Seminar at Bank of Portugal

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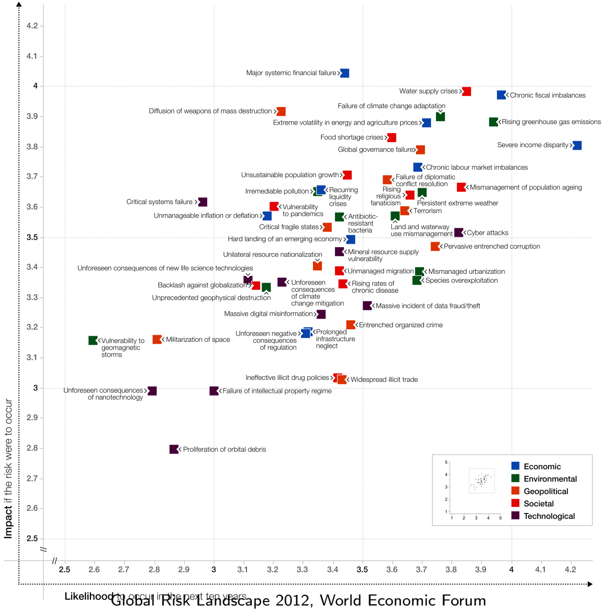
Systemic Risk in Financial Network

March 30, 2013



Global Risk Landscape 2012, World Economic Forum

Systemic Risk - Global Perspectives

- US crisis, EU sovereign crisis

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- US crisis, EU sovereign crisis The Perfect unforseen Storm !

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- US crisis, EU sovereign crisis The Perfect unforseen Storm !
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Two major issues of:

- **System Design**

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Two major issues of:

- **System Design** Network architecture, resilience, liquidity flow

Systemic Risk - Global Perspectives

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Two major issues of:

- **System Design** Network architecture, resilience, liquidity flow
- **Mechanism Design**

Systemic Risk - Global Perspectives

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Two major issues of:

- **System Design** Network architecture, resilience, liquidity flow
- **Mechanism Design** Coordination failure at global scale and lack of a global governance:

Systemic Risk - Global Perspectives

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Two major issues of:

- **System Design** Network architecture, resilience, liquidity flow
- **Mechanism Design** Coordination failure at global scale and lack of a global governance:
  - market concentration
  - unbalance in the representation of interests
  - massive conflicts of interest, moral hazard
  - excessive risk taking and socialization of downside risks

JM Paper 1

Market Procyclicality and Systemic Risk
coauthored with Battiston S.

Introduction

To model appeals to two ingredients:

1. **Procyclical Capital Requirements**;
2. **Positive feedback loop leverage-asset price**.

References:

- Adrian, T. and Shin, H. (2008a). Financial intermediaries, financial stability, and monetary policy. Brookings Panel on Economic Activity, September, Federal Reserve Bank of Kansans City Symposium at Jackson Hole.
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- Adrian, T. and Shin, H. (2008c). Liquidity and financial contagion. FSR Financial, page 1.
- Adrian, T. and Shin, H. (2010). Liquidity and leverage. Journal of Financial Intermediation, 19(3):418-437.
- Adrian, T. and Shin, H. (2011). Procyclical leverage and value-at-risk. FRB of New York Staff Report No. 338.
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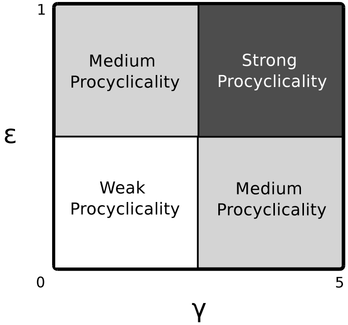
Research Question

How does systemic risk depend on the interplay between:

1. The **intensity** of the banks to tie their economic capital to VaR (ε)
2. Asset-market **liquidity** ($1/\gamma$)

In the presence of an Exognous Undiverisable asset-price **shock** ?

Research Question



Hypothesis: System exhibits a knife-edge dynamics

- Weak compliance with capital requirements + liquid asset market ⇒ No effect on systemic risk ;
- Strong compliance with capital requirements + illiquid asset market ⇒ Increase systemic risk.

Paper Layout

1. Interbank Network

Paper Layout

1. Interbank Network
2. A Balance Sheet Approach

Paper Layout

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2. A Balance Sheet Approach
3. The Behavioral Rule

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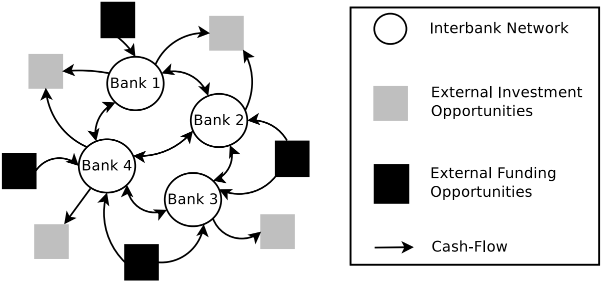
5. Financial Network in Mean-Field
6. Systemic Risk
7. Perturbation Analysis
8. Results

Paper Layout

Interbank Network

Interbank Network

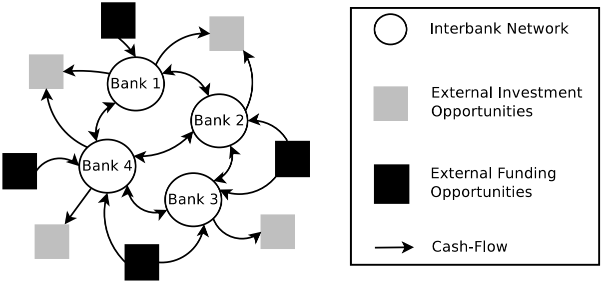
Interbank Network. The Setting



Asset Market Assumption

- External Assets/Invest. opportunities are indistinguishable, uncorrelated and have the same initial value.

Interbank Network. The Setting

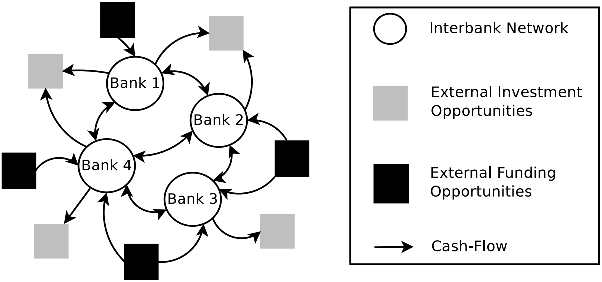


Asset Market Assumption

- External Assets/Invest. opportunities are indistinguishable, uncorrelated and have the same initial value.

$\Longrightarrow$ **the equally-weighted portfolio is the optimal inv. strategy**.

Interbank Network. The Setting



Network Assumption

Banks are tightly connected and have homogenous balance sheet structures:

- Similar investment and risk management strategies;
- Similar nominal total obligations and comparable market power.

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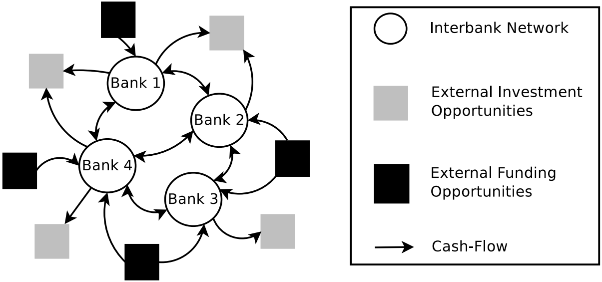

Network Assumption

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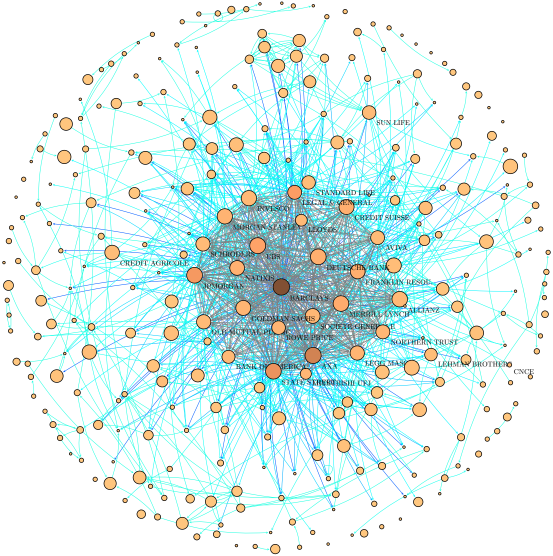
$\Longrightarrow$ **Banks have similar proportion of debt to asset ratio (leverage)**

Interbank Network. A core-periphery structure.



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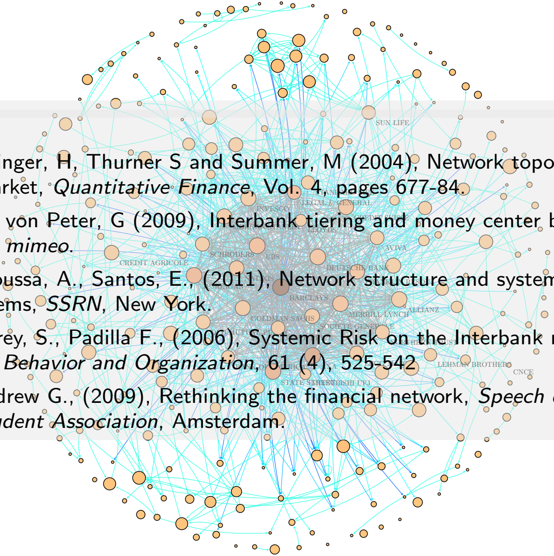

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- Iori, G., Jafarey, S., Padilla F., (2006), Systemic Risk on the Interbank market, *Journal of Economic Behavior and Organization*, 61 (4), 525-542
- Haldane, Andrew G., (2009), Rethinking the financial network, *Speech delivered at the Financial Student Association*, Amsterdam.

A Balance Sheet Approach

A Blance Sheet Approach

A Balance Sheet Approach

Tot. assets: $a_i := \sum_j Q_{il} s_l + \sum_j W_{ij} \hbar_j$
Balance-sheet Identity: $a_i \equiv h_i + b_i + e_i$

Bank-i balance-sheet

Assets	Liabilities
$\sum_j Q_{il} s_l$	h_i
$\sum_j W_{ij} \hbar_j$	b_i
	e_i

- Q_{il} (≥ 0): quantity of the external marketable asset l held by i
- s_l: price of the external assets l
- W_{ij} (≥ 0) (with $W_{ii} = 0$): quantity of debt issued by j and held by i
- $\hbar_j := h_j \left[1 + r_j\right]^{-\hat{t}}$ present market value of bank j's debts
- r_j: rate of return on $\hat{t}$-years maturity obligations
- h_i: book value of bank i's obligations to other banks
- b_i: book value of bank i's external funds
- e_i: equity value

A Balance Sheet Approach. Leverage

$$\phi_i := (h_i + b_i)/a_i \ \in (0,1] \tag{1a}$$

$$= (h_i + b_i)/\left(\sum_l Q_{il} s_l + \sum_j W_{ij} \hbar_j\right) \tag{1b}$$

$$= (h_i + b_i)/\left(\sum_l Q_{il} s_l + \sum_j W_{ij} h_j \left[1 + r_j\right]^{-\hat{t}}\right) \tag{1c}$$

A Balance Sheet Approach. Leverage

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$$= (h_i + b_i)/\left(\sum_l Q_{il} s_l + \sum_j W_{ij} h_j \left[1 + r_j\right]^{-\hat{t}}\right) \tag{1c}$$

$$\mathbf{r_j} := \mathbf{r_f} + \boldsymbol{\beta\phi_j}\ , \tag{2}$$

- r_f: risk-free rate
- $\beta \in (0,1)$: factor loading on j's leverage

A Balance Sheet Approach. Leverage

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$$\phi_i = (h_i + b_i)/\left(\sum_l Q_{il} \mathbf{s_l} + \sum_j W_{ij} h_j \left[1 + r_f + \beta\phi_{\mathbf{j}}\right]^{-\hat{t}}\right) \tag{3}$$

The Behavioral Rule

The Behavioral Rule

The Behavioral Rule. Target Leverage

The behavioral rule:

Banks adjust their balance-sheet to keep their economic capital (equity) equal to total VaR *:

$$\mathbb{P}(a_i < \bar{a}_i - VaR) \leq 1 - c \equiv \mathbb{P}(Loss > VaR) \leq 1 - c$$

(*) Shin, H. (2008) Liquidity Risk in a System Context, *Journal of Financial Intermediation*, 17(3):315–329.

The Behavioral Rule. Target Leverage

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$$\mathrm{e}_i = V \times a_i(s_l^*) \implies a_i(s_l^*) - (b_i + h_i) = V \times a_i(s_l^*) \implies$$

$$\mathrm{b}_i + h_i = a_i(s_l^*) \times (1 - V) \implies \frac{b_i + h_i}{a_i(s_l^*)} := \phi(s_l^*) = (1 - V) \implies$$

$$\phi(s_l^*) := \phi^* = (1 - V) \,,$$

where ϕ^*= financial reporting leverage, V=VaR per unit of assets.

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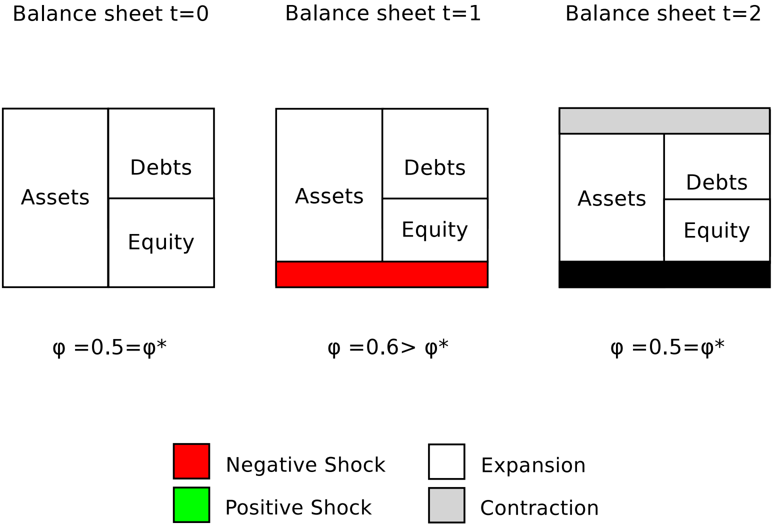
The Behavioral Rule. Target Leverage

...according to this behavioral rule any common (**non-diversifiable**) shock that deviates s_l from s_l^* deviates the market equity from VaR and ϕ from ϕ^*. This event triggers an accounting reaction to track back ϕ to ϕ^*.

Assumptions:

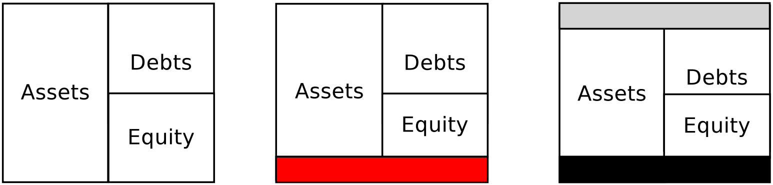
1. Equity rationing and Debt overhang
2. $h_{(t)} = h$ for all $t \geq 0$: constant nominal value of interbank obligations
3. Banks shrink or enlarge their balance sheets by adjusting their portfolio of external assets and their portfolio of external funds.

The Behavioral Rule. Target Leverage



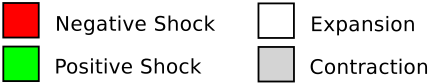
The Behavioral Rule. Target Leverage

Balance sheet t=0 | Balance sheet t=1 | Balance sheet t=2



φ =0.5=φ* | φ =0.6> φ* | φ =0.5=φ*

$$s_l < s_l^* \implies \phi > \phi^*$$



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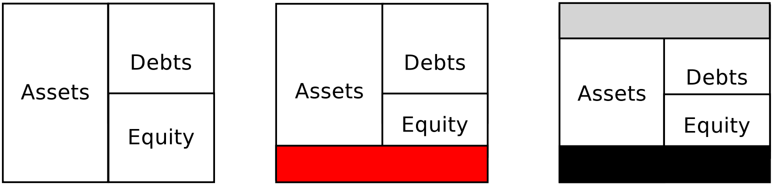


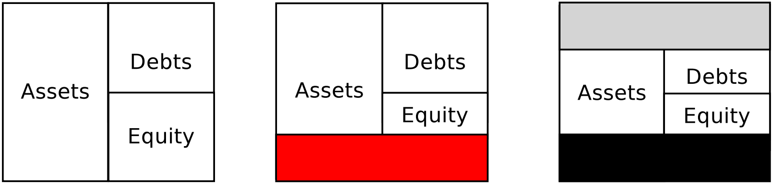
φ =0.5=φ* | φ =0.7> φ* | φ =0.5=φ*

$$s_l < s_l^* \implies \phi > \phi^*$$

Negative Shock
Positive Shock
Expansion
Contraction

The Behavioral Rule. Target Leverage

Balance sheet t=0 Balance sheet t=1 Balance sheet t=2



φ =0.5=φ* φ =0.8> φ* φ =0.5=φ*

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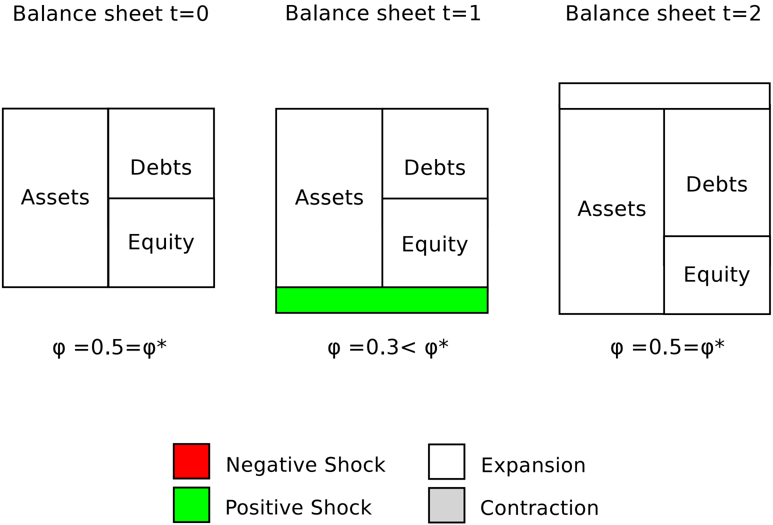
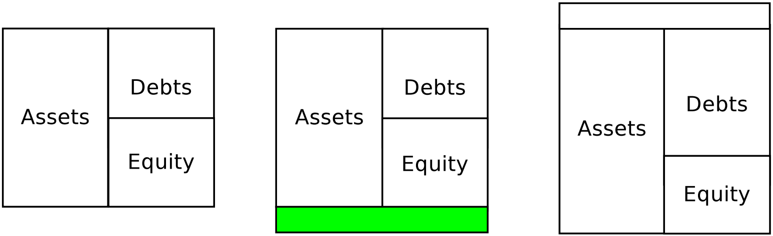


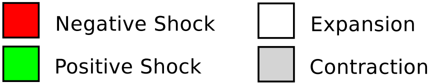
The Behavioral Rule. Target Leverage

Balance sheet t=0 | Balance sheet t=1 | Balance sheet t=2



φ =0.5=φ* | φ =0.3< φ* | φ =0.5=φ*

$$s_l > s_l^* \implies \phi < \phi^*$$



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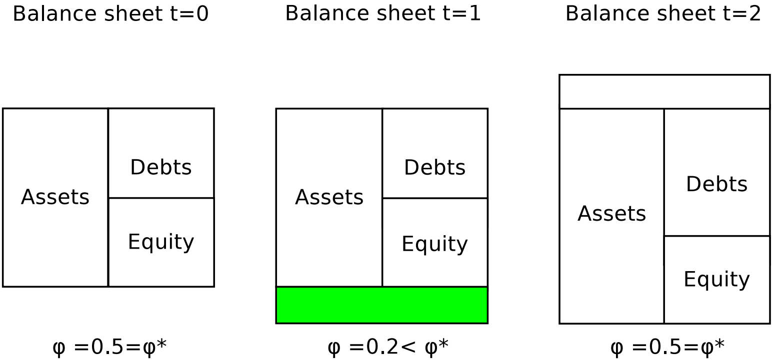

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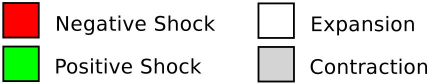

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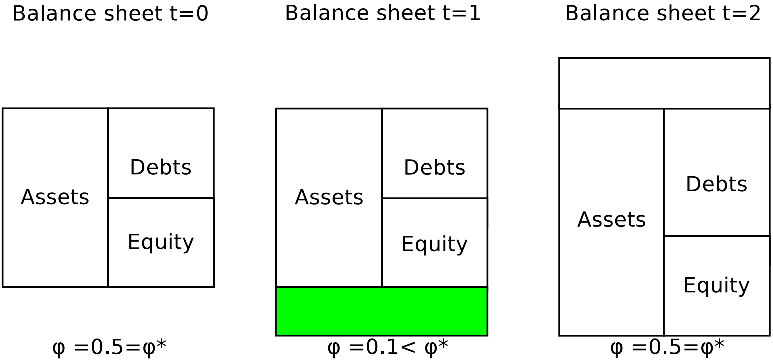

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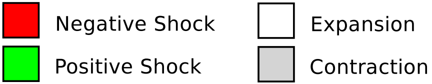


The Behavioral Rule. Target Leverage

Accounting Rule based on Target Leverage *:

$$\frac{db_i}{b_i} = \left(\frac{\varepsilon_{\mathbf{i}}}{\kappa_i \phi_i}\right)\left(\frac{\phi_i^* - \phi_i}{1-\phi_i^*}\right) , \tag{4}$$

$$\frac{dQ_{il}}{Q_{il}} = \left(\frac{\varepsilon_{\mathbf{i}}}{\alpha_{il}}\right)\left(\frac{\phi_i^* - \phi_i}{1-\phi_i^*}\right) . \tag{5}$$

- Accounting constraints: $Q_i = \sum_l Q_{il}$; $dQ_i \geq -Q_i$; $db_i \geq -b_i$.
- $\varepsilon_i \in (0,1]$: promptness of i in pursuing the target leverage ϕ_i^*
- $\kappa_i := b_i/(b_i + h_i) \in (0,1]$: ratio of external funds to total debts
- $\alpha_{il} := Q_{il}s_l/(\sum_l Q_{il}s_l + \sum_j W_{ij}\hbar_j) \in (0,1]$: ratio of the external asset l to total assets

(*) We formalize the idea from Adrian, T. and Shin, H., Liquidity and Leverage, Journal of Financial Intermediation.

Leverage-Price Cycle

Leverage-Price Cycle

Leverage-Price Cycle

Assumptions:

1. External asset price dynamics are driven by a standard GBM:

$$\frac{ds_l}{s_l} = \mu_l dt + \sigma_l dB_l \ , \quad \forall l \in \Omega_M \ . \tag{6}$$

2. Linear relationship between asset returns and trading volume[a]

$$\mathbb{E}\left(\frac{ds_l}{s_l}\right) = \gamma_l \left(\frac{dQ_l}{Q_l}\right) \ . \tag{7}$$

γ_l: market impact (average price response to bank trades)[b]

[a] In this case we are able to isolate the non-linear of the dynamic balance-sheet management on the asset price dyunamics

[b] $1/\gamma_l$ measures the market liquidity.

The Financial Network in Mean-Field

The Financial Network in Mean-Field

Financial Network. A Mean-Field Approximation

$$
\begin{cases}
\frac{ds}{s} & = \gamma\left(\frac{dQ}{Q}\right) dt + \sigma dB \\
\frac{dQ}{Q} & = \left(\frac{\varepsilon}{\alpha}\right)\left(\frac{\phi^* - \phi}{1-\phi^*}\right) dt \\
\frac{db}{b} & = \left(\frac{\varepsilon}{\kappa\phi}\right)\left(\frac{\phi^* - \phi}{1-\phi^*}\right) dt \\
\phi & = \frac{h(\beta-1)+\beta b - Q\,s + \left(4\beta(b+h)Q\,s + (h-\beta(b+h)+Q\,s)^2\right)^{1/2}}{2\beta Q\,s}
\end{cases}
\tag{8}
$$

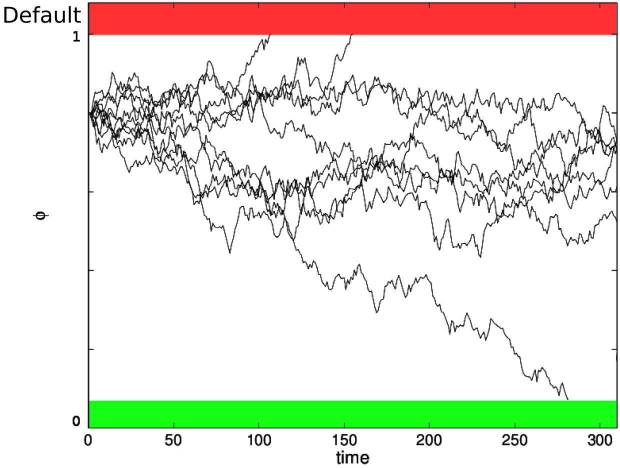
Financial Network. A Mean-Field Approximation

$$\begin{cases} \frac{ds}{s} &= \gamma\left(\frac{dQ}{Q}\right) dt + \sigma dB \\ \frac{dQ}{Q} &= \left(\frac{\varepsilon}{\alpha}\right)\left(\frac{\phi^*-\phi}{1-\phi^*}\right) dt \\ \frac{db}{b} &= \left(\frac{\varepsilon}{\kappa\phi}\right)\left(\frac{\phi^*-\phi}{1-\phi^*}\right) dt \\ \phi &= \frac{h(\beta-1)+\beta b-Q\,s+\left(4\beta(b+h)Q\,s+(h-\beta(b+h)+Q\,s)^2\right)^{1/2}}{2\beta Q\,s} \end{cases} \longrightarrow \phi = \phi(\varepsilon,\gamma) \qquad (8)$$

Systemic Default

Systemic Default

Systemic Default



Systemic Default

Systemic Default Event (DE)

The event that at any time $t > 0$, the leverage ϕ is equal or bigger than one is classified as a "systemic default event"

$$\mathrm{DE} \Leftrightarrow \phi(\epsilon, \gamma)_t \geq 1 \quad \forall t > 0$$

iff $\phi(\epsilon, \gamma)_t \geq 1$ and $\nexists\ t' < t$ s.t. $\phi(\epsilon, \gamma)_{t'} \geq 1$ or $\forall\ t' < t$, $\phi(\epsilon, \gamma)_{t'} < 1$.

Probability of Systemic Default

$$\mathbb{P}[DE] \approx dt/\bar{\tau}\,[\phi(\epsilon, \gamma)]$$

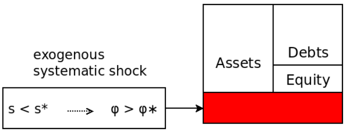
with $dt << \bar{\tau}$ where $\bar{\tau}$ is the mean time to default.

Iff the set of banks is populated at a constant rate at each interval dt.

Perturbation Analysis

Perturbation Analysis

Perturbation Analysis. Exogenous Price shock



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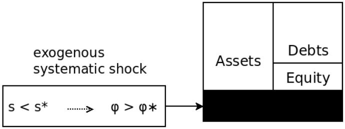

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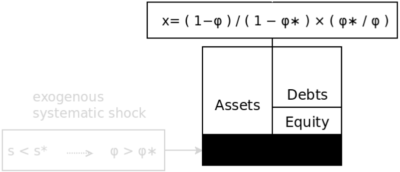

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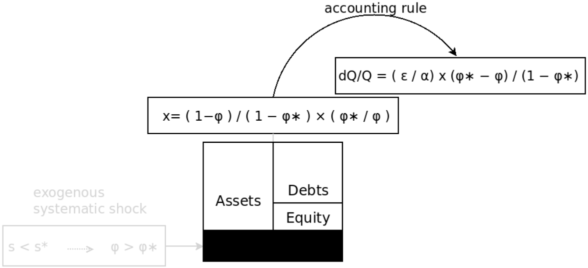

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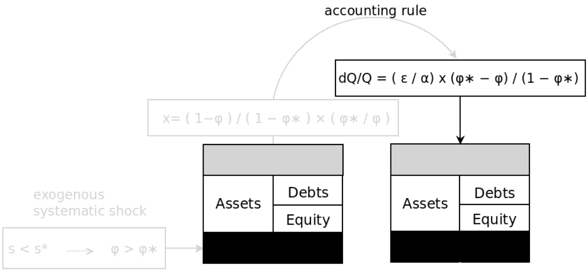
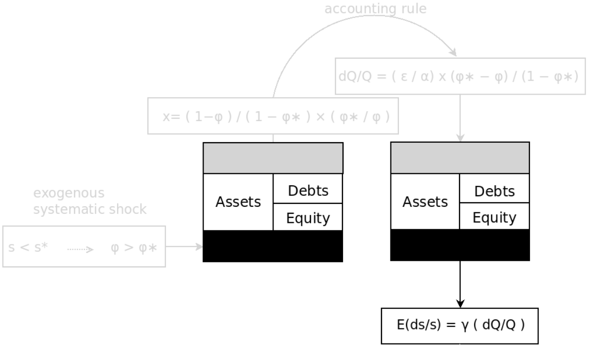


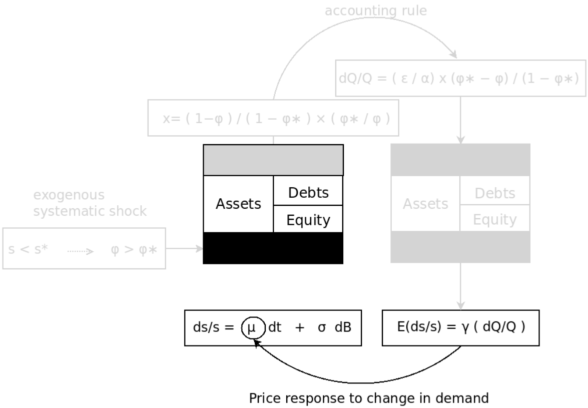
Perturbation Analysis. Exogenous Price shock



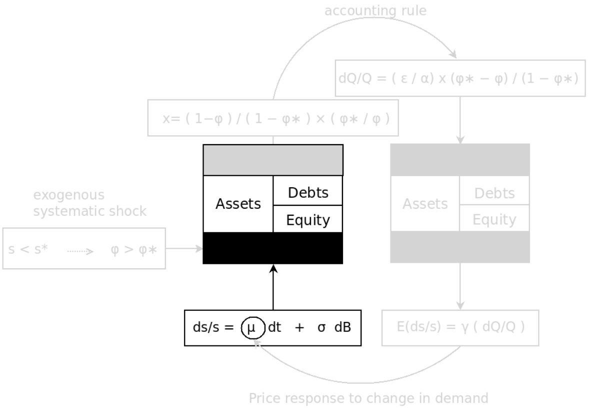
Price response to change in demand

$\gamma \geq 0$: captures the responsiveness of the price to changes in quantity.

Perturbation Analysis. Exogenous Price shock

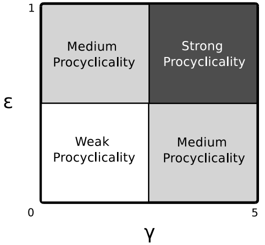


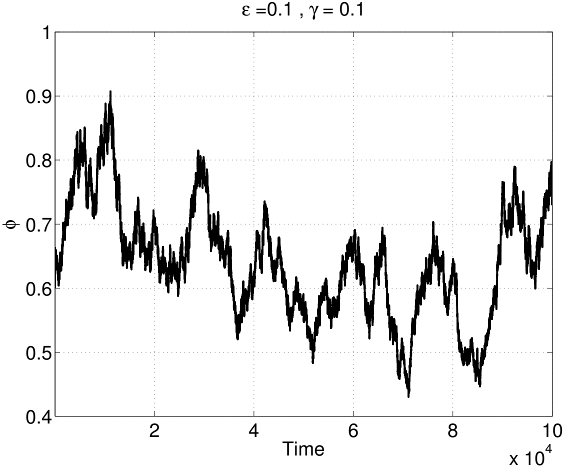
Perturbation Analysis. Exogenous Price shock



Perturbation Analysis. Exogenous Price shock

The Leverage-Price Cycle may amplify the initial shock ! The amplification depends on the interplay between (ε, γ)





Perturbation Analysis. Exogenous Price shock

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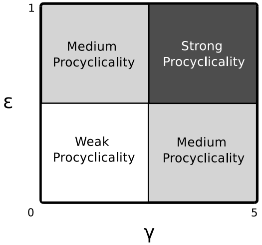
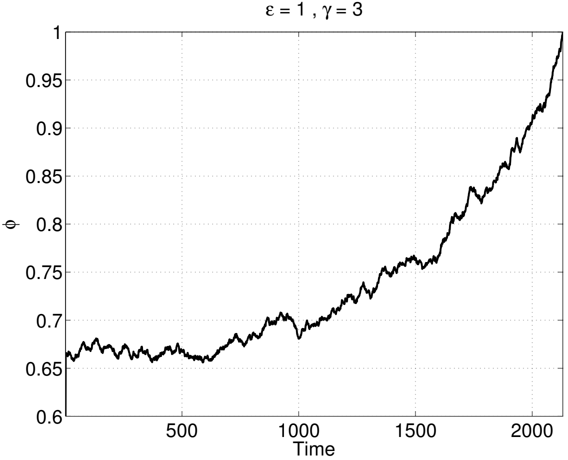
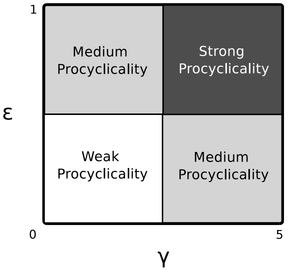


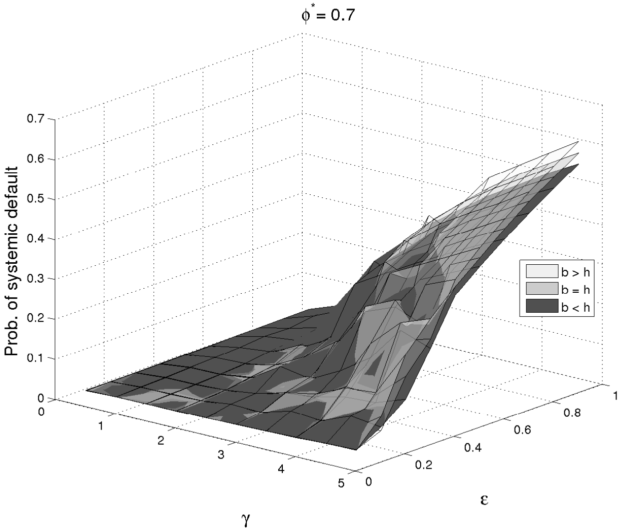


Results

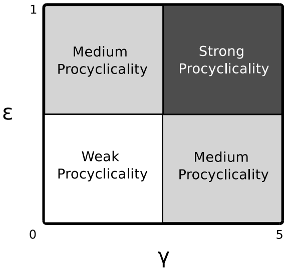
Results

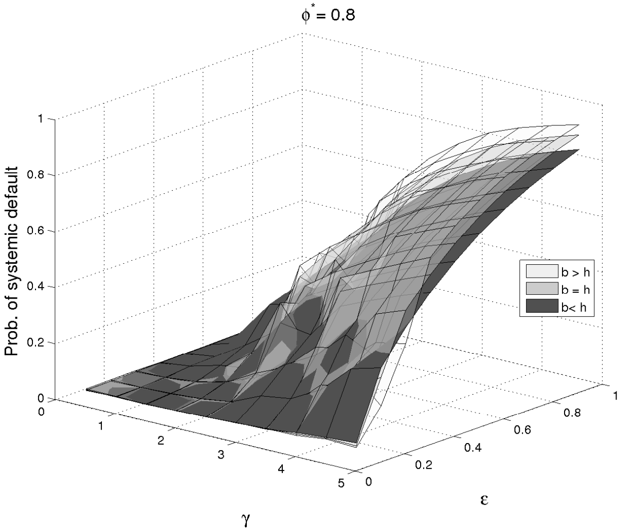
Results


1
Medium Procyclicality
Strong Procyclicality
ε
Weak Procyclicality
Medium Procyclicality
0
5
γ

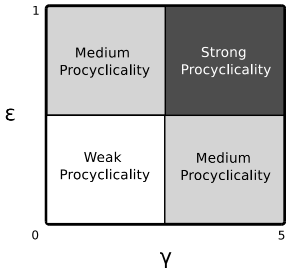

ϕ* = 0.7
Prob. of systemic default
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
γ
0
0.2
0.4
0.6
0.8
1
ε
b > h
b = h
b < h

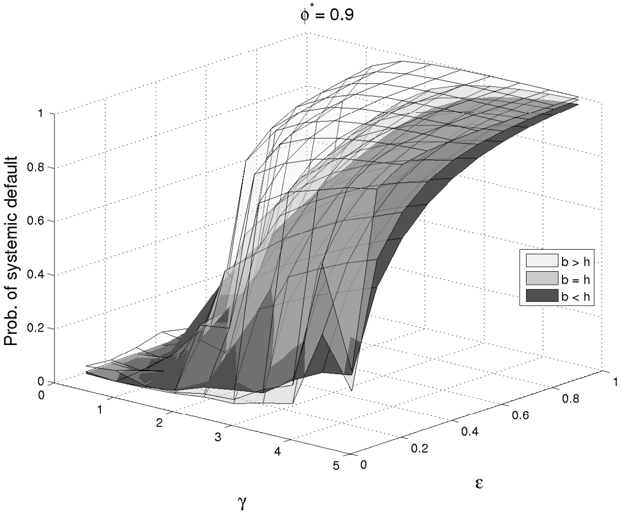
Results


1
Medium Procyclicality
Strong Procyclicality
Weak Procyclicality
Medium Procyclicality
ε
0
5
γ


$\phi^* = 0.8$
Prob. of systemic default
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
γ
0
0.2
0.4
0.6
0.8
1
ε
b > h
b = h
b< h

Results


1
ε
0
5
γ
Medium
Procyclicality
Strong
Procyclicality
Weak
Procyclicality
Medium
Procyclicality


$\phi^* = 0.9$
Prob. of systemic default
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
γ
0
0.2
0.4
0.6
0.8
1
ε
b > h
b = h
b < h

Results

Results.

In the presence of an aggregate asset-price shock:

- A strong compliance with capital requirements, usually alleged to be procyclical, does not increase systemic risk unless the asset market is illiquid.
- When the asset market is illiquid, even a weak compliance with capital requirements increases significantly systemic risk.

Conclusions

Conclusions

Conclusions

Conclusions

- Policy makers should employ macro-prudential supervisory risk assessment policies in coordination with monetary policies to compensate for the effect of market-wide liquidity in the presence of aggregate shocks.

JM Paper 2

DebrRank: Too Central to Fail? Financial Networks, the FED and Systemic Risk

coauthored with Battiston S., Puliga M., Kaushik R., Caldarelli G.

Introduction

DebtRank is a novel indicator to identify

1. Systemically Important Financial Institutions (SIFI);
2. Group of SIFI

DebtRank overcomes some limitations in

1. standard stress-test tecniques at central banks;
2. standard complex network measures (betweeness, centrality, etc.)

References:

- Haldane, A. G. and May, R. M., (2011), Systemic Risk in banking ecosystems, Nature 469: 351–355
- Vitali, S., Glattfelder, J. and Battiston, S. (2011), The network of global corporate control, PloS one, 6-10
- Nicosia, V., Criado, R., Romance, M., Russo, G., and Latora, V., (2012), Controlling centrality in complex networks, Nature Sci. Rep., 2
- Schweitzer, F. et al (2009), Economic Networks: the new challenges, Science, 325, 422–5

DebtRank. Impact Matrix and the Logic behind

- $\mathbf{A_{ji}}$: Exposure of j to i, e.g. amount invested (lended) by j in (to) i.
- $\mathbf{A_j} = \sum_\mathbf{i} \mathbf{A_{ji}}$: Total investment of j in interbank funding activities.
- $\mathbf{E_j}$: Core capital of j (Tier 1 capital).
- $\mathbf{W_{ij}} = \min\{1, A_{ji}/E_j\} \in (0, 1]$: Impact matrix of i on j, $\mathbf{W}$.
- $\mathbf{v_j} = \mathbf{A_j}/\sum_\mathbf{l} \mathbf{A_l}$: Relative economic value of j w.r.t to the total interbank market value, i.e.: market share of node j.

DebtRank: the logic behind

- A node i is more central if it has a strong impact (large W_{ij}) on many other central nodes (large v_j): recursive!
- Each node propagates its distress only once (we tame reverberations)

All formulas at doi:10.1038/srep00541

Widgets and infographics at: `http://ethz.focproject.net:8080/widget`

DebtRank. Impact of i to its indirect successors

- **Feedback Centrality**: Adapting notion of Feedback Centrality to financial distress: a node is more important if it impacts on many high value and important nodes

$$I_i = \sum_j W_{ij} v_j + \beta \sum_j W_{ij} I_j$$

where $\beta < 1$ is a damping factor.

$$I = (I - \beta W)^{-1} W v$$

As long as $\lambda(W) < 1/\beta$!!

- **Problem**: by imposing row-stochasticity we could not compare values across time. Because $W_{ij} > 0$ and $W_{ji} > 0$, the impact of i to j may hit back to i. Multiple Cycles $\Longrightarrow$ impact > 1 !!
- **Solution**: keep impact matrix as it is and tame cycles by excluding walks already visited once

DebtRank. Solution in details

State Variables of distress:

- $h_i \in [0,1]$: 0= healthy, 1=default.
- $s_i \in \{U, D, I\}$: U=Undistressed, D=Distressed, I=Inactive.

For all i and for $t \geq 2$:

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_j W_{ji} h_j(t-1)\right\}, \text{ where } j \,|\, s_j(t-1) = D$$

$$s_i(t) = \begin{cases} D & \text{if } h_i(t) > 0 \;\&\; s_i(t-1) \neq I \\ I & \text{if } s_i(t-1) = D \\ U & \text{otherwise,} \end{cases}$$

- $h_i(1) = \psi \in [0,1]$, where ψ = initial parameter of distress/shock.
- Update order: all h_i are updated in parallel before all s_i.
- a node in D at time t moves to I at time $t+1 \implies$ no cycles !!
- After T steps, the dynamic stop when all nodes are in I or U states

DebtRank. Formula

DebtRank Formula

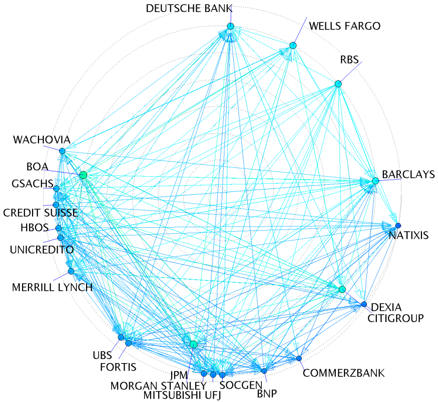
$$R_i(t) = \sum_j h_j(T)v_j - \sum_j h_j(1)v_j$$

R_i measures the distress induced in the system, excluding the initial distress.

Application: an exercise on FED data + BvD data

- Take banks' investment in each others equity share as a proxy of banks' **exposures**. Data from Bureau van Dijk's ORBIS database (http://www.bvdep.com/orbis.html).
- Focus on the largest borrowers from the FED in 2008-2010
  - 22 inst., peak lending 1.2 USD trillions, total assets 20 USD trillions
- Incorporate dynamics of core capital (take market capitalization as a proxy of core capital)

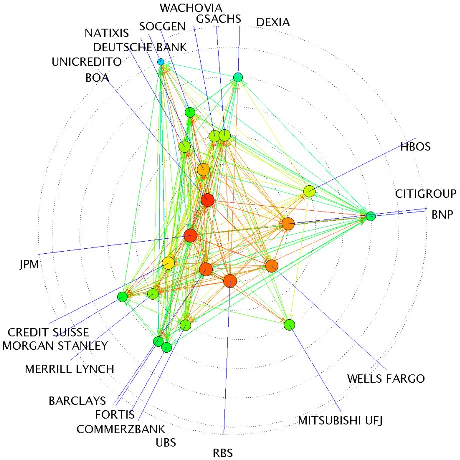
DebtRank



- more **central**/**red**/**big** the node is: more systemically important is the institution.

Evolution of DebtRank over time (Aug.2007–Jun. 2010).

DebtRank



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- not just a ranking but **monetary value** of systemic loss

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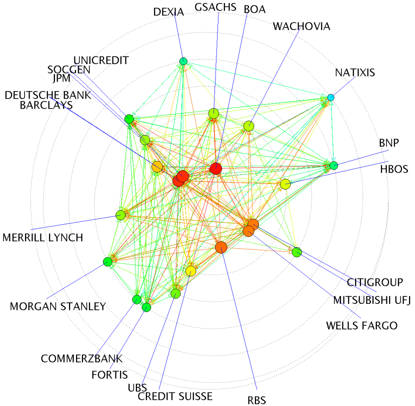
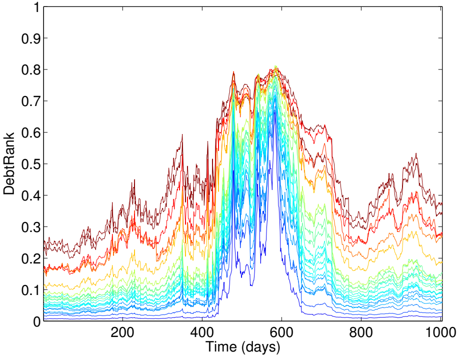
DebtRank



- more **central**/**red**/**big** the node is: more systemically important is the institution.
- not just a ranking but **monetary value** of systemic loss
- **overcomes limitations** of state-of-the art approaches on default-only algo, eigenvec centrality, impact centrality, hub and authorities and akin

Evolution of DebtRank over time (Aug.2007–Jun. 2010).

DebtRank vs Other Measures

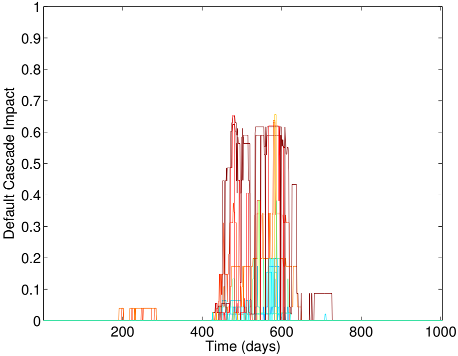


DebtRank (Aug.2007–Jun. 2010).

Advantages of DebtRank w.r.t other measure of feedback centrality and default cascade impact:

1. individual and **groups**
2. impact vs **vulnerability**
3. complement to **Early Warning System**
4. extentions towards **VaR** and **ES**

DebtRank vs Other Measures



Default Cascade Impact (Aug.2007–Jun. 2010).

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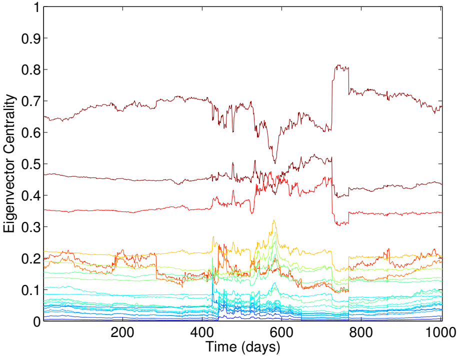
DebtRank vs Other Measures



Eigenvector Centrality (Aug.2007–Jun. 2010).

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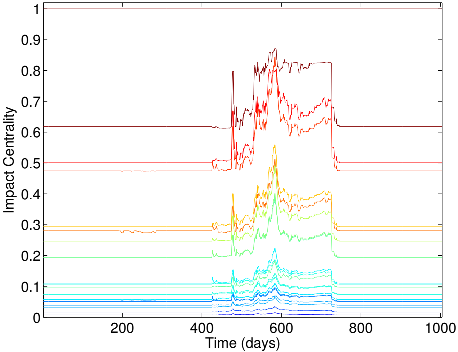
DebtRank vs Other Measures



Impact Centrality (Aug.2007–Jun. 2010).

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3. complement to **Early Warning System**
4. extentions towards **VaR** and **ES**

Conclusions

- Network effects matter for distress propagation: SIFI and counterparty risk
- DebtRank is a centrality-inspired algorithm to assess SIFI in network context, overcoming some limitations of state-of-the-art stress-testing
- From Too-Big-to-Fail to Too-Central-to-Fail
- Currently: a new method to evaluating VAR and ES in a network context

Applications: SIFI

Run DebtRank and GroupDebtRank to assess systemic impact of one or more institutions

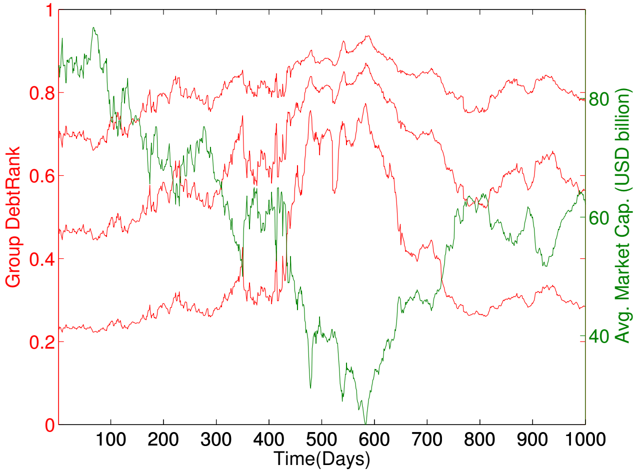
- FED data (the Sci Rep paper)
- In several countries, Central Banks maintain databases:
  - Balance sheet interlocking exposures, external assets, core capital
- ECB, Bank of Italy, Deutsche Bundesbank, Bank of Brazil, Bank of Japan,...

Group DebtRank

Recipe

1. A selected group of institutions is hit by a shock: for each a certain fraction $\phi_i < 1$ of equity vanishes
2. Propagate distress according to impact matrix as before (closed walks traversed only once)
3. Test various values of ϕ and impact scaling factor α

GroupDebtRank



GroupDebtRank