



Collective Decisions in Multi-Agent Systems

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Overview

- 1 Collective decisions
 - Decision making
 - Consensus versus coexistence
- 2 Voter models
 - Linear and non-linear VM
 - Simulations of VM
 - VM with memory effects
- 3 Propagation of opinions
 - Spreading of minority opinions
 - Decisions in hierarchical organizations
 - Local versus global trends
- 4 Decisions with continuous alternatives
- 5 Conclusion

Decision Making

- decision making: selection among alternatives
 - ▶ *basic* process in social and economic systems
- individual perspective of social actor (“agent”):
 - ▶ decision outcome \Rightarrow increase private utility
- classical approach: *rational agent*
 - ▶ calculation of utility function
 - ▶ common knowledge assumption
 - ▶ dissemination of information: fast, loss-free, error-free
- problems
 - ▶ incomplete (limited) information \Rightarrow *bounded rationality*
 - ▶ how to quantify private utility in social systems? (public votes)
 - ▶ ambiguous solutions, conflicts (“frustrated system”)

Social elements ...

... reduce the risk of making wrong decisions

- *imitation strategies*
 - ▶ biology, cultural evolution: adapt to the community
 - ▶ economy: copy successful strategies
- “information contagion”, herding behavior
 - ▶ agents more likely do what others do
 - ▶ examples: financial markets, mass panics, fashion, ...

Collective Decisions

- aggregated outcome of many individual decisions
 - ▶ most individual implications are averaged out
 - ▶ interaction among agents play crucial role
 - ▶ system utility (social welfare) $\neq \sum_i U_i^{\text{indep}}$
- *our focus:*
 - ▶ prediction of global/system quantities, not of individual decisions
 - ▶ role of local/neighborhood effects in collective decisions
 - ▶ influence of social elements (herding behavior)

Consensus versus Coexistence

Public polls \Rightarrow collective decision processes

- examples from Europe (2005):
 - ▶ May 29: French vote for/against Europ. constitution (45/55)
 - ▶ June 5: Swiss vote for/against Schengen (54.6/45.4)
- characteristic features
 - ▶ two alternatives: YES/NO (binary decision)
 - ▶ no simple utility maximization
 - ▶ hard to predict ($\sim 50/50$)
- find minimalistic agent models to explain generic dynamics

Voter Models

- simple model of opinion formation with consensus
- population of agents: $i = 1, \dots, N$
- each agent i : spatial position i , “opinion” $\theta_i(t) \Rightarrow \{0, 1\}$
- “decision”: to keep or change opinion $\theta_i(t)$

$$\theta_i(t+1) = \begin{cases} \theta_i(t) & \text{keep} \\ 1 - \theta_i(t) & \text{change} \end{cases}$$

- rate to change opinion depends on other agents

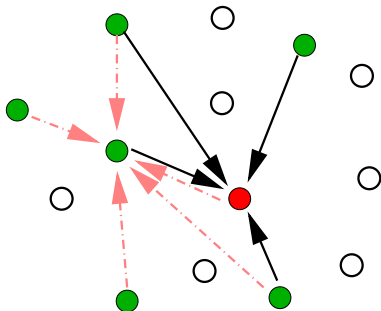
$$w(1 - \theta_i | \theta_i) = \kappa(f) f_i^{1-\theta_i}$$

- ▶ $0 \leq f_i^{1-\theta_i} \leq 1$: frequency of agents with *opposite* opinions in “neighborhood” of agent i
- ▶ $\kappa(f)$: nonlinear response to frequency of other opinions

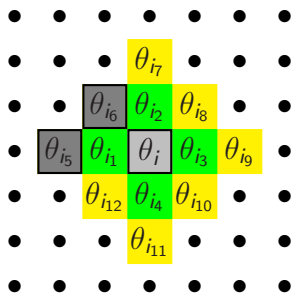
- ↳ Voter models

- ↳ Linear and non-linear VM

- neighborhoods are defined by an adjacency matrix C_{ij}
 \Rightarrow network structure



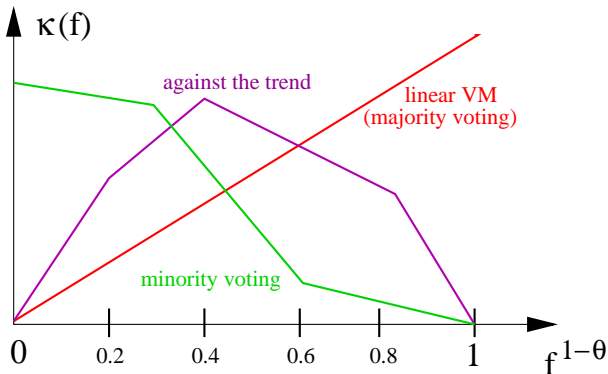
- simplified geometry: regular grid



└ Voter models

└ Linear and non-linear VM

Nonlinear response $\kappa(f)$

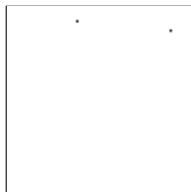
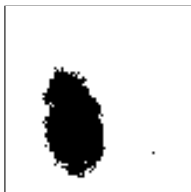
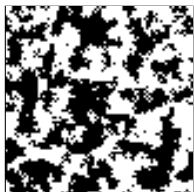


Results of computer simulations

1. Linear voter model

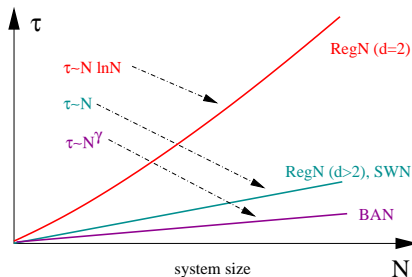
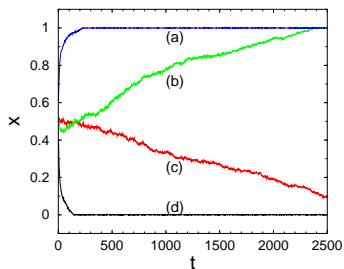
- stochastic simulation, $w(1 - \theta|\theta) = f^{1-\theta}$
- initially $x = 0.5$, random distribution
- results:
 - ▶ coordination of decisions on medium time scales
 - ▶ asymptotically: “no opposition” (\rightarrow equilibrium)

Online Simulation



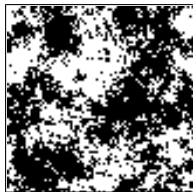
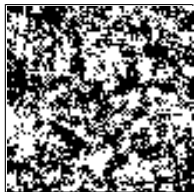
$t = 10^1, 10^2, 10^3, 10^4$

Time to reach consensus τ



Coexistence? \Rightarrow 2. Non-linear voter model

- **Online simulation 1**:
 - ▶ coexistence, but no spatial coordination
- **Online simulation 2**:
 - ▶ small perturbation for $f^{1-\theta} = 1$ ($\rightarrow \varepsilon = 10^{-4}$)
 - ▶ coordination of decisions on long time scales
 - ▶ asymptotically: coexistence, but non-equilibrium

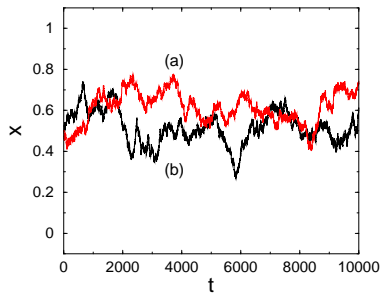


$\varepsilon = 10^{-4}$

$t = 10^1, 10^2, 10^3, 10^4$

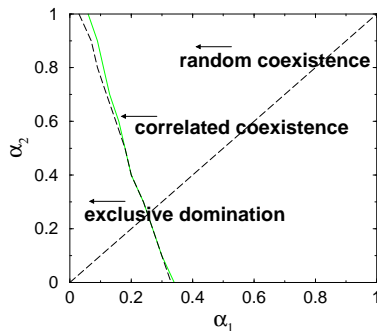
- └ Voter models

- └ Simulations of VM



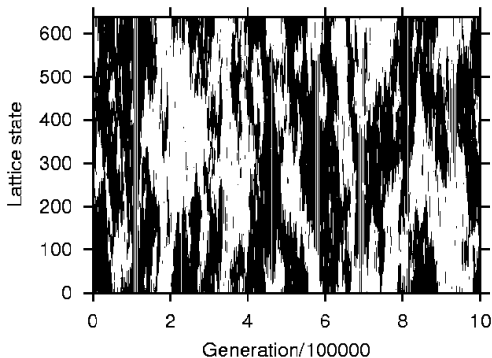
(a) $\varepsilon = 10^{-4}$, $\alpha_1 = 0.2$,
 $\alpha_2 = 0.4$
 (linear VM)

(b) $\varepsilon = 10^{-4}$, $\alpha_1 = 0.25$,
 $\alpha_2 = 0.25$



Phase diagram for coexistence

1d CA:



- long-term nonstationarity
- only *temporal* domination of one opinion

Results:

- collective decisions \Rightarrow nonlinearity in the voter model
- consensus:
 - ▶ time scale?, symmetry of outcomes?
- coexistence:
 - ▶ non/stationarity? spatial correlations?, different attractors?¹
- missing
 - ▶ memory effects, various opinions
 - ▶ influence of social structure, agent's utility

¹Schweitzer, F.; Zimmermann, J.; Mühlenbein, H.: Coordination of Decisions in a Spatial Agent Model, Physica A 303/1-2 (2002) 189-216

Including memory effects

- $\nu_i(\tau_i)$: reluctance of agent i to change opinion θ_i
 - ▶ persistence time τ_i (opinion was *not* changed) \Rightarrow “history”
 - ▶ reflects local experience with agents in neighborhood

$$\frac{d\nu}{d\tau} = \mu\nu(1 - \nu) \quad \Rightarrow \quad \nu_i = \frac{1}{1 + e^{-\mu\tau_i}}$$

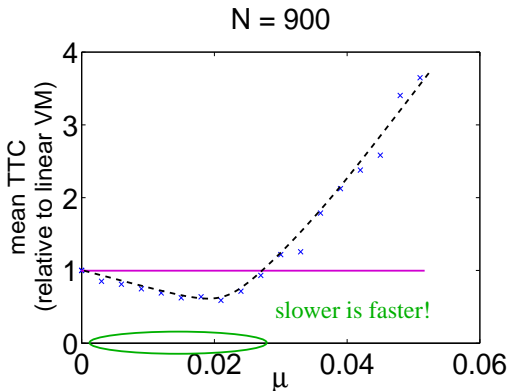
- decision dynamics:

$$w(\theta'_i|\theta_i) = [1 - \nu_i(\tau_i)] f_i^{\theta'_i}$$

- ▶ $\mu > 0$: slowing down of opinion dynamics
- consensus vs. coexistence of opinions ??
 - ▶ decision between 3 opinions: $\{-1, 0, +1\}$

Simulation Video

Time to reach consensus



- *heterogeneity* of agents important:
 - ▶ local groups of “confident” agents convince an indifferent neighborhood

Do not change the Status Quo

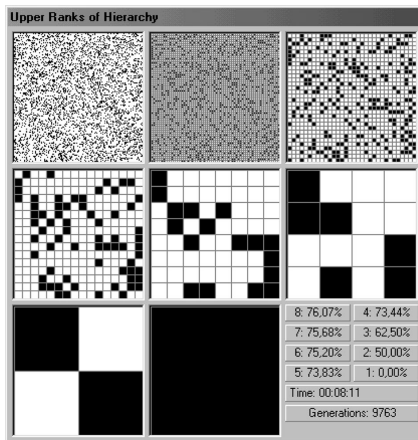
- conservative society: if you are in doubt, stay to the established opinion (Galam 2000, 2002)
- N agents with $\theta_i \in \{-1, +1\}$; ruling opinion $\theta_G = +1$
- government proposal $\Rightarrow N_+$ supporters, N_- objectors
? how much support needed to accept the proposal?
! depends on mechanism of collective opinion formation!

example: local interaction between 4 agents

- majority rule: $\{4+, 0-\} \rightarrow 4+$, $\{3+, 1-\} \rightarrow 4+$,
 $\{1+, 3-\} \rightarrow 4-$, but: $\{2+, 2-\} \rightarrow 4+$
- n consecutive random interactions

- └ Propagation of opinions

- └ Spreading of minority opinions



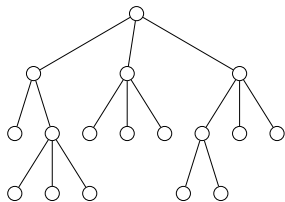
initial condition: 24% supporters
(black), 76% objectors
result: after 7 iterations or voting
levels \Rightarrow 100% support
 \Rightarrow *minority wins*
(Galam 2000, 2002)

Decisions in hierarchical organizations

Problem: propagation of new ideas through organization

- initialization on lowest level \Leftrightarrow conviction at the top level??
- depends on acceptance threshold f_c and social structure
 - ▶ asymmetry of C_{ij}
 - ▶ reporting/authority links

$$\theta_i(t+1) = \Theta \left[f_i^{(1)}(t) - f_c \right]$$



Online simulation

Local versus global trends

- agents exploit two different information
 - local*: “do what your neighbors do”
 - global*: “do not follow the trend”
- dynamics: N agents on a lattice, two opinions $\theta_i \in \{-1, +1\}$

$$\theta_i(t+1) = \begin{cases} +1 & \text{with } p = \frac{1}{1 + \exp\{-2\beta h_i(t)\}} \\ -1 & \text{with } 1 - p \end{cases}$$

$$h_i(t) = \sum_{j \in NN} J_{ij} \theta_j - \alpha \theta_i \left| \frac{1}{N} \sum_j \theta_j \right|$$

Online Simulation²

²(Bornholdt 2001, cond-mat/0105224)

Summary

Baseline model:

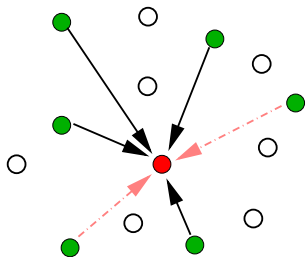
- decision between discrete alternatives: $\{0, 1\}$, $\{-1, 0, 1\}$...
- consideration of local neighborhood: \mathcal{C}_{ij} (network, grid, ...)
- agent's utility: maximize consensus with neighborhood
 - ▶ decision: adopt opinion of local majority
- global/systems dynamics: consensus versus coexistence

Advanced model:

- consider social relations (friends/foes)
- spectrum of opinions
- more complex agent's utility

Continuous alternatives

- alternatives: $\theta_i(t) \in [0, \dots, 1]$ (social behavior)
- different social relations of agent i :
 - ▶ *ingroup*: friends \Rightarrow try to reach *consensus* (attraction)
 - ▶ *outgroup*: foes \Rightarrow try to depart (repulsion)
 - ▶ neutral \Rightarrow no relation
 - ▶ $t = 0$: probabilities p_{in} , p_{out} (decrease with distance)
 \Rightarrow adjacency matrix: C_{ij}



K_i : size of ingroup

L_i : size of outgroup

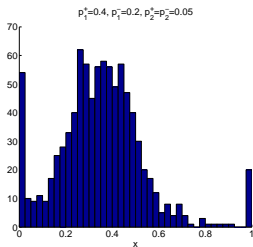
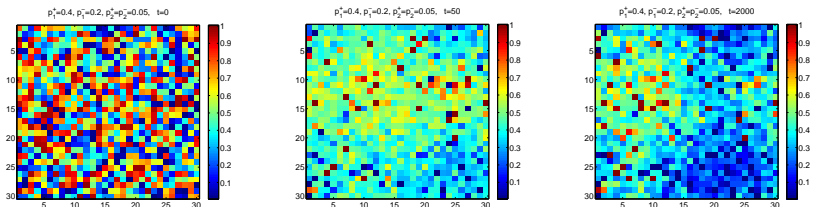
- agent's decision: adopt opinion θ_i which maximizes private utility

$$U(\theta_i^{t+1}) = -\alpha \times (\theta_i^{t+1} - \theta_i^t)^2 + \\ + (1 - \alpha) \times \left[- \sum_{k \in I(i)} (\theta_i^{t+1} - \theta_k^t)^2 + \sum_{l \in O(i)} (\theta_i^{t+1} - \theta_l^t)^2 \right]$$

- ▶ α : weights between importance of own opinion θ_i and opinions of "others"

Simulation Video

Results of computer simulations with $N = 900$ agents

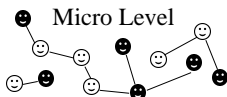


heterogeneous social behavior

- coexistence
- spatial concentration
- stationarity (slow dynamics)

Conclusions

- collective decisions \Leftrightarrow aggregated individual decisions??
- theory of complex systems:
 - ▶ *How are the properties of the elements and their interactions (“microscopic” level) related to the dynamics and the properties of the whole system (“macroscopic” level)?*



- approach: multi-agent models
 - ▶ agent: “intermediate” internal complexity $\rightarrow \theta_i$
 - ▶ simple update dynamics: non-linear VM, utility maximization, ...
 - ▶ interaction: local neighborhood $\rightarrow \mathcal{C}_{ij}$: topology, in/outgroups

- *minimalistic agent models*:
 - ▶ cover generic features of collective decisions
e.g. influence of hierarchies, memories, lobbies,
 - ▶ fitting with data within reach
 - ▶ but: will not predict your next “Volksabstimmung”
- KISS (*Keep It Simple, Stupid*) principle
 - ▶ details: not as much as possible, only as much as necessary
 - ▶ systematic understanding: role of parameters, feedbacks ...
 - ▶ abstract modeling level: elucidates dynamic key features
- Where is the “Social” ??
 - ▶ N.Gilbert: *Putting the social into social simulation*
(Wednesday, 17:15)

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- Stefan Bornholdt
- ...