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Swarming of Brownian Agents

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What is it about?

Blue Planet: Seas of Life, Discovery Channel 2001

- ... a swarm ... ... and ...
- “vortex swarming” of self-propelled animals

Why do physicists care?

- what is physics? "swarm" of gas molecules in a container
- *passive* vs. *active* motion
  - physics: energy from initial conditions
    or from *thermal fluctuations* $\Rightarrow$ Brownian motion
  - biology: energy take-up, storage, conversion
    (metabolism)
- *undirected* vs. *directed* (correlated) motion
  - physics: driven by convection, currents, external fields
  - biology: "intentional" (preferred direction, avoid
    collisions, ...)

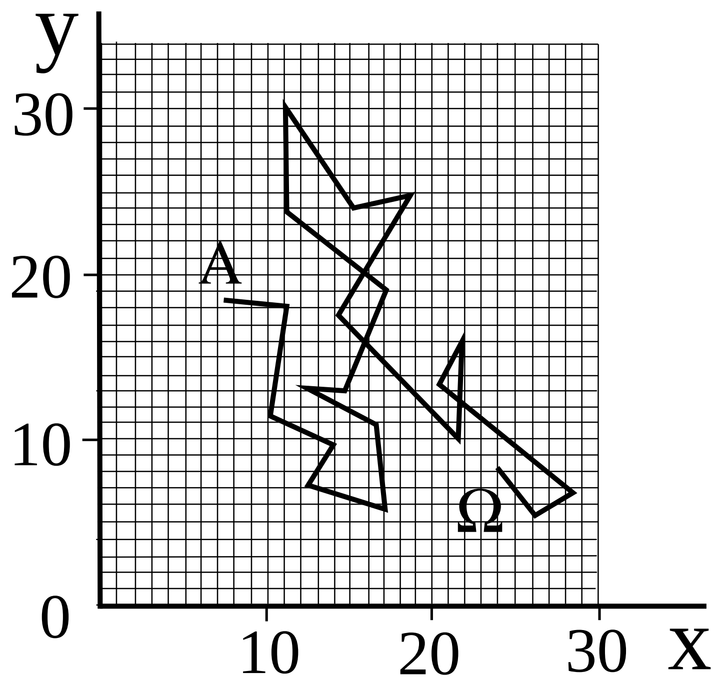
Our aims:

- Can we find a *minimal model* of biological swarming?
  ... reproduce "stylized facts"
- Can we obtain an *analytical description* of the phenomenon?
  $\Rightarrow$ derive a macroscopic dynamics from microscopic interactions?
  $\Rightarrow$ estimate relevant parameter ranges ? (no blind simulations)
  $\Rightarrow$ predict sudden changes in dynamic behavior? (phase transitions)
- Can we *bridge the gap* between "physics" and "biology"?
  ... derive common principles ...
  ... self-organization, non-equilibrium phase transitions, ...

Passive Motion

- *undirected motion:* driven by thermal noise, random impacts
  $\Rightarrow$ Brownian motion
- Fluctuation-Dissipation Theorem:

$$S = \gamma \frac{k_B T}{m}$$



Active Motion

- *active Brownian particles*★ with internal energy depot $e_i(t)$
- equations of motion:

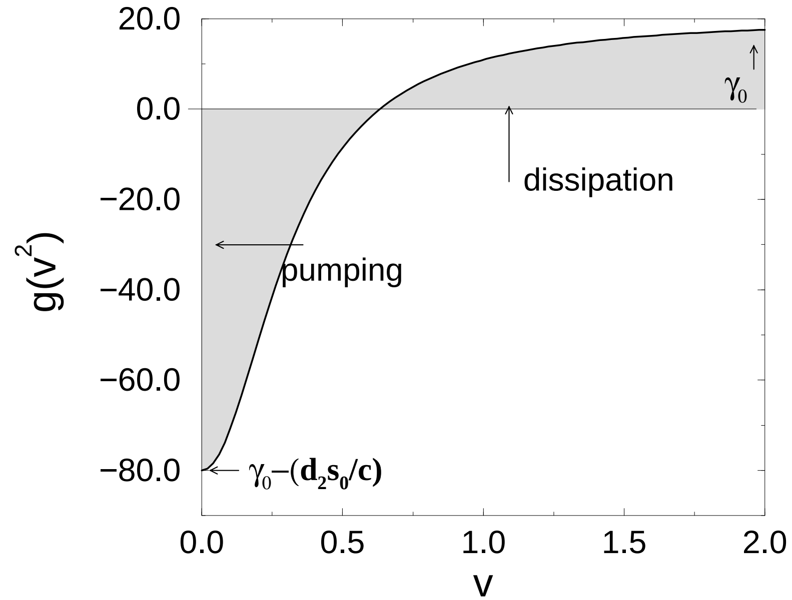
$$\dot{\boldsymbol{v}} = -\gamma_0 \boldsymbol{v} + d_2 e(t) \boldsymbol{v} + \mathcal{F}^{\text{stoch}}(t)$$

$$\dot{e}(t) = s_0 - c\, e(t) - d(\boldsymbol{v})\, e(t)$$

- friction function:

$$e_0 = \frac{s_0}{c + d_2 v^2}$$

$$g(v^2) = \gamma_0 - \frac{s_0\, d_2}{c + d_2 v^2}$$



★ F.S. *Brownian Agents and Active Particles*, Springer Series in Synergetics, 2003

Equation of Active Motion:

$$\frac{d\boldsymbol{v}}{dt} = -\left(\gamma_0 - -\frac{s_0\, d_2}{c + d_2 v^2}\right)\, \boldsymbol{v} + \sqrt{2S}\, \boldsymbol{\xi}(t)$$

Supercritical take-up of energy: $s_0 d_2 > \gamma_0 c$

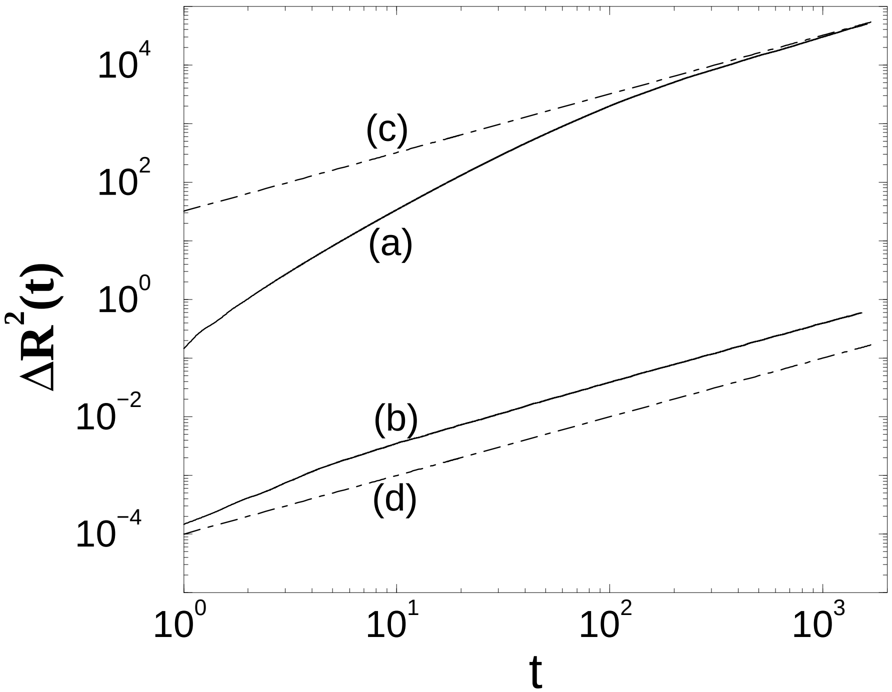
$$v_0^2 = \frac{s_0}{\gamma_0} - \frac{c}{d_2} > 0\ ; \qquad v_0^2 \gg \frac{d}{2} k_B T$$

Start Online-Simulation: Motion of Free Swarm

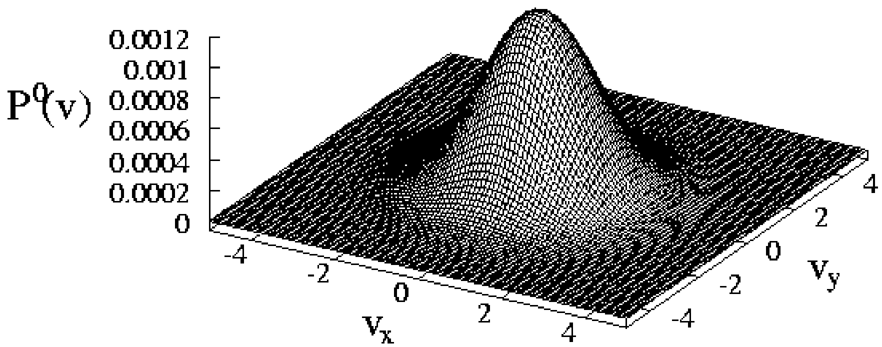
- mean squared displacement:

$$\Delta R^2(t) = 4D_r t$$

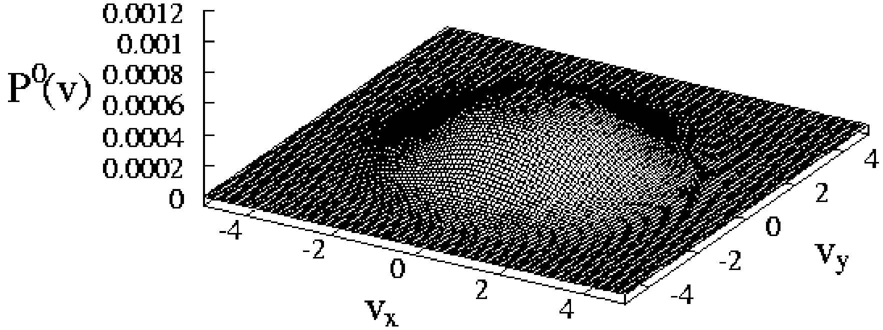
$$D_r^{\text{eff}} = \frac{v_0^4}{2S} = \frac{1}{2D}\left(\frac{s_0}{\gamma_0} - \frac{c}{d_2}\right)^2$$



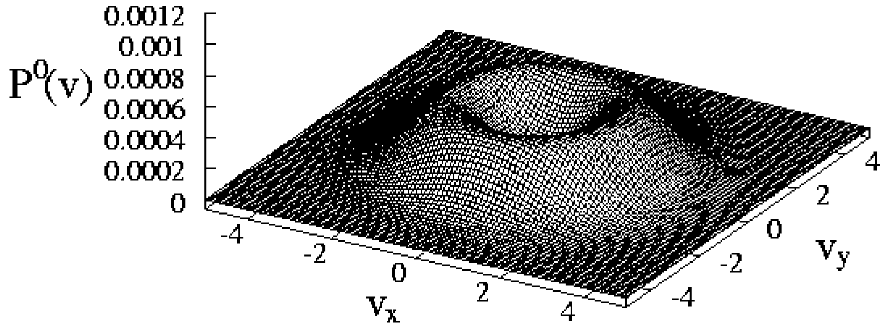
Velocity Distribution



$d_2 = 0.07$



$d_2 = 0.2$



$d_2 = 0.7$

Harmonic Swarms

- parabolic interaction potential $U(\boldsymbol{r}_i, \boldsymbol{R}) = \frac{a}{2}\,(\boldsymbol{r} - \boldsymbol{R})^2$
- equation of motion

$$\begin{aligned} \dot{\boldsymbol{v}}_i &= -g(v_i^2)\,\boldsymbol{v}_i - \boldsymbol{\nabla} U(\boldsymbol{r}) + \sqrt{2\,S}\;\boldsymbol{\xi}_i(t) \\ &= -g(v_i^2)\,\boldsymbol{v}_i - \frac{a}{N}\sum_{j=1}^{N}(\boldsymbol{r}_i - \boldsymbol{r}_j) + \sqrt{2\,S}\;\boldsymbol{\xi}_i(t) \end{aligned}$$

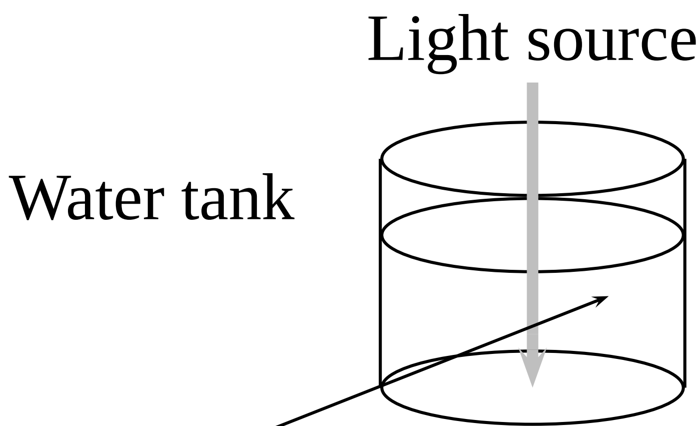
- supercritical take-up of energy: $s_0 d_2 > c\gamma_0$
  $\Rightarrow$ *active motion* $\Rightarrow$ spatial *dispersion* of the swarm
- mutual interaction $\Rightarrow$ spatial *concentration*
- **Result:** complex coherent motion

Film

Superposition of two motions:

- motion of the center of mass $(\dot{\boldsymbol{R}}, \dot{\boldsymbol{V}})$
- motion *relative to* the center of mass
  - $\boldsymbol{V} = 0:$ motion of free particles in parabolic potential
- *dependence on initial conditions:*
  - $v_i^2(0) < v_0^2 \Rightarrow$ rotational mode
    $V^2(t) \to 0,\, v^2(t) \to v_0^2,\, r_0 = |v_0|\, a^{-1/2}$
  - $v_i^2(0) > v_0^2 \Rightarrow$ translational mode
    $V^2(t) \to v_0^2,\, v_i^2(0) > v_0^2$

Swarming of Daphnia



- *Daphnia* (water flea) inserted into water tank

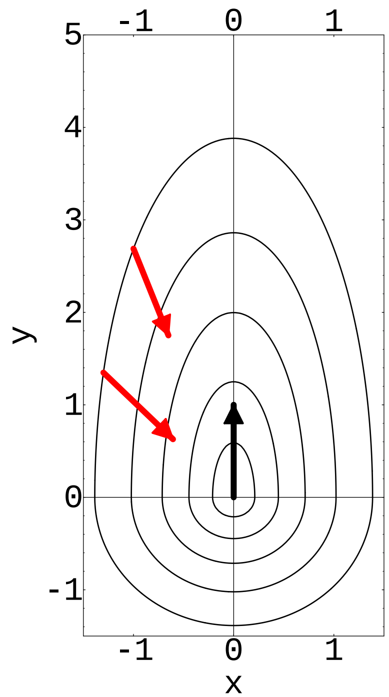
Videos* Single animal Several animals

* Courtesy of Anke Ordemann, Center for Neurodynamics, University of Missouri

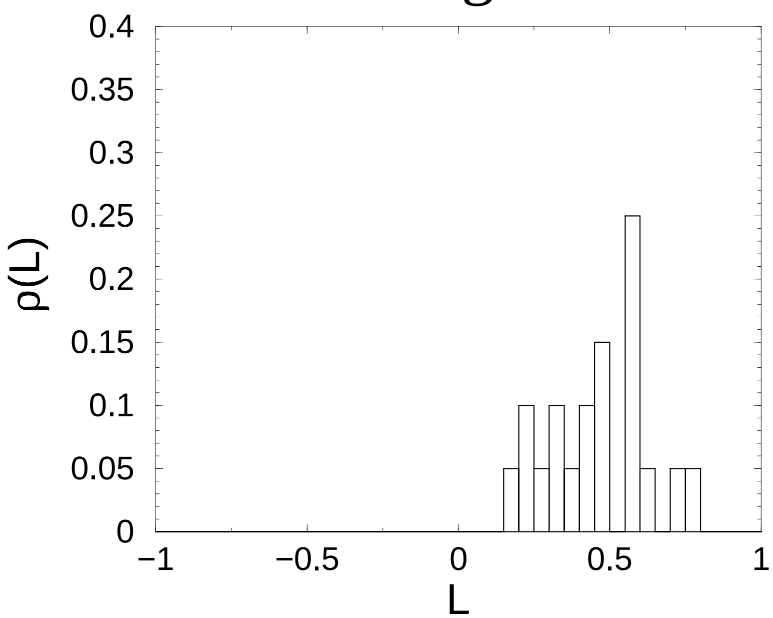
Interaction via Avoidance Potential

$$\mathcal{F}_i^{\text{ext}} = -\nabla U(\boldsymbol{r}) = -a\boldsymbol{r} \; ; \quad \mathcal{F}_i^{\text{int}} = -\sum_{j \neq i} \nabla V\Big(R_i(\boldsymbol{r}_{ij}, \boldsymbol{v}_i, \boldsymbol{v}_j)\Big)$$

Simulation: 20 agents



distribution of angular momenta



Two different modes of swarm motion:

- translation/rotation
  - transitions possible dependent on noise, initial conditions
- *translational mode:*
  particles heading into the same direction, $V^2(t) \neq 0$

Film: Translation of Swarm

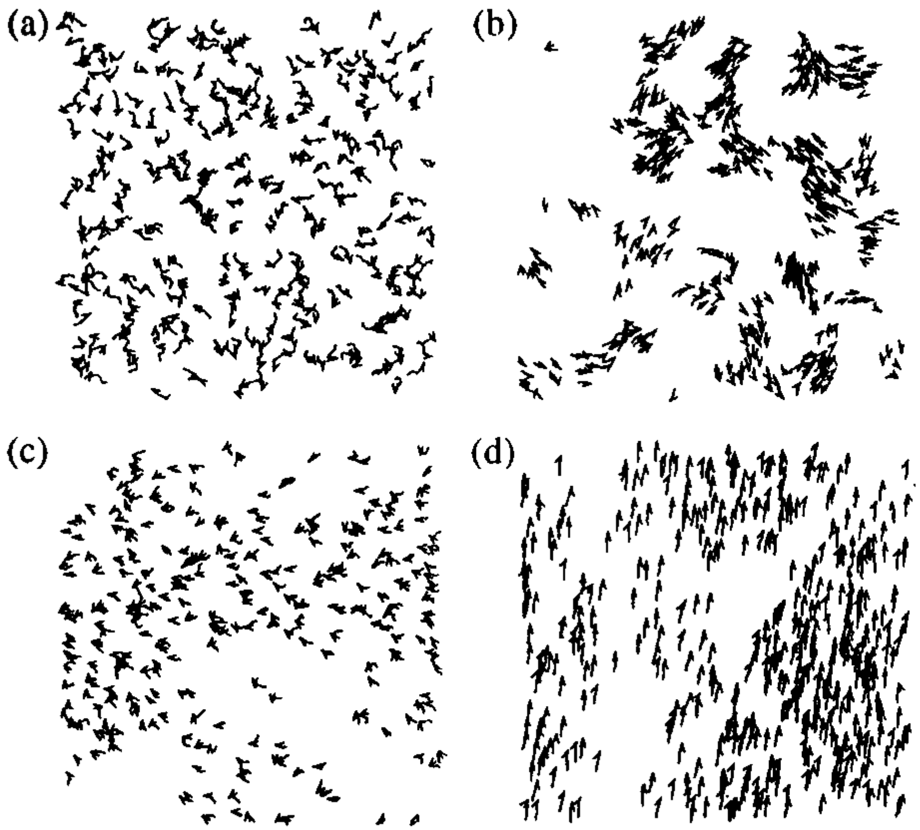
Fish Bird

- *rotational mode:*
  particles moving around a center, $V^2(t) \to 0$

Swarm of fish Simulation Fish

Interaction via local alignment

- local orientation of particle: $\theta_i(t) = \langle \theta(t-1) \rangle_r + \Delta\theta$
  T. Vicsek et al PRL 75 (1995) 1226



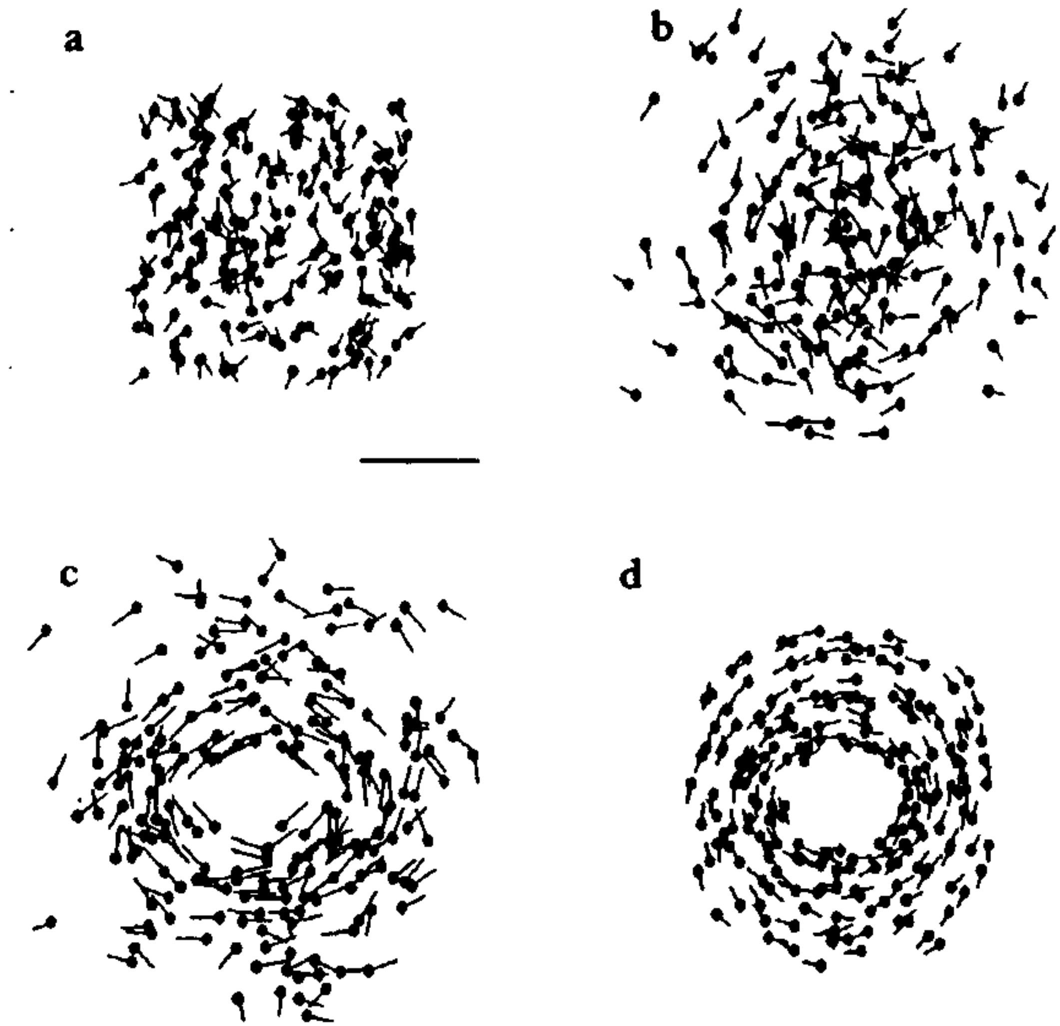
H. Levine et al PRE 63 (2001) 017101

- long-range attraction, short-range repulsion

$$U = \sum_{j \neq i} \left\{ C_a \exp\left(- \left|x_i - x_j\right| / l_a\right) - C_r \exp\left(- \left|x_i - x_j\right| / l_r\right) \right\}$$

- alignment $f_i = v_j \exp\left(- \left|x_i - x_j\right| / l_r\right)$, or $f_i = v_i$
- equation of motion:

$$\dot{v}_i = \alpha f_i - \gamma v_i - \nabla U \big|_{x_i}$$


a
b
c
d

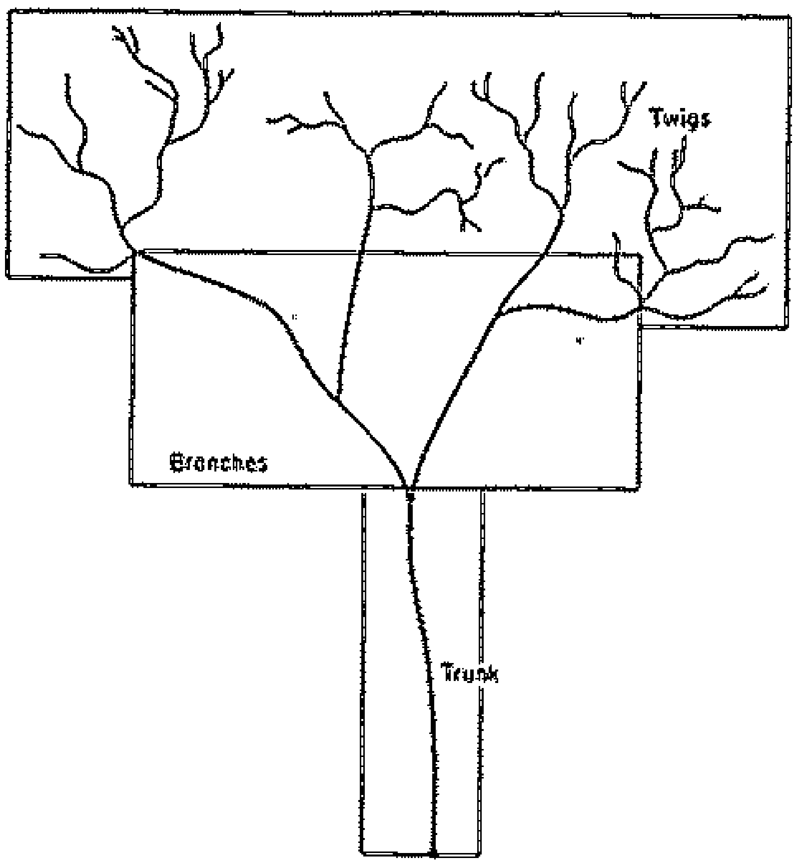
Swarming by means of chemical communication

- bacteria, slime molds, amoebae, social insects ...
  use different kind of chemicals for *local* communication
- spatio-temporal communication field

$$\frac{\partial}{\partial t} h_\theta(\boldsymbol{r},t) = \sum_{i=1}^{N} s_i\, \delta_{\theta,\theta_i}\, \delta(\boldsymbol{r}-\boldsymbol{r}_i) - k_\theta h_\theta(\boldsymbol{r},t) + D_\theta \Delta h_\theta(\boldsymbol{r},t)$$

- multi-component scalar field reflects:
  - existence of *memory* (past experience)
  - *exchange of information* with *finite* velocity
  - influence of *spatial distances* between agents
    $\Rightarrow$ *weighted* influence (space, time)

Example: Foraging Route of Ants



Schematic representation of the complete foraging route of *Pheidole milicida*, a harvesting ant of the southwestern U.S. deserts. Each day tens of thousands of workers move out to the dendritic trail system, disperse singly, and forage for food.

Hölldobler, B. and Möglich, M.: The foraging system of *Pheidole militicida (Hymenoptera: Formicidae)*, *Insectes Sociaux* **27/3** (1980) 237-264

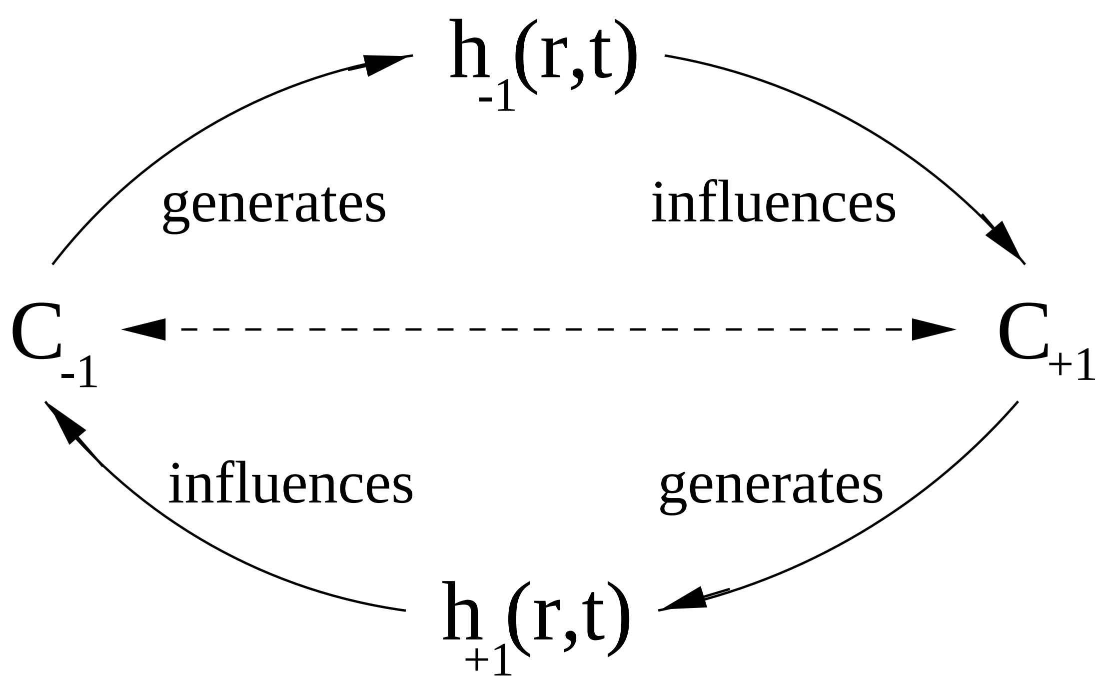
Brownian Agents

- two state variables: position r_i
  internal parameter $\theta_i \in \{-1, +1\}$ (transitions possible)
- dynamic equation for r_i:

$$
\begin{aligned}
\frac{d\boldsymbol{r}_i}{dt} &= \frac{1}{\gamma_0}\boldsymbol{\nabla_i} h^e(\boldsymbol{r}, t) + \sqrt{2D}\boldsymbol{\xi}_i(t) \\
\boldsymbol{\nabla_i} h^e(\boldsymbol{r}, t) &= \frac{\theta_i}{2}\Big[(1+\theta_i)\,\boldsymbol{\nabla_i} h_{-1}(\boldsymbol{r}, t) \;-\; (1-\theta_i)\,\boldsymbol{\nabla_i} h_{+1}(\boldsymbol{r}, t)\Big]
\end{aligned}
$$

- state dependent production rate $s_i(\theta_i, t)$

Non-linear feedback:



➤ **Result:** exploitation of food sources Film

Conclusions

- *physical* models of swarm dynamics
  - based on generalized Langevin equations
    - $\Rightarrow$ derivation of macroscopic dynamics for distributions
    - $\Rightarrow$ stability analysis: prediction of critical parameters, ...
- turn passive into *active motion*
  - take-off, storage, conversion of energy
- local or global interactions $\Rightarrow$ *coherent motion*
  - coupling to invariants of motion or local averages
  - attractive, repulsive interaction
  - chemical communication
- *Result:* translational / rotational modes of swarming
  “meaningful behavior”