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Statistical inference for letter network of Reformation

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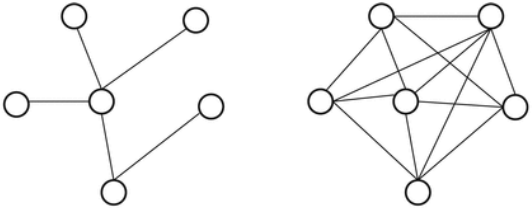
The problem: unobserved data

- Studying system involves **unobserved data**
- Resulting network is biased
- Network reconstruction is hard, unsolved problem
- Intermediate step: **inference**



➔ Which factors drive the network?

The European Reformation (1517-1648)

- Transformative movement
  - Division of Catholic Church
  - Major changes in the socio-political system

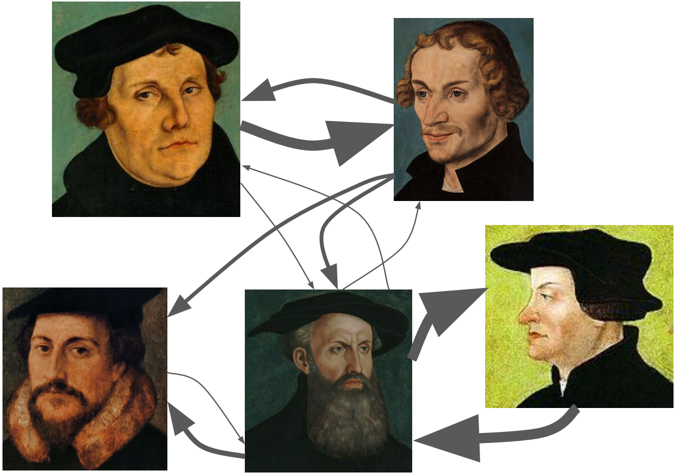
! Letters were the main means of communication.

! Use them to study the social system in 16th century Europe

Martin Luther's posting of his 95 theses to the church in Wittenberg (1517)

The letter correspondence network of reformers

- **Data**: 20,000 letters, 3,000 people, sending- and (receiving) dates + locations, 1510 - 1575
- **Network**: directed multi-edge network of interactions
  - nodes: reformers
  - edges: letters



Schematic representation of a sample from the letter correspondence network

Back to the problem

- Which factors drive the network?
- **Use ERGM?**
  - **Problem**: Only for binary edges
- **Use regression?**
  - $\mathbf{y} = \beta_0 + \beta_1 \mathbf{x}_1 + \ldots + \beta_p \mathbf{x}_p + \varepsilon$
  - E.g. $\mathbf{y}$: number of letters per reformer pair, $\mathbf{x}_i$: social relations, age, etc.
  - **Problem 1**: Networks do not meet independence assumption
  - **Problem 2**: GLMs do not fix number of edges

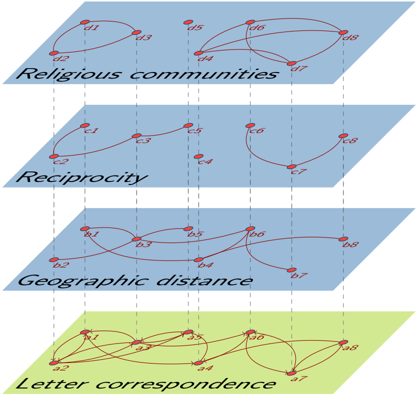
➔ Network regression:

infer interactions (letter connections) from relations

The role of geographic distance on letter correspondence

Research question

How does **geographic distance** affect the letter correspondence, i.e. the network topology?

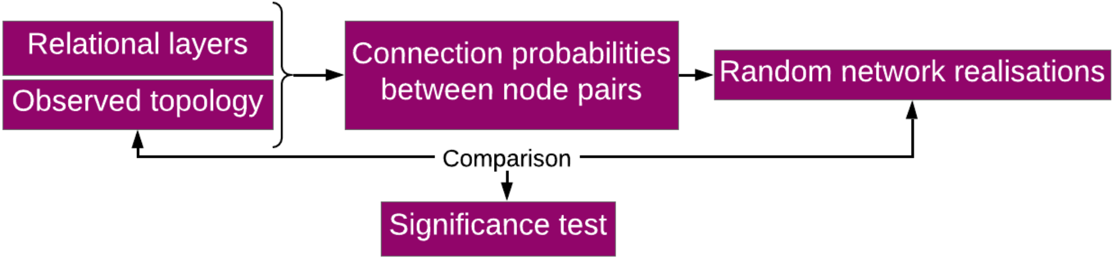


Social relations (**R**) between sender and receivers to be tested:

- **Geographic distance** (tested):
  Long distances: letters are convenient but costly;
  Short distances: letters are inconvenient but cheap
- **Reciprocity** (control):
  Social norm of rewarding kind actions
- **Religious communities** (control):
  Support for same/different religious denominations
  E.g. Lutherans, Reformed, Calvinists, Baptists, etc.

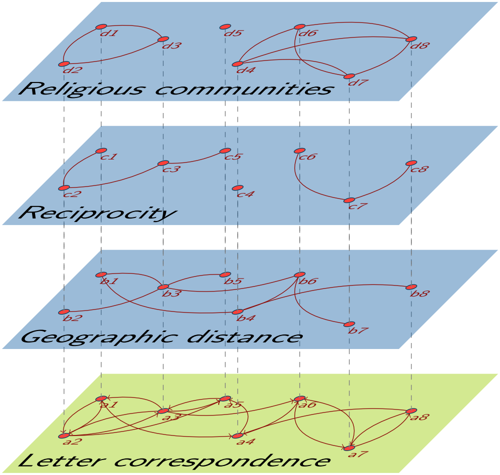
Network regression Casiraghi, 2017; Casiraghi et al., 2016

- Statistical model based on **generalised hypergeometric network ensembles** (gHypE)



Network regression output

- **Regression coefficients** β_k
  - Quantify importance of relational layers
- **Propensity matrix** $\mathbf{\Omega}$
  - $\mathbf{\Omega} := \prod_{k=1}^{K} \mathbf{R}_k^{\beta_k}$
    where each relational layer corresponds to one $\mathbf{R}_k$
  - **Odds ratio** $\mathbf{\Omega}_{ij}/\mathbf{\Omega}_{mn}$: How much more likely are nodes i and j to be connected compared to nodes m and n?



Predictor construction

❶ Geographic distance

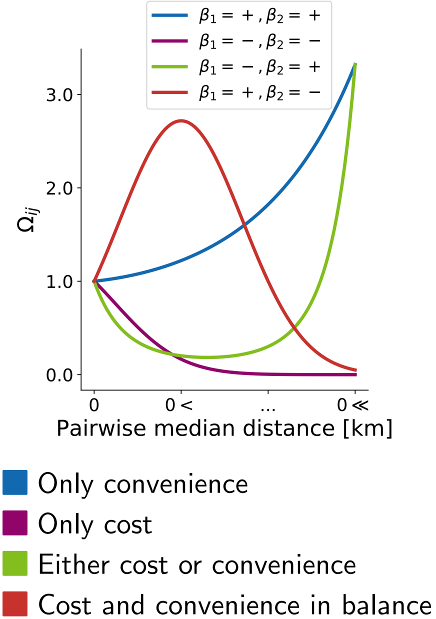
- **cost** (distance ↑, #letters ↓);
  convenience (distance ↑, #letters ↑)
- $\mathbf{R}_{ij}^{(1)} = e^{dist_{ij}}$, $\mathbf{R}_{ij}^{(2)} = e^{dist_{ij}^2}$
- $\mathbf{\Omega} = \mathbf{R}^{(1)\beta_1} * \mathbf{R}^{(2)\beta_2}$: Covers all possible combinations of cost and convenience

❷ Reciprocity

- $\mathbf{R}^{(3)} = \mathbf{A}^T$ (change statistic Snijders, 2006)
- $\mathbf{R}_{ij}^{(3)}$: number of letters i would have to send to j in order to answer each letter of j to i

❸ Religious communities

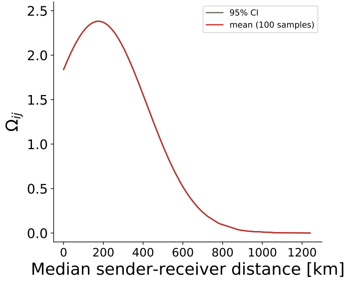
- Assume homophily
- Same: $\mathbf{R}_{ij}^{(4)} = 10$, different: $\mathbf{R}_{ij}^{(4)} = 1$

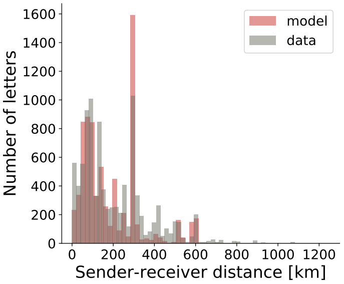


Results: reduced model $\Omega_{ij} = (e^{dist_{ij}})^{\beta_1} * (e^{dist_{ij}^2})^{\beta_2}$

	reduced
Distance	
Linear distance	7.885 (0.159)***
Quadratic distance	−17.918 (0.405)***
AIC	43427.830
McFadden $pseudo - R^2$	0.009

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$





- **Optimal intermediate distance**: At 168km people are most likely to send letters.
- **Odds ratio**: $\Omega_{168km}/\Omega_{0km} = 1.29$, $\Omega_{168km}/\Omega_{1000km} = 28809$

Results: full model

	reduced	full
Distance		
Linear distance	7.885 (0.159)***	−3.354 (0.176)***
Quadratic distance	−17.918 (0.405)***	5.032 (0.388)***
Controls		
Reciprocity		0.461 (0.004)***
Religious homophily		0.276 (0.016)***
AIC	43427.307	33989.210
McFadden $pseudo - R^2$	0.009	0.224

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

- The **full model is better** than the reduced as the smaller AIC shows.
- The **sign flip** of the distance predictors shows that the controls are essential.

Summary

1 Insights on the letter correspondence network of reformers

- People are likely to write letters if they live close to or far away from each other

2 Benefits of network regression

- Multi-edges, interdependence, fixed edge count
- Can deal with missing data ($\mathbf{R}_{ij} = 1$ ➔ β has no effect)
- Construction of predictors is not restricted: Use any kind of quantifyable relation, test hypotheses.

Take home message

Network regression:
Relations explain interactions

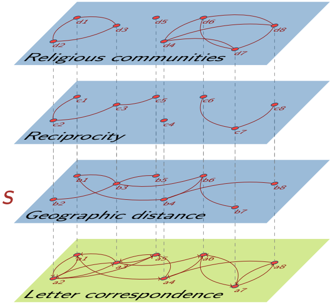
3 Outlook

- Address instability of model
- Tailor predictor selection towards specific theories of historical research
- Use ensemble approach for edge reconstruction (goodness-of-fit needed)

Network regression

gHypE depends on four $N \times N$ matrices

- **Adjacency matrix $\mathbf{A}$**: given
- **Combinatorial effects matrix $\boldsymbol{\Xi}$**: covered by configuration model
- **Propensity matrix $\boldsymbol{\Omega}$**: to be computed from predictor matrices $\mathbf{R}'s$



$$\boldsymbol{\Omega} := \prod_{k=1}^{K} \mathbf{R}_k^{\beta_k}$$

- **Odds ratio $\boldsymbol{\Omega}_{ij}/\boldsymbol{\Omega}_{mn}$**: How much more likely are nodes i and j to be connected compared to nodes m and n?
- Each **predictor matrix $\mathbf{R}_k$** encodes one relational network layer
- $\mathbf{R}_{ij}$ can quantify the relation directly or encode some specific assumptions
- The larger $\mathbf{R}_{ij}$ the larger the propensity to be connected of node pair ij
- β_k are the estimated regression coefficients quantifying the importance of one layer

Collinearity causes sign flip

	Reciprocity	Religion
Distance		
Linear distance	$-3.758\ (0.172)^{***}$	$8.283\ (0.164)^{***}$
Quadratic distance	$5.584\ (0.381)^{***}$	$-18.552\ (0.410)^{***}$
Controls		
Reciprocity	$0.457\ (0.004)^{***}$	
Religious homophily		$0.219\ (0.016)^{***}$
AIC	34229.532	43271.460
McFadden $pseudo - R^2$	0.219	0.012

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

- Corr(linear distance, reciprocity) = 0.265
- Corr(quadratic distance, reciprocity) = 0.268
- Corr(linear distance, religion) = -0.022
- Corr(quadratic distance, religion) = -0.021
- Corr(reciprocity, religion) = -0.002