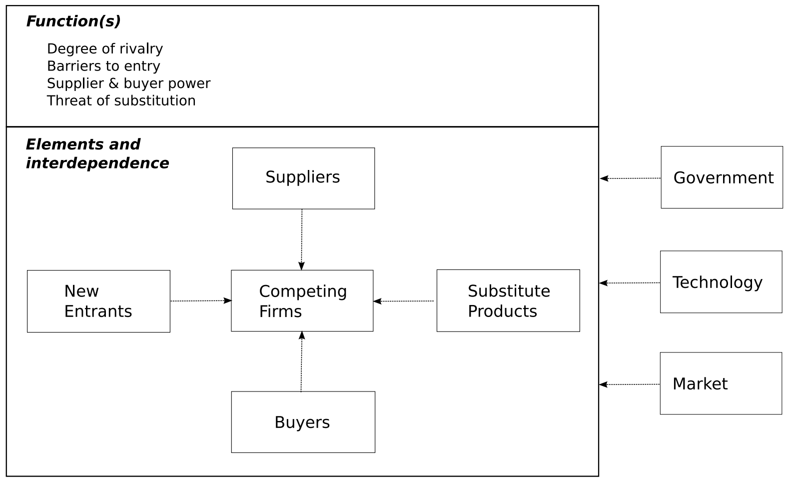
Y: Yield (Gross Domestic Product), C: Consumption, I: Investments, G: Government Expenditure, Ex: Export, Im: Import

Hierarchical Systems

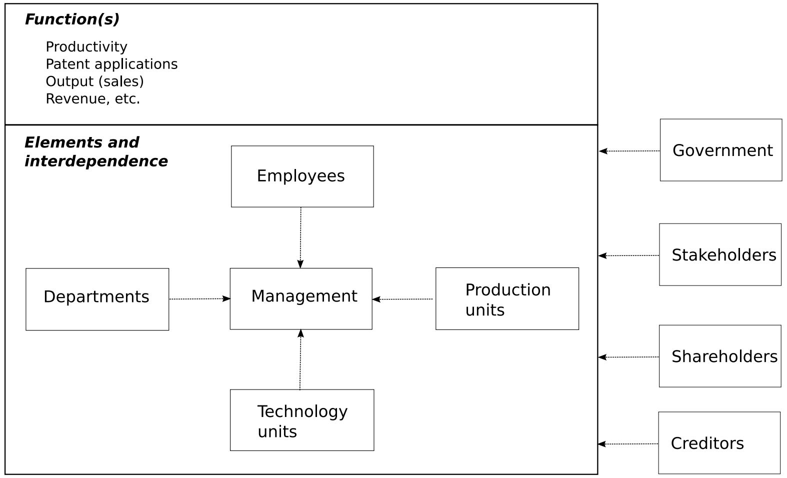


- systems comprise subsystems (parts)
- systems can be part of other (super)systems
- examples: human society (individual – family – tribe – nation), ecosystem, nuclear plant, airport, ...

Example: The system of an industry (with firms as subsystems) vs the system of a firm



The system of a firm



Complex systems perspective: Bottom-up

- focus: *collective* effects
  - ensembles of companies: $i = 1, ..., N$
  - simple characterization: company "size" $x_i(t)$
    - income, output, employees, ...
- focus: *dynamics*

$$\frac{dx_i}{dt} = \mathcal{F}_i(?)$$

  - aggregated outcome for different assumptions for $\mathcal{F}_i$
- schedule:
  - I. growth of companies ⇒ size *distribution*
  - II. interaction of companies ⇒ network structure

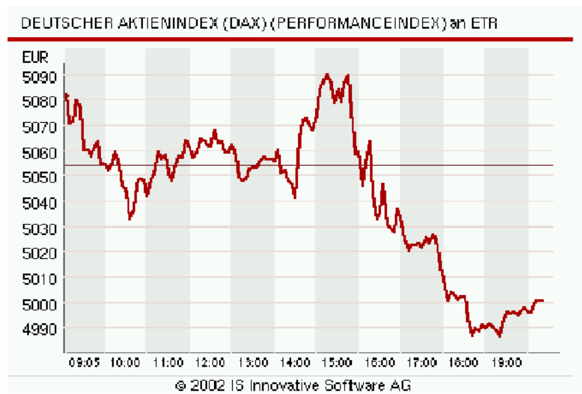
Most simple assumption: Random growth

- growth rate: $dx_i/dt = \mathcal{F}_i$
- $\mathcal{F}_i(t)$ is a random force:
  - $\langle \mathcal{F}_i(t) \rangle = 0$
  - $\langle \mathcal{F}_i(t)\mathcal{F}_i(t') \rangle = S\delta_{ij}\delta(t-t')$

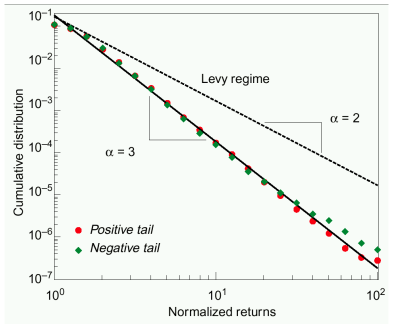
$$x_i(t+\Delta t) = x_i(t) + \sqrt{S\Delta t}\,\xi_i$$

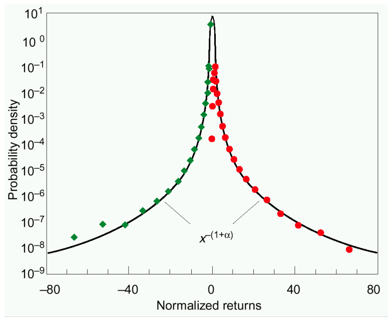
  - growth as random walk????

- Louis Bachelier: *Théorie de la spéculation* (1900)
  - PhD Thesis (supervisor Henri Poincaré)
- random walk of asset prices



- developed the mathematics of Brownian motion





Normalized log-returns $r_\tau(t) = \log\{p(t+\tau)/p(t)\}$ of 1.000 US companies (1994-1995), τ=5 min (Plerou *et.al.*, 1999)

- short term ($\tau <$ month) fluctuations are non-gaussian
  - power law $f(r) \sim \langle r \rangle^{-\alpha}$, $\alpha \approx 3$
- "volatility clustering": positive correlations ...

Conclusions for modeling

- surprising regularities on the aggregated level (distribution)
- simple random models neglect 'fat tails' (extreme events)

Gibrat dynamics of firm growth

- $\dot{x}_i = \mathcal{F}_i = f(x_i) + \ldots = b_i\, x_i$
  - no interactions between firms
  - $b_i(t)$: independent of i, no temporal correlations (random noise)
    $\Rightarrow$ *multiplicative stochastic process*
- "Law of proportionate growth" (Gibrat, 1930)

$$x_i(t+\Delta t) = x_i(t)\Big[1 + b_i(t)\Big]$$

- *growth "rates"*: $g(t) = x(t+1)/x(t)$, $t \gg \Delta t$, $\ln(1+b) \approx b$

$$\ln g(t) = \sum_{n=1}^{t} b(n)$$

$\Rightarrow$ random walk for $\ln g(t)$ $\Rightarrow$ log-normal distribution for $x_i(t)$

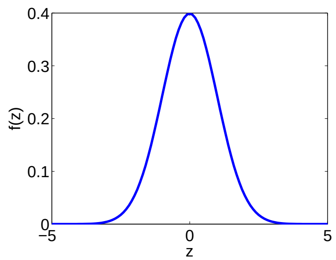
Normal vs log-normal distribution

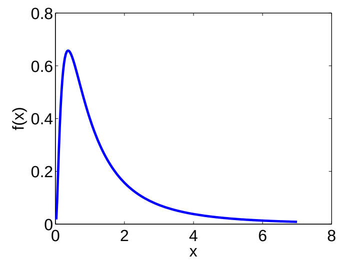
- normal distribution $P(z)$ for $z = \ln x$

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left\{\frac{-z^2}{2}\right\}$$

- log-normal distribution $P(x)$ for x ($\mu = 0, \sigma = 1$)

$$f(x) = \frac{1}{\sqrt{2\pi}}\frac{1}{x} \exp\left\{\frac{-(\ln x)^2}{2}\right\}$$



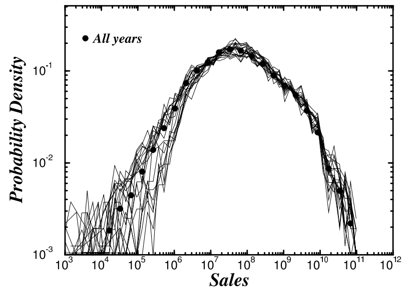


Empirical Evidence?

- log-normal distribution of company sizes

$$P(x) = \frac{1}{\sqrt{2\pi}\,\sigma\, x} \exp\left[\frac{(-\ln x - \mu)^2}{2\sigma^2}\right]$$

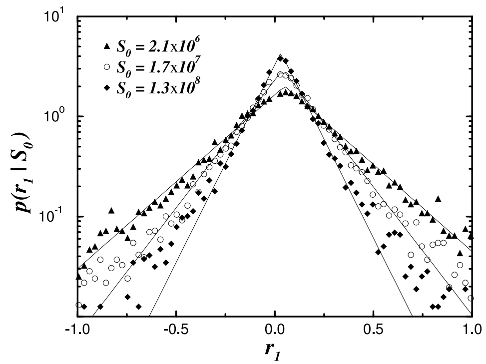
Fig. 2(a)



Empirical distribution of company sizes (1974-1993) (Amaral et al, 1997)

- Empirical distribution of growth rates
  $\Rightarrow$ depend on size $\longrightarrow$ tent-shape, exponential distribution

Fig. 3



(Amaral et al, 1997)

Possible Explanation

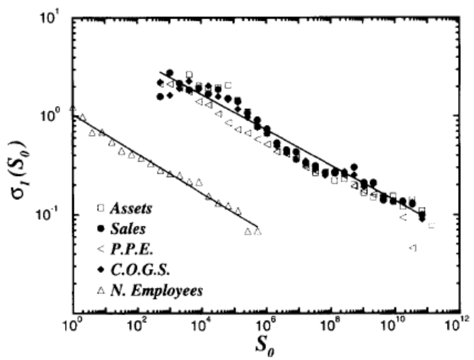
- correlations in the growth rates
  company is attracted to an "optimal size"

$$\frac{x_{t+\Delta t}}{x_t} = \begin{cases} k e^{\varepsilon_t}, & x_t < x^* \\ \frac{1}{k} e^{\varepsilon_t}, & x_t > x^*, \end{cases}$$

- result:

$$P(r_1|x_0) = \frac{1}{\sqrt{2}\,\sigma_1(x_0)} \exp\left[-\frac{\sqrt{2}\,|r_1 - \bar{r}_1(x_0)|}{\sigma_1(x_0)}\right]$$

- Empirical distribution of standard deviation of growth rates
  $\Rightarrow$ depend on size, power-law distribution $\sigma_1(x_0) \sim x_0^{-\beta}$



L.A. Amaral, S. Buldyrev, S. Havlin, P. Maass, M. A. Salinger, H. E. Stanley, M. H. Stanley: Scaling behaviour in economics: the problem of quantifying company growth, *Physica A* **244** (1997) 1-24

Possible Explanation

- growth depends on properties of management hierarchies
  n levels, z mean branching ratio, decisions on higher level are followed with prob π

$$\beta = \begin{cases} -\ln(\pi)/\ln(z) & \text{if} \quad \pi > z^{-1/2} \\ 1/2 & \text{if} \quad \pi < z^{-1/2} \end{cases}$$

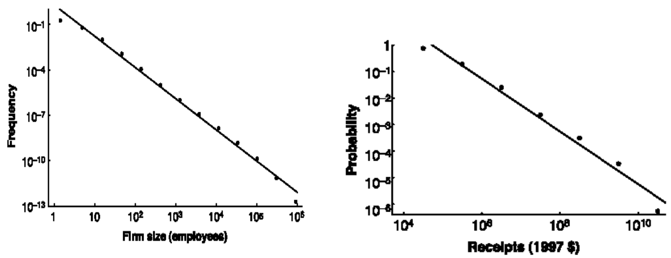
- result:
  - $\sigma_1(x_0) \sim x_0^{-\beta}$; $\beta < 0.5$
  - β decreases in time $\Leftrightarrow$ companies better coordinated

Zipf Distribution of Firm Sizes

- alternative candidate, different names: Pareto, Zipf, power law, ...

  $P(x, b, a) = ab^{\mu} x^{-(1+\mu)}$

  - log-log plot shows a straight line with descent $\alpha = -(1 + k)$



Axtell, R.: Zipf Distribution of U.S. Firm Sizes. *Science*, **293** (1997) 1818–1820

Stylized facts about firm size

- firm sizes follow a *skewed distribution* $P(x)$
- nature of $P(x)$ depends on economic sectors, aggregation level, etc
- log-normal or power law distributions good candidates

Stylized facts about firm growth

- growth rates follow a *Laplacian distribution*
- variance of growth rates decreases with firm size (and age)

Conclusions for modeling

- surprising regularities on the aggregated level (distribution)
- multiplicative stochastic processes as candidate framework
- additional ingredients needed

From log-normal to power-law distributions

- *mathematical idea: add more noise!* (Kesten '73)

  $$x(t+1) = x(t)\big[1 + b(t)\big] + a(t)$$

  - b, a positive, independent random variables
  - $a(t)$: prevents firm from bankruptcy
    - ★ reasons: internal (inhouse production), external (subsidies)
    - ★ dynamics: "effective repulsion" from zero
    - ★ assumption here: $a = \mathrm{const.} > 0$
- some economic interpretation: $b(t) = r(t)q(t)$
  - firm invests a portion $q(t)$ of its net asset in its growth
  - $r(t)$: stochastic return on investment (RoI) ($r(t) > -1$)
  - choose $q(t)$ dependent on predicted RoI
  - assumption here: $q(t) = q_0 = \mathrm{const.}$
- Question: What is the most probable size x_{mp} asymptotically?

Framework of multiplicative processes

- individual process with $\eta(t)$ as stochastic variable

$$\Delta x(t) = \eta(t) G[x(t)] + F[x(t)]$$

- stationary probability distribution

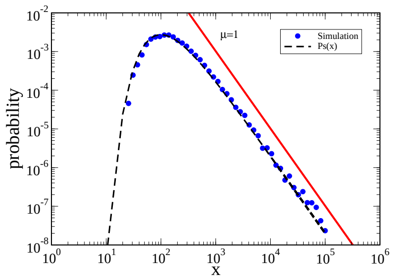
$$P_s(x) = \frac{1}{G^2(x)} \exp\left(\frac{2}{D}\int^x \frac{F(x')}{G^2(x')}dx'\right)$$

- our example: $F(x) = a$, $G(x) = x$
  $D = \left\langle (\log(1+b))^2 \right\rangle - \langle \log(1+b) \rangle^2$, $\mu = -2\langle \log(1+b)\rangle / D$

$$P_s(x) \propto x^{-2} \exp(-2a/Dx)$$

- for large x: $P_s(x) \propto x^{-(1+\mu)}$ with $\mu = 1$

Richmond, P.: Power Law Distributions and Dynamic Behaviour of Stock Markets. *The European Physical Journal B* **20(4)** (2001) 523–526.



assumption: binary stochastic return distribution $r(t) = B\{-1, 1\}$

- for $\langle r \rangle = 0$, $\langle log(1+b)\rangle \approx 0$ and small values of q_0:

$$x_{\rm mp} \approx \frac{a}{q_0^2 \langle r^2 \rangle}$$

  - $\langle r^2 \rangle = 1$ for $B(-1,1)$, $\langle r^2 \rangle = 1/3$ for $U(-1,1)$, $\langle r^2 \rangle = \sigma^2$ for $N(0,\sigma)$

J. E. Navarro, R. Cantero, J. Rodrigues, F. Schweitzer: Investments in Random Environments, *Physica A* **387** (2008) 2035-2046

- **econonomy with N non-interacting firms**

$$x_i(t+1) = x_i(t)\left[1 + r(t) q_{i_0}\right] + a$$

  - every firm is forced to 'grow' $\rightarrow$ investment q_{i_0}
  - bancrupcy prevented ($\rightarrow a$) $\Rightarrow$ constant number of firms
  - overall economy is growing (on intermediate time scales)
  - dilemma: invest less – get more: $x_{i_{\rm mp}} \propto q_{i_0}^{-2}$
  - chances for larger size do not increase with q_0: $P \propto x^{-(1+\mu)}$ (note: μ does not change with q_0)

Conclusions for modeling

- random growth assumptions work well $\Rightarrow b(t)$, $a(t)$
- bridging between log-normal and power law behavior
- 'problematic' relation between growth and investment

Proportionate growth with constant resources

- firms competing for ressources (customers, material, ...)

$$\frac{dx_i(t)}{dt} = f(x_i) = \alpha_i x_i(t)$$

- $y_i = x_i / \sum x_i$: relative market share of firm i, $\sum_i^N y_i = 1$
- growth rate of market share i: $\alpha_i = E_i - k$
  - E_i: quality (fitness) of product produced by firm i
  - k: 'dissipation' rate (constant for all firms)
- conservation of market requires:

$$\sum_{i=1}^{N} \frac{dy_i}{dt} = 0 \;\; ; \qquad k = \frac{\sum_i E_i\, y_i(t)}{\sum_i y_i(t)} = \langle E_i(t) \rangle$$

- result: Fisher-Eigen dynamics ("the winner takes it all")

$$\frac{dy_i}{dt} = y_i \left[E_i - \langle E_i(t) \rangle\right] \qquad \langle E_i(t) \rangle = \frac{\sum_i E_i\, y_i}{\sum_i y_i}$$

Simple competition scenario

- derivation ingredients:
  - *(i) positive feedback*: all firms grow, albeit at different rate
  - *(ii) conservation law*: limited resource (market)
- indirect (weak) competition: through relative market share
  - market share grows only if E_i above average $\langle E \rangle$
  - $\langle E(t) \rangle$ increases over time $\longrightarrow$ more and more firms loose
  - "survival of the fittest"
- problems:
  - E_i is fixed (winner can be predicted), what if $E_i(t)$?
  - what is the economic meaning of E_i?
  - is the outcome *realistic*? $\Rightarrow$ distribution of market shares
  - is the outcome *desirable*? (competitors as resources of innovations)

What it the economic meaning of 'fitness'?

- explanation linked to economic theory $\Rightarrow$ Karl Marx: *Capital* (1867)
  - aim: *explain the objective 'laws of motion' of the capitalist system*
  - *reveals the causes and dynamics of the accumulation of capital, the growth of wage labour, the concentration of capital, competition, the tendency of the rate of profit to decline, ...*
- idea: i firms produce same good, sell it on the same market
  - da_i: quantity per time interval produced by firm i
  - ω_i: 'value' (effort, expressed in working time), $1/\omega_i$: efficiency
  - $z_i = da_i/dt$: production velocity
- **(i) conservation law** $\Leftrightarrow$ law of exchange-value

$$\sum_i \omega_i da_i = p \sum_i da_i \Rightarrow p = \langle \omega \rangle = \frac{\sum \omega_i z_i}{\sum z_i}$$

  - exchange process (market): sets price for sum of 'values'

This explanation follows the work of R. Feistel (1977). For more details see: W. Ebeling, R. Feistel, *Physik der Selbstorganisation und Evolution*, Berlin: Akademie-Verlag (1982), or: F. Schweitzer, G. Silverberg: Konkurrenz, Selektion und Innovation in ökonomischen Systemen, in: Irreversible Prozesse und Selbstorganisation (Hrsg. Th. Pöschel, H. Malchow, L. Schimansky-Geier), Berlin, Logos-Verlag (2006) pp. 361-373

(ii) positive feedback on growth

- firms receive $p\, da_i \Rightarrow$ pay production costs $\kappa_i da_i$, profits m_i remain
  - κ_i: costs for labour (variable capital) and machinery (constant capital)

$$\langle \omega \rangle\, da_i = \kappa_i da_i + m_i \;\; \Rightarrow \;\; m_i = da_i(\langle \omega \rangle - \kappa_i)$$

- fraction α_i of profit used to extend production (at constant costs)
  - linear effect on production velocity

$$\frac{dz_i}{dt} = \alpha_i \frac{dm_i}{dt} = \alpha_i z_i \left(\langle \omega \rangle - \kappa_i \right)$$

- relative market shares $y_i = z_i / \sum z_i$, $\frac{dy_i}{dt} = \frac{1}{\sum_j z_j} \frac{dz_i}{dt} - z_i$

$$\frac{dy_i}{dt} = y_i \Big[\langle \omega \rangle \left(\alpha_i - \langle \alpha \rangle \right) + \langle \alpha \kappa \rangle - \alpha_i \kappa_i \Big]$$

- for $\alpha_i = \alpha$ (same fraction of profit reinvested in growth)

$$\frac{dy_i}{dt} = \alpha y_i \Big[\langle \kappa \rangle - \kappa_i \Big]$$

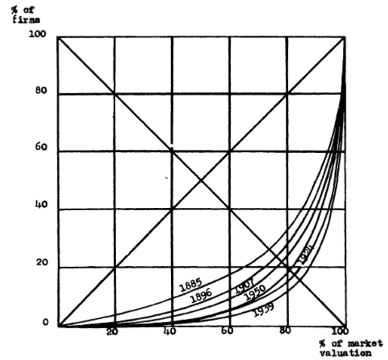
Conclusions for modeling

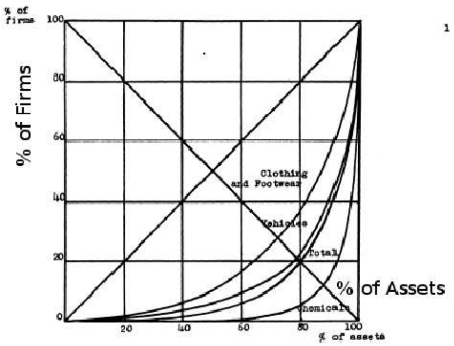
- competition scenario for free-market capitalism
- cost κ_i (labor, machinery) plays role of fitness value
- economic insights into growth: $p = \langle \omega \rangle > \kappa_i$
- ways to increase competitiveness ($\kappa_i(t)$):
  - decrease labour costs (globalization)
  - increase efficiency ($1/\omega_i$)
  - nonlinear effects: $da_i/dz_i < 0$: hyperselection
- increasing efficiency reduces price $\longrightarrow$ new pressure on κ_i
  - viscious cycle

Distribution of market shares

- market share of a firm: $y_i(t) = x_i(t)/\sum_{j=1}^{N} x_j(t)$
  - x_i can be firm 'size', but also 'market valuation' (number of stocks times stock price),
- *'concentrated' industry*: uneven distribution of market shares
  - *monopoly:* highly concentrated industries likely to induce big firms to exploit market power at the expense of consumers
- graphical representation of inequality (size, wealth): *Lorenz curve*
  - developed by Max O. Lorenz in 1905 for income distributions
  - applies to a set of ordered elements $x_1 < x_2 < x_3 < \ldots < x_n$
  - relation between two cumulative properties:
    - ⋆ x-axis: cumulative proportions of ordered elements
    - ⋆ y-axis: cumulative proportions of their size
  - Example: 5% of all firms control 60% of market valuation

- **Example: UK-operating companies (1885-1950)**
  - data: market valuation (different time periods, different sectors)
  - shows increasing market concentration over time for pre-war period



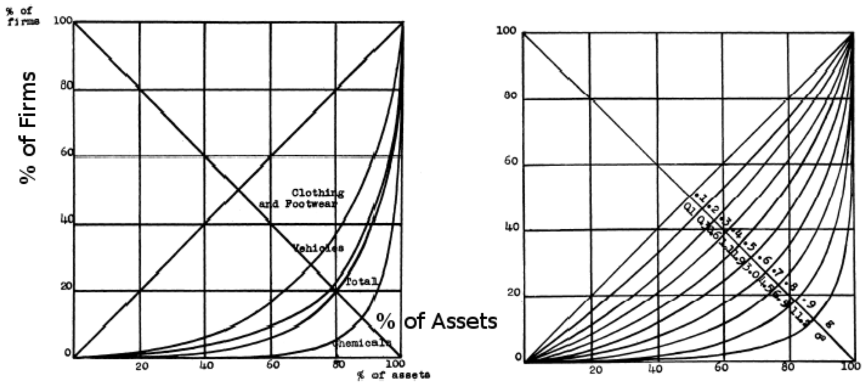


Hart, P. E. and Prais, S.: The analysis of business concentration: A statistical approach. *Journal of the Royal Statistical Society*, 119(2) (1956) 150–191
Size classes constructed in geometric progression, with the upper interval limit equals 2 times the lower interval limit

- **Symmetrical Lorenz curves**
  - unequality, yet symmetry –
    top 5% of firms constitute 20% of total market valuation, then bottom 20% of firms account for 5% of total market valuation
  - symmetrical Lorenz curve ⇒ underlying distribution is **log-normal**
- **Lorenz curve and Gini coefficient**
  - straight diagonal line (line of equality) ⇒ all elements of same size
  - *Gini coefficient* ⇒ (area below Lorenz curve)/(area below line of equality)

$$g = \frac{2\sum_i^N i y_i}{n\sum_i^N y_i} - \frac{n+1}{n} \; ; \quad y_i \leq y_{i+1}$$

  - $g = 0$: all elements are equal, $g \to 1$: increasing inequality



(left) Lorenz curves for different industries
(right) Gini coefficients g for different σ^2 of the log-normal distribution

Conclusions for modeling

- inequality in relative market shares persists
  - slight increase over time
- symmetric Lorenz curve indicates log-normal size distribution
- why don't we observe a 'winner-takes-all' scenario?
  - entry/exit dynamics: number of firms change over time
  - firms have to cooperate to survive $\dot{x}_i = f(x_j, x_k)$

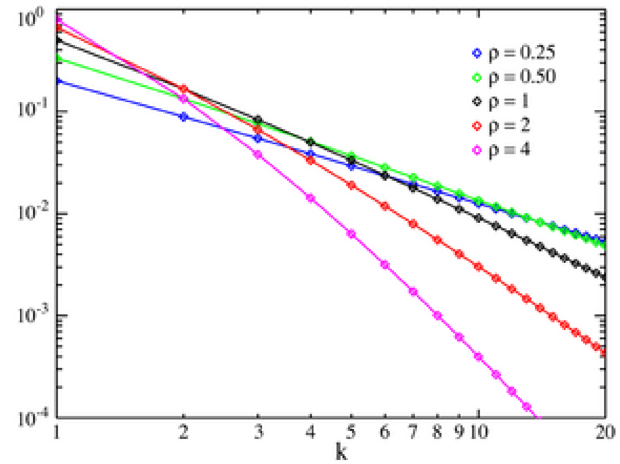
Entry/Exit Dynamics of Firms

- number of firms is not constant
  - new firms enter the market
  - existing firms disappear (bankruptcy, merger)
- simple entry model (Herbert Simon *et al.*, 1955, '58, '64, '67)
  - existing firms grow proportional to size
  - new firms are born into smallest size class at constant rate
- result: **Yule-Simon Distribution** (instead of log-normal)

$$P(x) = \rho\, B(x, \rho+1) = \frac{\rho\, \Gamma(\rho+1)}{(x+\rho)^{\rho+1}}$$

  - discrete probability distribution: $x = 1, 2, 3, \ldots \Rightarrow$ *rank*, or "size"
  - $B(x, \rho)$: Beta function, $\Gamma(\rho)$: Gamma function
  - $\rho \Rightarrow G/(G-g)$, where G is net growth in assets of all firms and g is the growth part of the new firms

Yule-Simon distribution for different values of ρ



Result:

- for large x:

$$P(x) \approx \frac{\rho\, \Gamma(\rho+1)}{x^{\rho+1}} \propto \frac{1}{x^{\rho+1}}$$

  - distribution follows *Zipf's Law*: $P(x) \propto x^{-\rho-1} \Rightarrow$ *power law*
- $\alpha = g/G = 0.1$: new firms account for 10% of growth in assets $\Rightarrow$ $\rho = 1/(1-\alpha) = 1.1$
  - assumption: α is constant over time
- empirical result: UK: ρ=1.11, US: ρ=1.23
  - 9.9% (UK) and 18.7 % (US) of growth in assets accounted by new firms

Simon, H. A. and Bonini, C. P.: The size distribution of business firms. *The American Economic Review* 48(4) (1958) 607–617.

Conclusions for modeling

- different data suggest different forms of *skewed distributions*
- Gibrat's dynamics of proportionate growth is a robust framework
  - predicts log-normal distribution of firm sizes
- modifications in different directions
  - additional growth (fix, stochastic) ⇒ power laws
  - entry dynamics ⇒ Yule-Simon distribution
  - correlations between growth rates in different years ⇒ Yule-Simon distribution
- what is not included? ⇒ (direct) interaction of firms