Network Models of Cooperation

Frank Schweitzer

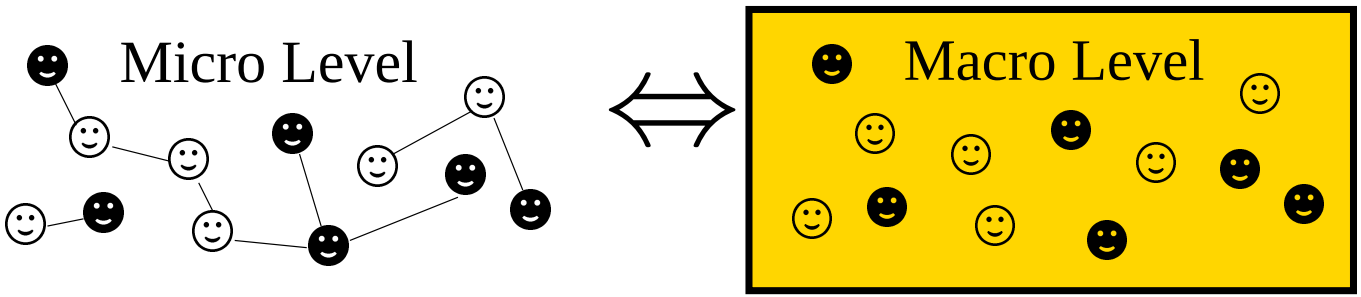
fschweitzer@ethz.ch

Overview

I. Network model of coalition formation
II. Network model of growth through cooperation

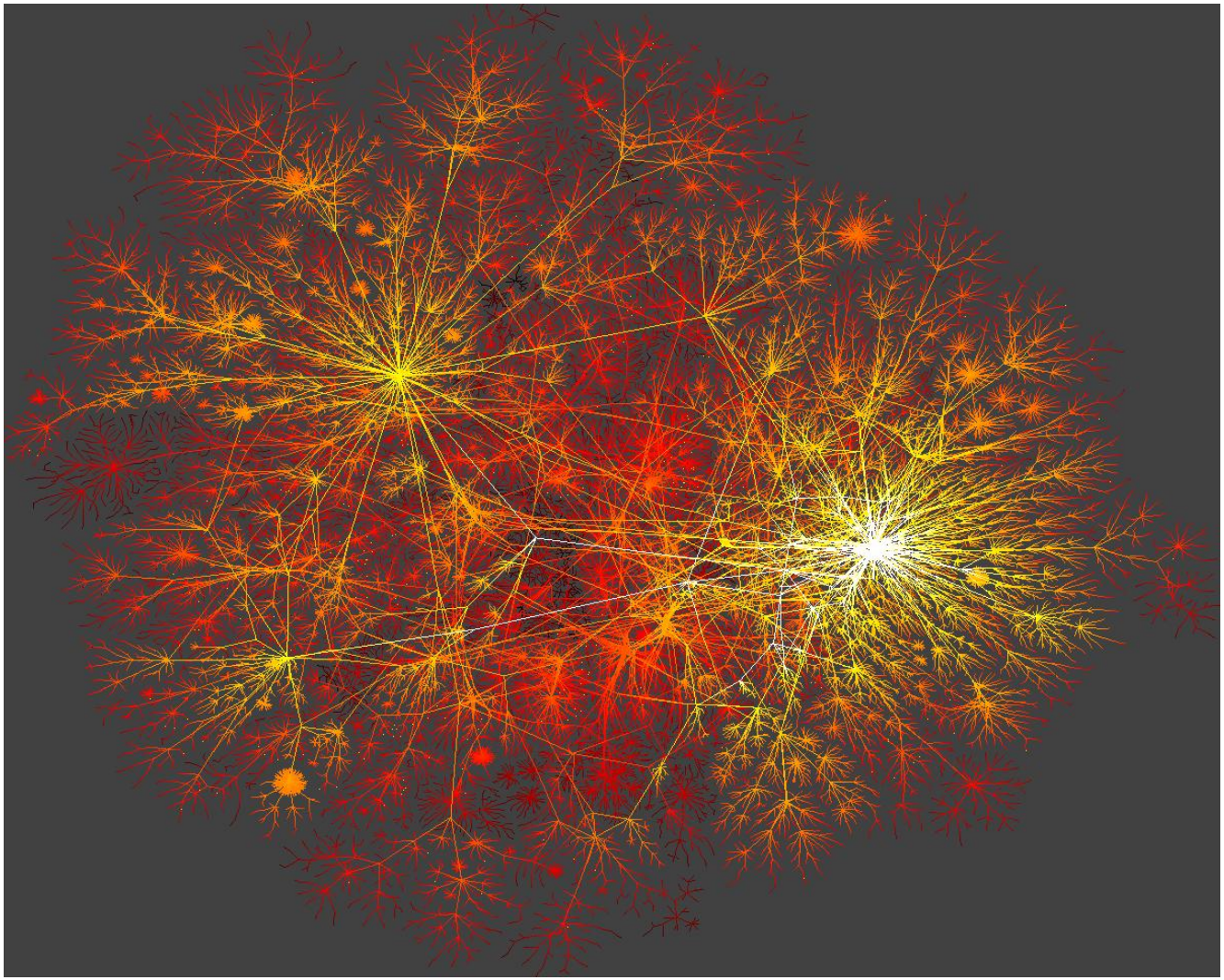
Challenge

- 21st-Century Center of Excellence Program:
  Creation of agent-based social systems sciences
  - bottom-up approach of formation/design of institutions
- key question for the theory of complex systems:



  - The micro-macro link:
    How are the properties of the elements and their interactions ("microscopic" level) related to the dynamics and the properties of the whole system ("macroscopic" level)?

Complex Networks



Structural versus relational features:

- nodes $\Rightarrow$ *agents*
  - non-linear *local* eigendynamics
- links $\Rightarrow$ *interaction*:
  - internet $\Rightarrow$ *structural features* $\longrightarrow$ WHAT?
    degree distributions, clustering coefficients, ...
  - social systems $\Rightarrow$ *relational features* $\longrightarrow$ WHY?
    quality of relations: support, friendship
- global quantities/qualities emerge from *self-organization* (no central control)
  - internet: robustness
  - social system: cooperation, welfare

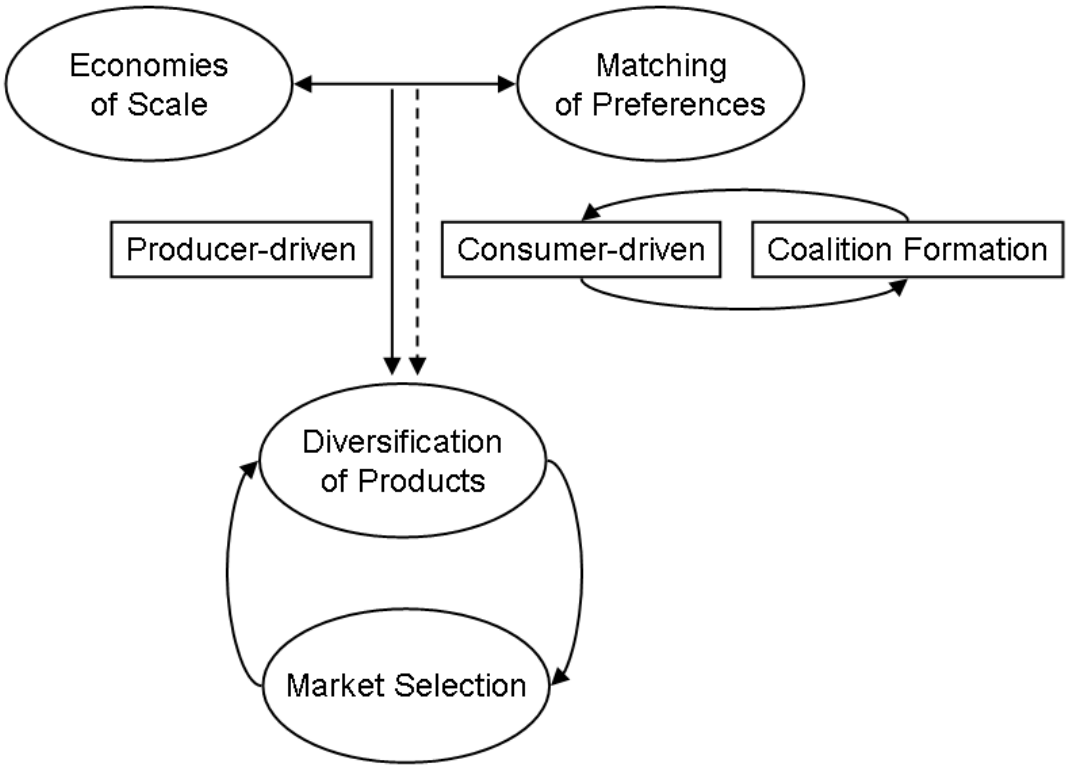
I. Network model of coalition formation

Coalition formation

- Internet: opportunity to form *spontaneous, location-independent communities*
  - emergence of services based on *social networking*
- Application: electronic markets
  - "Buying clubs" for e-commerce have been around for several years, but the concept itself has not really become popular.
  - advantage: economies of scale
    increase in quantity $\longrightarrow$ decrease in cost-per-item
  - Tsvetovat & Sycara (2000): formation of groups of buyers to obtain volume discounts from sellers

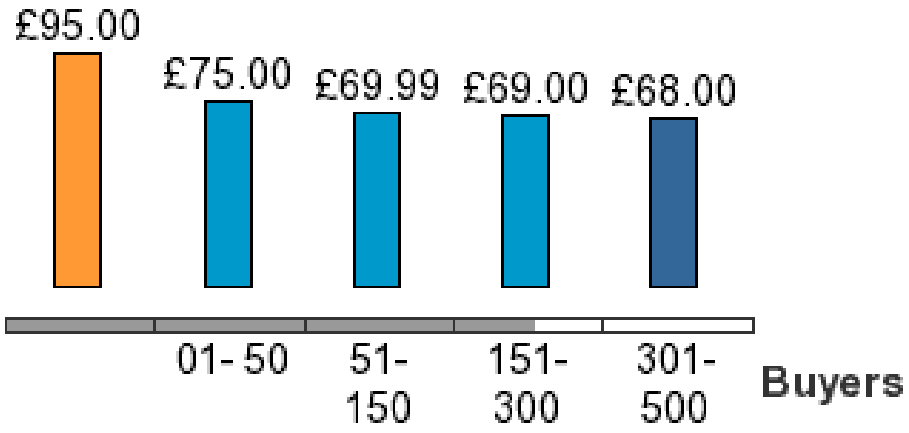
Coalition Formation:

- alternative to achieve trade-off between *economies of scale* and *matching of preferences*:



Examples

- buying clubs for food: few self-organised groups
- co-buying web sites such as *www.letsbuyit.com*:



Current Price: £69.00

End date: **17/03/2006**

Number of buyers: **231**

- I will buy at the current price £69.00
- Buy at the closing price
- Buy at the best price

Purchase

- Average retail price
- Best Price
- Number of buyers

Drawbacks and Incentives

- "more buyers, lower cost" principle based on limited selection of products ⇒ buyers have to *compromise*
- "buying clubs": waiting time and risk of not concluding a deal (additional overhead)

Buyers

- volume discount
- customised items: match of preferences
- sharing of search cost
- "bundle search"

Sellers

- better predictability of sales volumes
- customized items: increase of sales
- reduction of transaction costs

Model for Coalition Formation

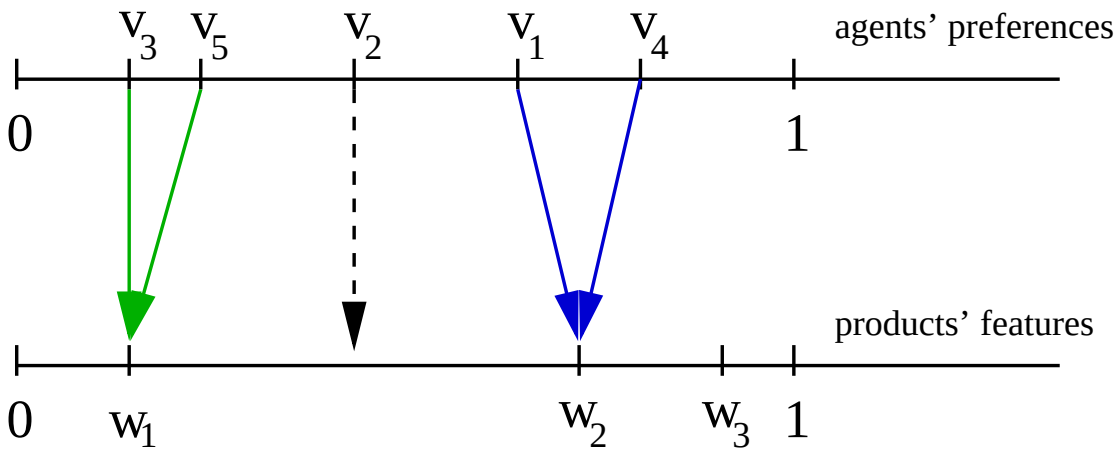
- agent-based model $\Rightarrow$ buyers B and sellers S (represented by their products)
- focus: dynamics of *creation*, *evolution*, and *breakup* of coalitions of buyers
- emphasis on:
  - effect of *heterogeneity* of agents' preferences $\Leftrightarrow$ *size*, *number* and *lifetime* of coalitions
  - existence of *stationary* and *non-stationary regimes* (stable and unstable coalitions), *transition* from one regime to the other

Heterogeneity:

- products j: vector of *features* $[w_{j,1}, \ldots, w_{j,k}]$
- buyers i: *preferences* for product features $\Rightarrow [v_{i,1}, \ldots, v_{i,k}]$

Example:

- buyer $i \longrightarrow v_i$, seller/product $j \longrightarrow w_j$, distributions $\mathcal{F}(v)$, $\mathcal{G}(w)$
- each agent buys only one product
  - different buying modes: individually, in coalitions, new demand



Utility

- agents: *rational* and *self-interested* $\longrightarrow$ maximise their private utility over time
- benefit of agent i from purchase of product j depends on:
  - distance between features w_j and preferences v_i: $\Delta_{ij} = |w_j - v_i|$
  - price of product j, which depends on quantity sold: $p_j = P/N_j^{\beta}$ (price elasticity: $\beta = 0.5$)
- agent's utility: compromise between cheap price and match of preferences

$$U_i = \frac{1}{p_j}\,[1 - |w_j - v_i|]$$

  - *indirect cost* for joining a coalition $\Rightarrow$ commitment unsuccessful coalition: $U_i = 0$ (risk of failure)

Agent's actions and decisions

1. purchase product j individually
   - advantage: get product immediately
   - disadvantage: pay higher price $p_i = P$

$$k_i^{\mathrm{ind}}(t) \propto \frac{1}{P}\left[1 - \Delta_{ij}\right]$$

2. join existing coalition j with a set of other buyers N_j
   - advantage: pay lower price $p_i = P/\sqrt{N_j}$
   - disadvantage: (i) waiting time until coalition has reached critical size $N_j \geq N_{\mathrm{thr}}$, (ii) risk of coalition failure

$$k_i^{\mathrm{coal}}(t) \propto \frac{\sqrt{N_j}}{P}\frac{N_j}{N_{\mathrm{thr}}}\left[1 - \Delta_{ij}\right]$$

1. initiate new coalition k and wait for other buyers to join
   - advantage: get product k according to preferences: $\Delta_{ik} = 0$
   - disadvantage: (i) risk of coalition failure: $N_k(t_0) = 1 \ll N_{\rm thr}$, (ii) waiting time until coalition has reached critical size

$$k_i^{\rm init}(t) \propto \frac{\sqrt{1}}{P}\frac{1}{N_{\rm thr}}$$

2. postpone decision
   - advantage: no commitment, open for future possibilities
   - disadvantage: (i) wait for product, (ii) uncertainty of future

$$k_i^{\rm wait}(t) \propto \exp\left(-\alpha t\right)$$

Stochastic decision dynamics

- each possible action has a certain weight k_i
- *decision:* stochastic draw among the weighted possibilities
  - path dependence: symmetry break
  - positive feedback: decision affects weights k_j of other agents
  - consequences for utility at $t \longrightarrow t_{\rm end} \Rightarrow$ affects strategy in repeated games

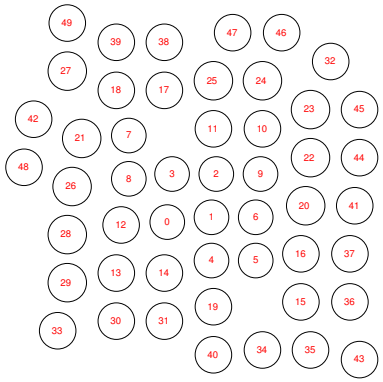
Scenarios of coalition formation

1. individual purchasing behaviour, i.e., no buyer-seller network exists among the agents,
2. formation of several heterogenous coalitions, i.e., a number of buyer-seller networks which are not connected,
3. condensation to a single giant coalition, i.e., a buyer-seller network involving all agents.

The transition between these scenarios is governed by

- heterogeneity of agents' preferences, η
- threshold for successful coalitions, $N_{\rm thr}$

Individual purchasing ($N_{\mathrm{thr}} = 50$)



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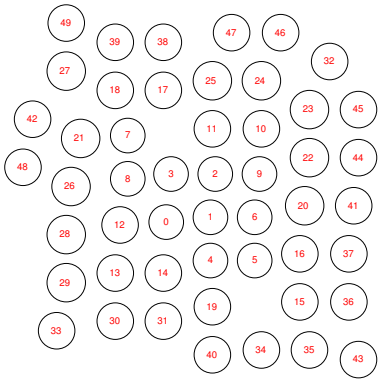

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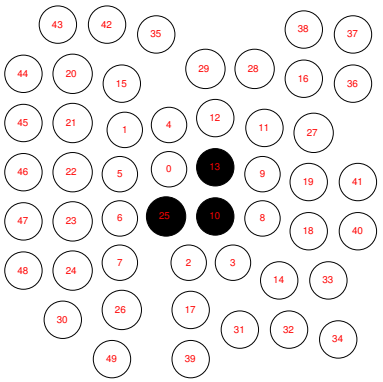

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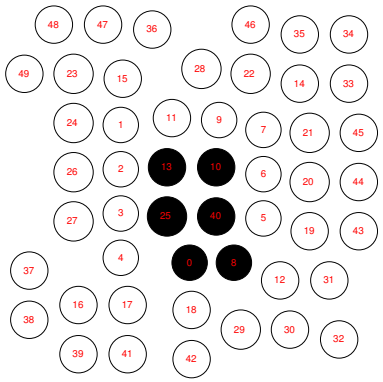

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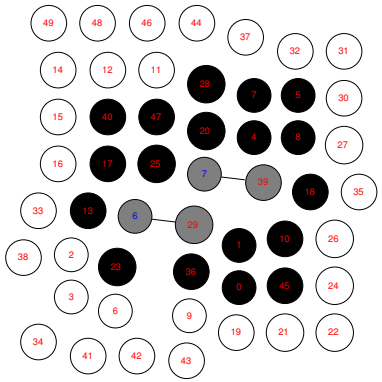

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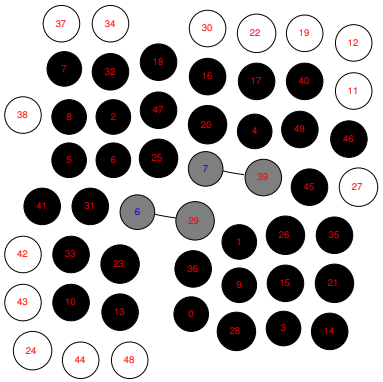

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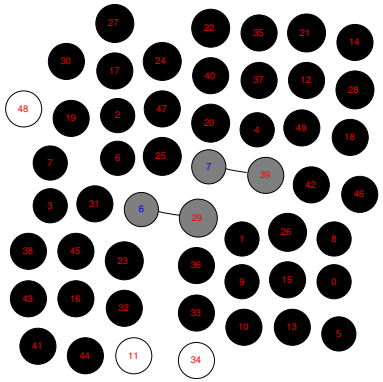

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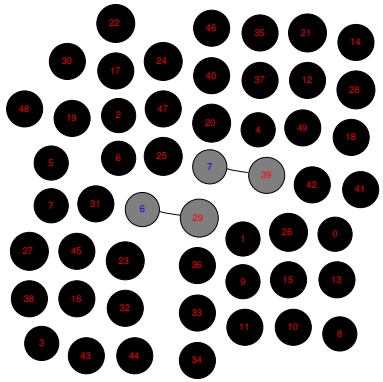

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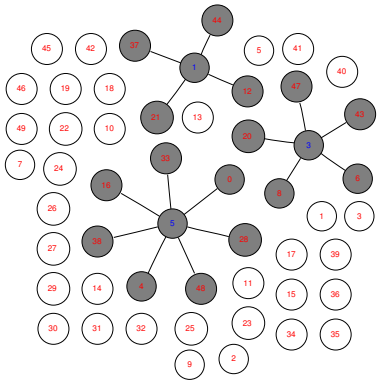
Several heterogeneous coalitions ($\varepsilon = 0.04$, $N_{\rm thr} = 5$)



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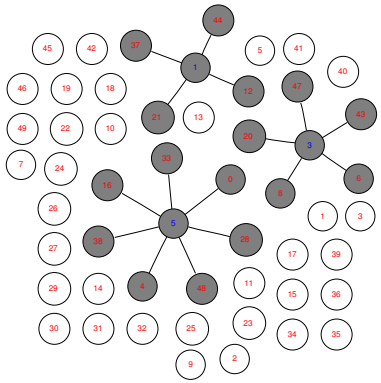

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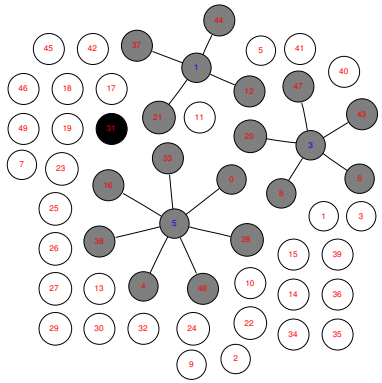

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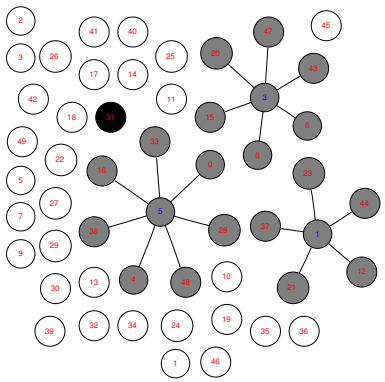

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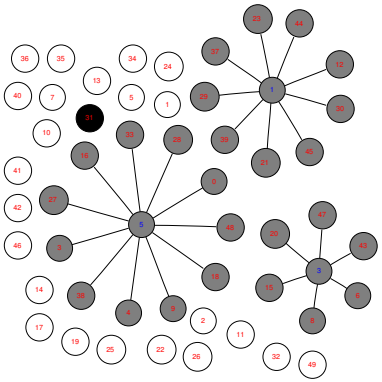

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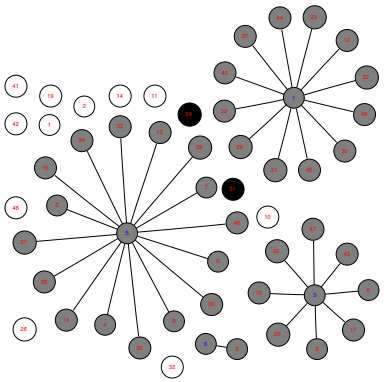

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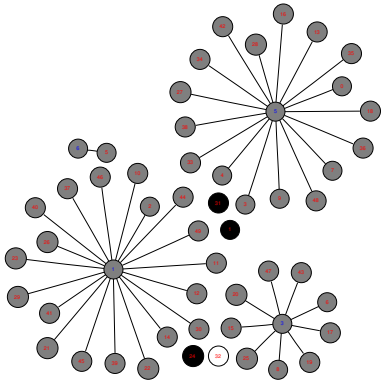

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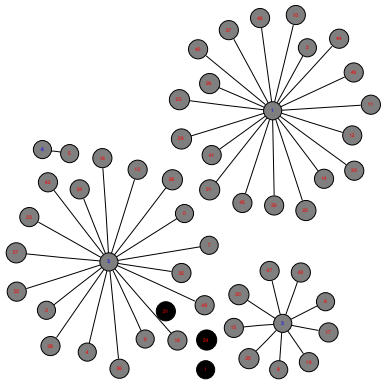

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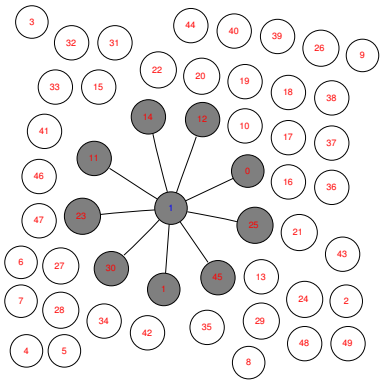
Single giant coalition ($\varepsilon = 0.05,\ N_{\rm thr} = 20$)



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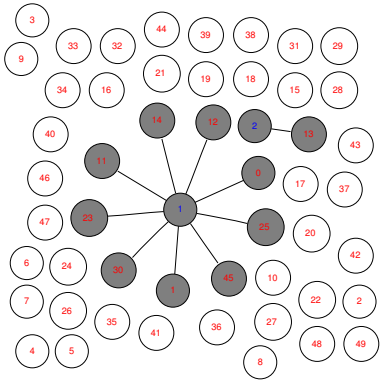

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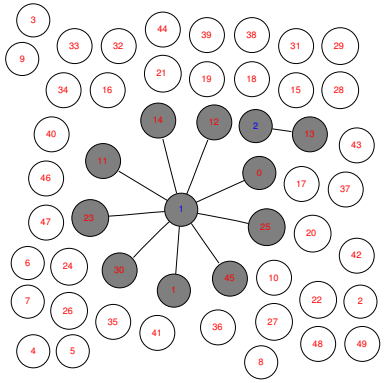

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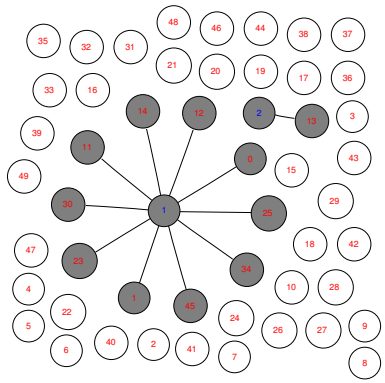

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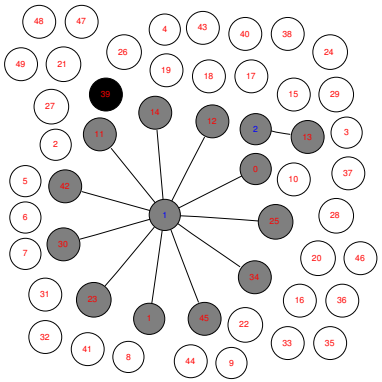

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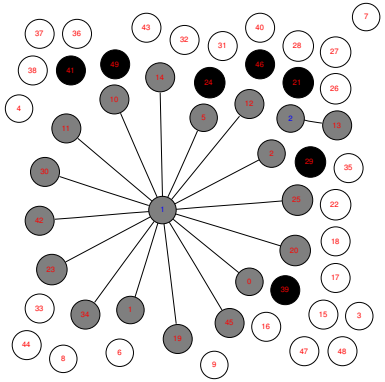

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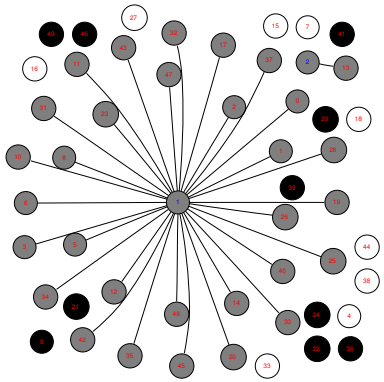

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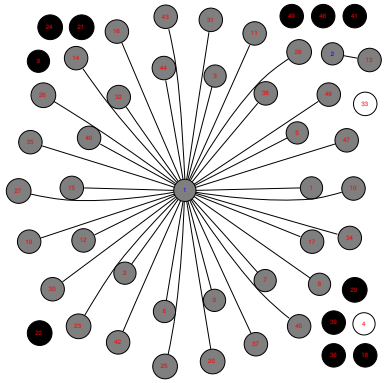

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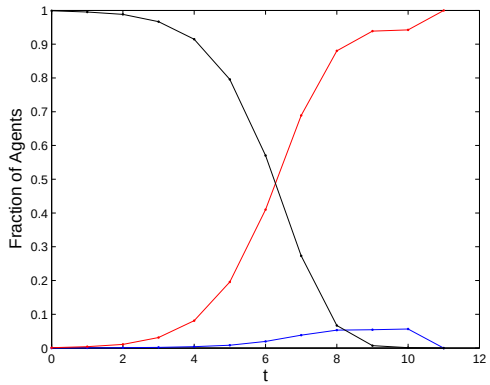
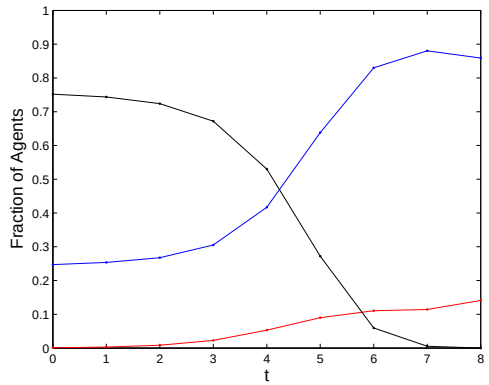
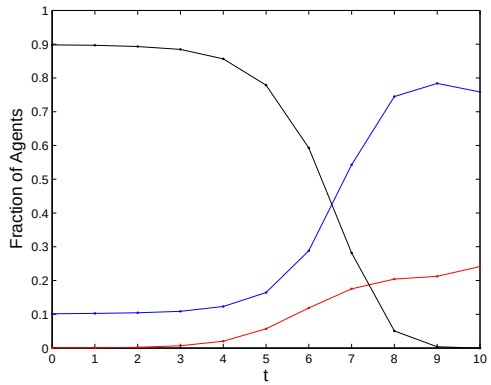
Fraction of agents in coalitions vs. time

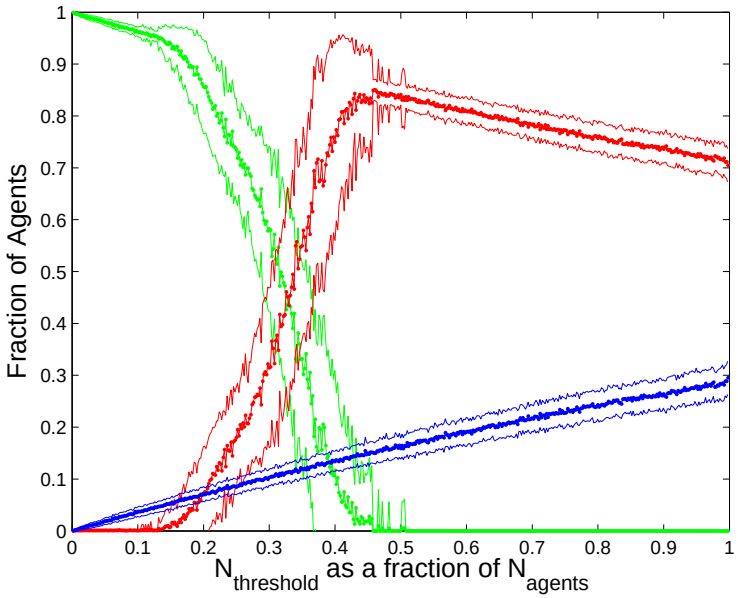


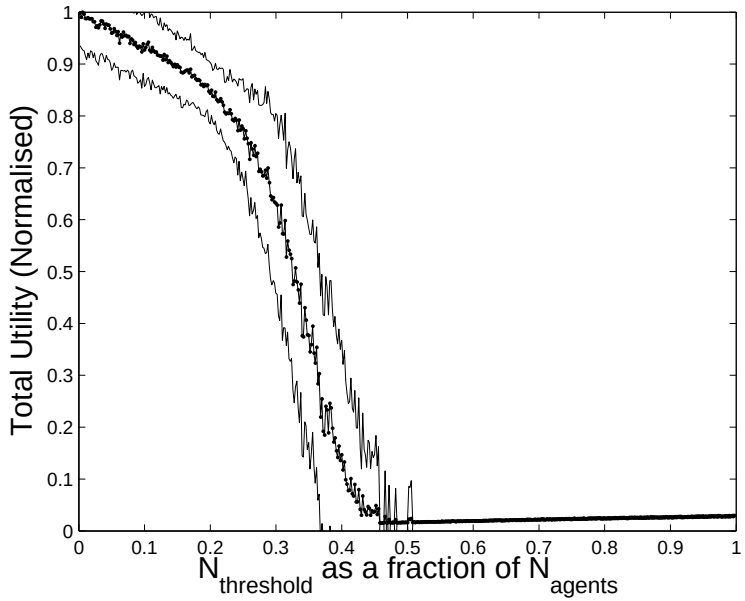




Utilities	indiv.p.	several c.	giant c.
Avg	0.43	1.25	1.01
StDev	0.04	0.24	0.36

Influence of coalition threshold N_{thr}





Extensions

- buyer's dynamics
  - different preferences (multidimensional case)
  - incomplete, bounded in time information about products
  - buy different products with limited budget $\longrightarrow$ competition
- seller's dynamics
  - products with different features (multidimensional case)
  - offer more than one product
  - limited production ressources $\longrightarrow$ competition
- repeated games
  - buyers: memory about the failure/success of coalitions
  - sellers: memory about agents $\Rightarrow$ loyalty reward
  - stationary/non-stationary coalitions

II. Network model of growth through cooperation

Growth through Network Effects

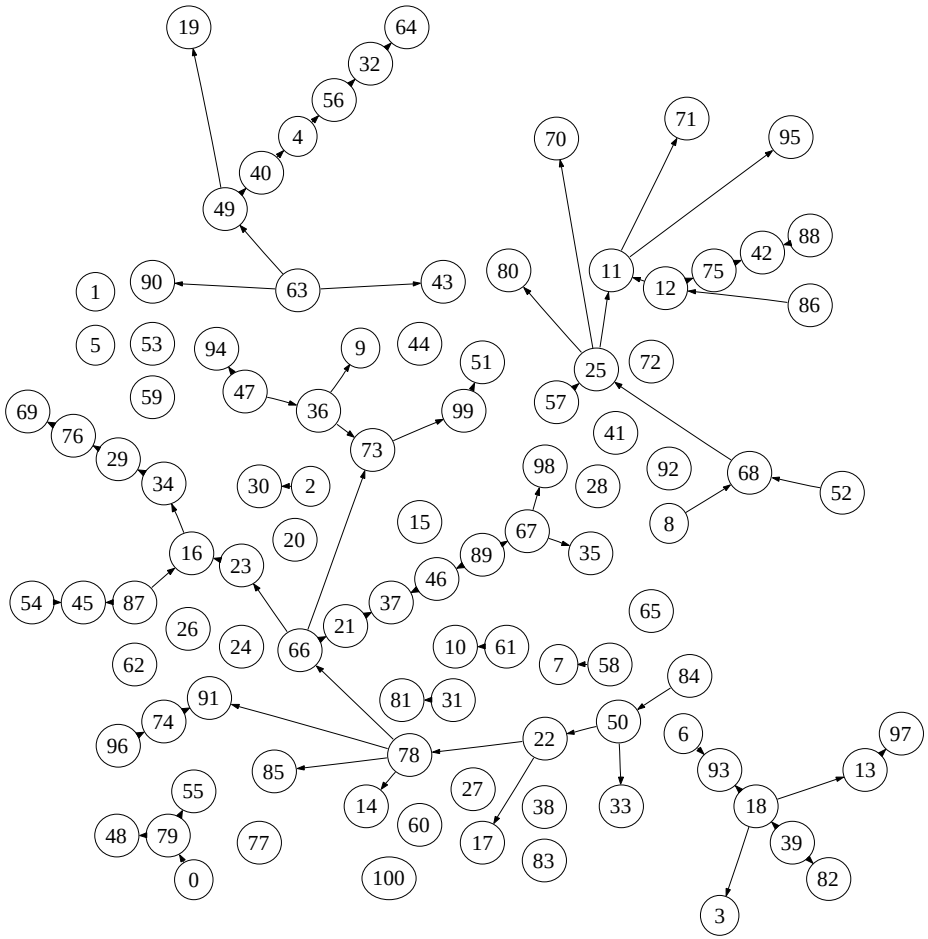
- $\dot{x}_i = \mathcal{F}_i = f(x_j, x_k) + \ldots$

$$\frac{dx_i}{dt} = \sum_{j=1}^{N} c_{ij} x_j - \Phi x_i$$

  - $c_{ij} \in \{0, 1\} \Rightarrow$ represents a directed network
    j catalyzes the growth of i, link probability p
    i is connected to $m = p(N-1)$ other agents (on average)

- *two time scales:*
  agent growth (fast), network dynamics (slow)
- *assumption:* extremal dynamics $\Rightarrow$ minimum performance threshold

- **Questions**:
  - Under which conditions do agents survive?
  - Which structures of cooperation networks emerge?
  - What happens if selection pressure is increased?
- **Results of computer simulations**:
  Emergence of a core of *cooperative* agents, and a *parasitic* pheriphery, considerable crashes and recovery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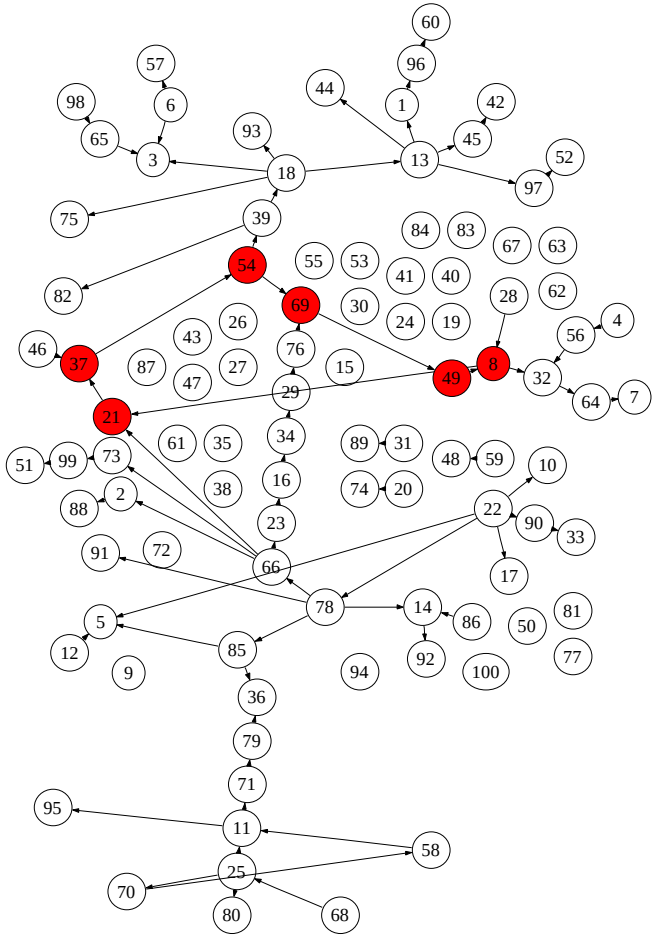
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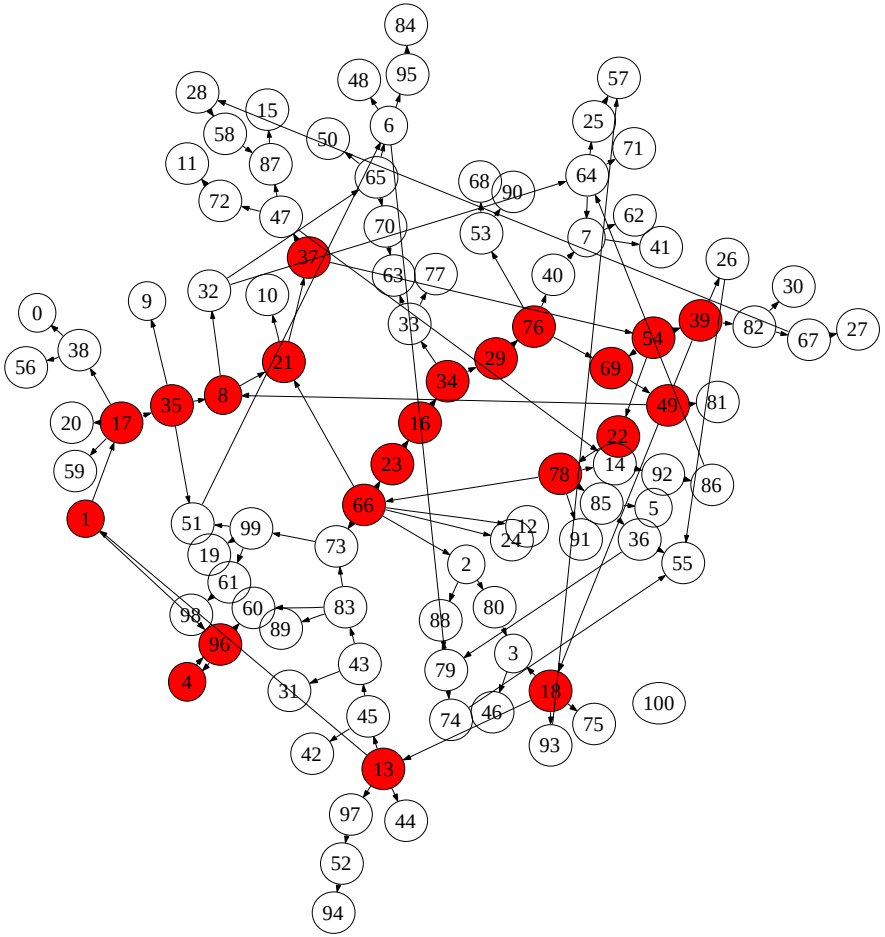
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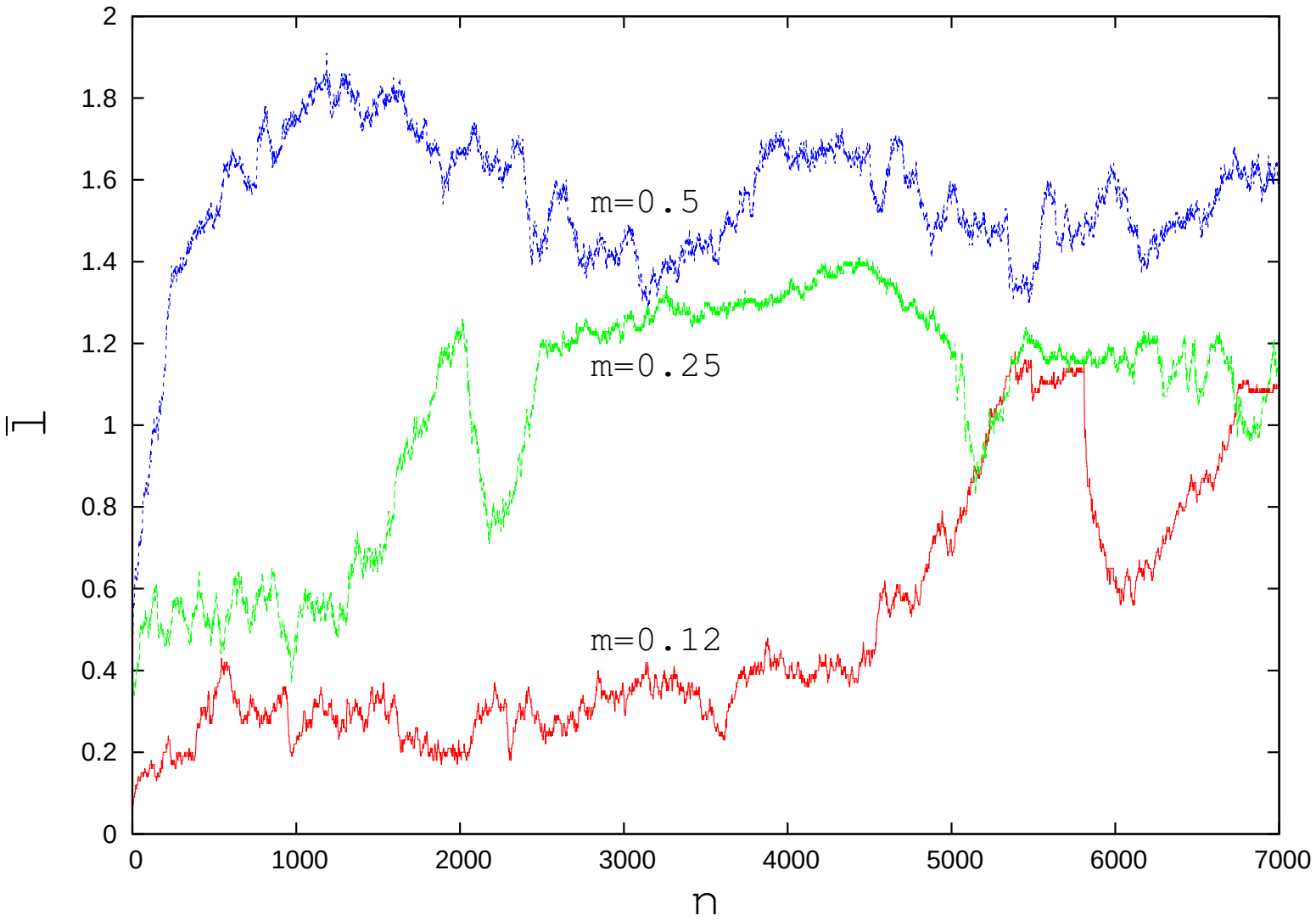
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1
0.8
0.6
0.4
0.2
0
L
0
1000
2000
3000
4000
5000
6000
7000
n
m=0.5
m=0.25
m=0.12

Advanced Growth Model

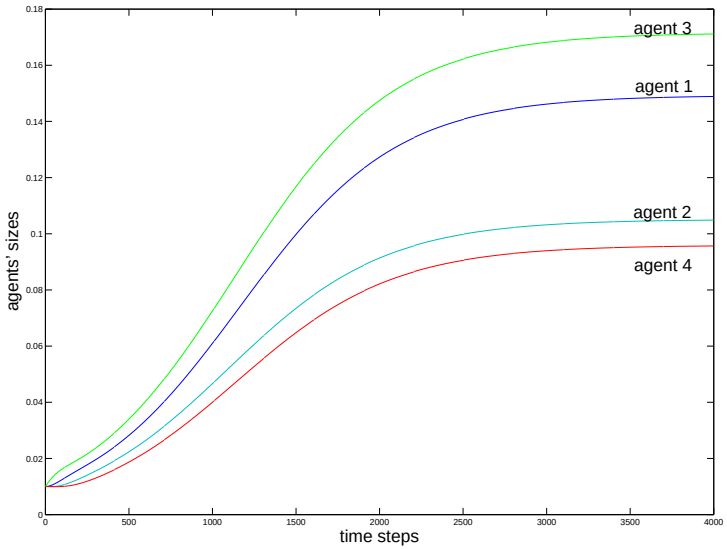
Extension of the basic model:

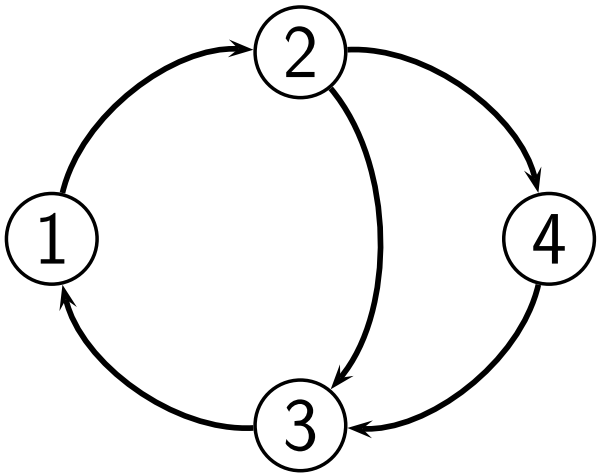
- agents take decisions with whom to interact (higher level of intelligence) without strategic interaction
- decisions are bounded rational
  - goal is to grow in size
  - locally bounded (no complete information on the system)
  - temporarily bounded (finite time horizon)
- opportunity costs for interaction
- growth (through interaction) reaches saturation

We consider a set of agents each having size x_i.

$$\dot{x}_i = -x_i + \sum_{j=1}^{n} a_{ji} x_j - c \underbrace{\left(\sum_{k=1}^{n} a_{ik} \right)^{\alpha}}_{d_{\rm out}(i)^{\alpha}} x_i^2$$

Example:





Network Dynamics

Increasing Levels of Intelligence

1. Least fit addition/removal of links[1]
2. Reactive acceptation/refusal of link changes
3. **Choosing which link to add/remove**[2]
4. Strategic interaction in game theoretic models[3]

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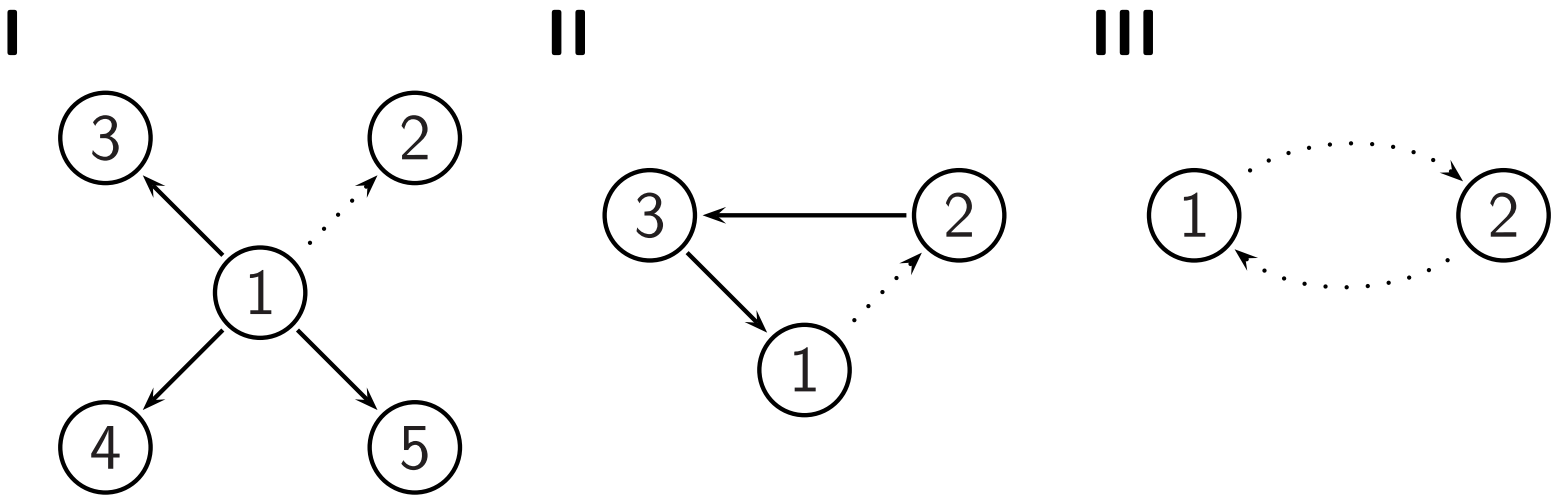
[1] S. Jain and S. Krishna (1998) Autocatalytic Sets and the Growth of Complexity in an Evolutionary Model *Phys. Rev. Lett. 81: 5684-5687*

[2] S. Bornholdt and H. Ebel. (2002). Evolutionary Games and the Emergence of Complex Networks *cond-mat/0211666*

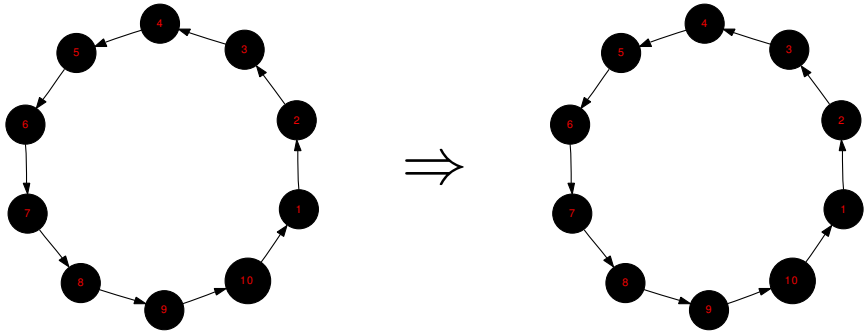
[3] V. Bala and S. Goyal. (2000). A Noncooperative Model of Network Formation *Econometrica 68, 5: 1181-1230*

- link update mechanisms
  - I choosing a link for deletion (finite time horizon)
  - II choosing a second neighbor for link creation (finite time horizon)
  - III mutual creation
- mutual links created $\Leftrightarrow$ both agents strictly benefit
- unilateral deletions $\Leftrightarrow$ source-agent strictly benefits

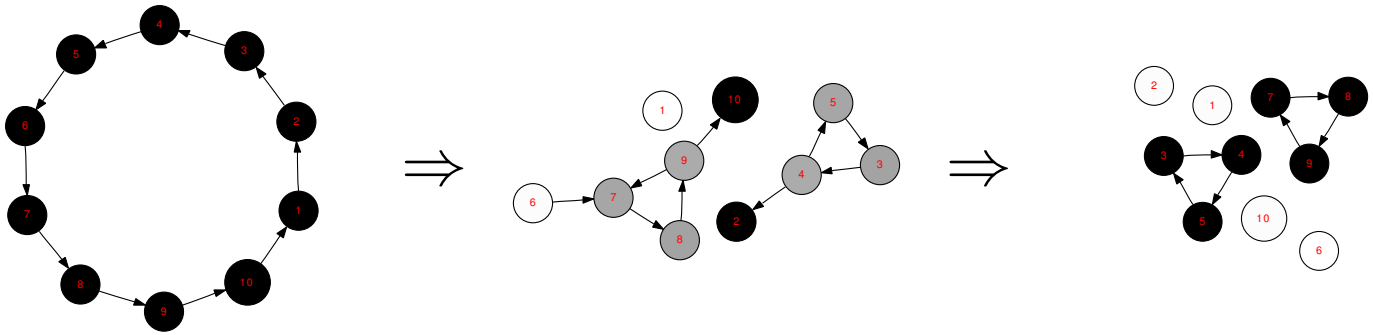


Examples

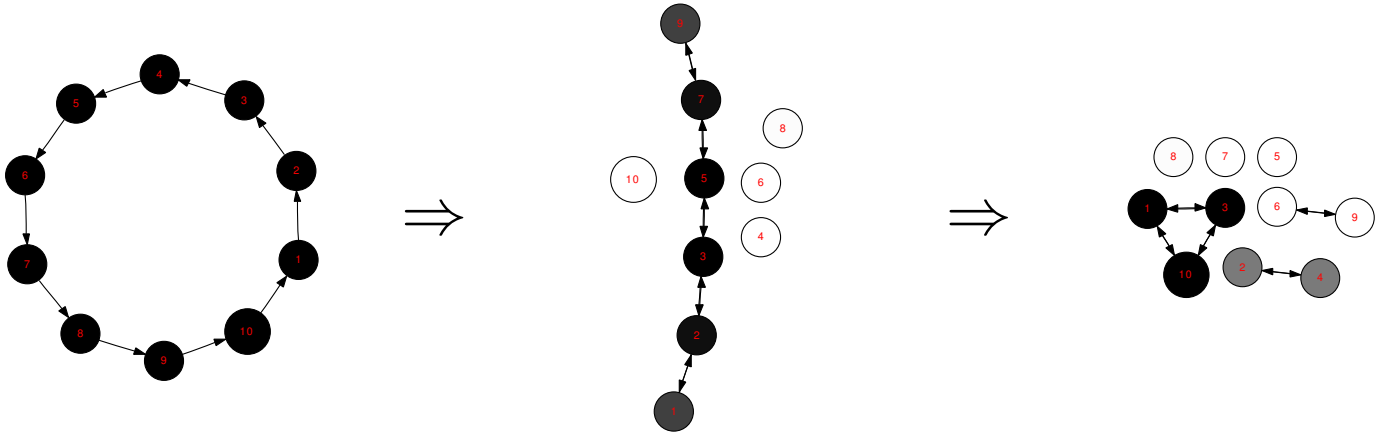
Choosing deletion (**I**)



Choosing deletion and creation with 2^{nd} *neighbors* (**I**+**II**)



Choosing deletion and mutual creation (I+III)



Observations:

- I mere link deletion would preserve C_{10}
- I+II link deletion & creation of links with 2^{nd} neighbor destroys C_{10}
- I+III link deletion & mutual link creation destroys C_{10}

Cooperation of a few can lead to destruction of cooperation of many.

Conclusions

- two models of network formation in social systems
  - coalition formation $\Rightarrow$ social network of agents to reach common goal
  - growth dynamics $\Rightarrow$ support network of agents to increase welfare (cooperation)
- local agent dynamics
  - driven by utility maximization
  - affects network/interaction structure: creation/removal of links
- "economic" conditions for emergence/stability of networks
  - coalitions: get customized products at a lower price
  - growth model: joining cooperating network prevents fall-out
- agent based modeling framework: testbed for interaction rules, critical constellations (costs)