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Identifying Spam in Reviews

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Synonyms

[Features](#); [Online shopping](#); [Opinion spam](#); [Review spammer](#); [Social networking](#); [Spam detection](#); [Spam review](#); [User-generated content](#); [Water army](#)

Glossary

Features	A set of attributes indicating the spamming behavior
Review	A malicious user who write
Spammer	fraudulent reviews
Spam	Identify spam reviews, users, or
Detection	groups

Spam	A deceptive review to manipulate the
Review	opinion about the product
Water	A group of ghostwriters paid to post
Army	fake reviews

Definition

The fake reviews target at promoting or demoting the sale of products in e-commerce sites, and attracting attention or triggering curiosity in social networking sites, by creating and spreading purposeful comments. Hence, the goal of spam detection is to identify spam objects, including review/opinion spam, spam users, and spammer groups, from reviews.

Introduction

Online reviews are actually a kind of user-generated content (UGC) and hence provide a voice for customers to praise or criticize products, services, and even offline shops. Nowadays, when one wants to buy a product or eat out, most probably, one will first read reviews about several online shops or nearby restaurants. Then, one will select one shop or restaurant that has a mass of positive reviews to consume. Unsurprisingly, online reviews are strongly correlated with product sales (Forman et al. 2008). Driven by these financial incentives, an increasing number of biased or

malicious reviews appear on e-commerce sites (e.g., Amazon, eBay, and Taobao), online-to-offline sites (e.g., Yelp and Dianping), and travel booking sites (e.g., TripAdvisor, hotels.com, and Ctrip). Some research reported that up to 6% of reviews on sites like Yelp and TripAdvisor may be deceptive (Ott et al. 2012), and roughly 8% URLs included in Twitter messages direct users to scams, malware, and phishing sites (Grier et al. 2010). These spam or misleading reviews are seriously threatening the sustainable development of online shopping and social networking. Therefore, fake reviews have been acknowledged as a critical challenge by both the research community and the e-commerce industry.

Initially, the content of spam reviews is often very similar with each other, since to create multiple fake reviews with different content is cost- and time-consuming (Heydari et al. 2015). These duplicate reviews can be easily generated and propagated by bots. In earlier research, the spam detection methods try to find similar content of a set of reviews by using text mining, and thus the reviews containing duplicate text could be identified as spam reviews (Jindal and Liu 2007a, b, 2008). Along this line, a great deal of content-based spam detection techniques are subsequently proposed (Ott et al. 2011; Lin et al. 2014a, b). These techniques have defined a variety of linguistic features based on review content and employed various classification models for detection.

Due to the rapid development of spam detection techniques, the spam objects to be detected are also gradually expanded. More and more work begins to detect spam users and even spammer groups, rather than reviews themselves. The nature of this problem has been clearly described as a classification problem (Fayazi et al. 2015). That is, most of the detection methods first try to define a set of features for indicating the abnormal spam behavior, and hence try to design a befitting machine learning model to train a classifier (Mukherjee et al. 2012, 2013a; Lam and Riedl 2004; Lim et al. 2010; Wang et al. 2011). It is noteworthy that features about different targets can usually be fused together. For example, to

detect spam users might use the linguistic features of review content (Mukherjee et al. 2013a). In contrast, to identify a group of fake reviews might use both the behavioral features of reviewers and the characteristics of multiple reviews (Mukherjee et al. 2012).

In recent years, crowdsourcing platforms have emerged as the broker for intelligently guiding large numbers of people to create fake reviews on major e-commerce platforms in exchange of a fee (Wu et al. 2015; Fayazi et al. 2015). As a result, both fake reviews and their authors are becoming more and more tricky, since fake reviews are carefully written by different users other than copying the existing fake reviews. Meanwhile, these users are always legitimate, i.e., having many true purchasing records and releasing many true reviews, but they occasionally create fake reviews when accepting the task on crowdsourcing platforms to make a profit. So, it is difficult to identify these spam reviews and users in clever disguise by using features only. To address this challenge, many recent studies (Fakhraei et al. 2015; Fayazi et al. 2015; Wu et al. 2013, 2015) propose to employ the relation for defining new topological features or design hybrid learning framework, in order to enhance the detection performance.

Established in a large number of abovementioned studies, this chapter has two goals: first, it aims to draw a clear roadmap of algorithms and ideas used for spam detection from reviews; second, it aims to sketch several important problems that are deserved for future research.

Key Points

Almost all of existing work targets to detect three kinds of fake or malicious objects (Heydari et al. 2015): review/opinion spam, spam users, and spammer groups. Deceptive reviews are superficial components of hidden spammers. So, these malicious targets to be detected are actually bound up with each other. Meanwhile, the detection techniques for three targets are also in common with each other. The spam detection problem is

usually modeled as a classification problem (Fayazi et al. 2015), where the goal is to assign a class label of *spam* or *normal* to a candidate object o based on a classifier c :

$$c : o \rightarrow \{\text{spam}, \text{normal}\}. \quad (1)$$

The object o might be a review, a user, or a group of reviews/users. To effectively build the classifier c , there exist several critical challenges including: (i) to identify valid ground-truth training data, (ii) to construct discriminative features based on various data sources, and (iii) to train the classification model by using various machine learning techniques.

Historical Background

Various spam comes with the development of mainstream components of the Web. In the early years, an extensive body of research is devoted to identify, characterize, and prevent email spam (Sahami et al. 1998; Pitsillidis et al. 2010). To filter email spam, a number of techniques have been developed including IP blacklisting, domain and URL blacklisting, and filtering on email contents. With the prevalence of Web search engines that usually adopt the link-based ranking methods, e.g., HITS and PageRank, many tricks have been attempted to boost page rankings (Zhou and Pei 2009). This kind of spam is termed as Web spam or spamdexing which is recognized as one of the key challenges for search engine industry (Cheng et al. 2011). So far, Web spam taxonomy has been clearly delineated (Gyongyi and Garcia-Molina 2005) and a vast number of detection algorithms have been presented. The cited review (Spirin and Han 2012) can help interested readers understand the work on Web spam detection quickly.

About 10 years ago, e-marketing sites as well as social Web sites started to become important components of the Web. Both kinds of sites rely on user-generated content (UGC) that makes them incredibly dynamic and tempting targets for spam (Heymann et al. 2007). In general, there are mainly two kinds of spam content in e-commerce

and social networking sites: (i) the malicious content with scams, malware, and phishing attacks (Jagatic et al. 2007; Grier et al. 2010); (ii) the purposeful comments or reviews for attracting attention or promoting the ranking/sale of products (Lee et al. 2010; Chen et al. 2013; Liu et al. 2013). According to the analysis on Twitter spam (Grier et al. 2010), the malicious content is usually wrapped by a spam URL, which points to some malicious pages, such as phishing websites, pages hosting malicious software, or attempting to exploit a user's browser, and websites advertising pharmaceuticals, software, or adult content. So, detecting spam URLs is very similar to detect email spam, and thus URL blacklisting becomes an effective technique for monitoring and detecting spam URLs.

The purposeful or unfair reviews try to deliberately mislead readers in order to promote or demote some target products (Mukherjee et al. 2012). The opinion spam is more tricky than the malicious content, since it is difficult to be judged by a single metric, such as the spam URL pointing to a malicious page. Indeed, detecting opinion spam and its related spam users or groups needs to construct multifold features based on review contents and user behavior, and transform the detection problem to a classification problem as summarized by Eq. 1. In what follows, we shall introduce the state-of-the-art methods in this area, including three key points: (i) feature construction, (ii) detection algorithms, and (iii) training data preparation.

Feature Construction

Finding right features largely determines the detection performance. However, feature construction is both *data-specific* and *task-specific*. That is, a right feature should be computable on the available data, and it can serve for the pre-defined detection task. A wide range of features that are likely linked with spamming have been presented in the literature. These features can be roughly summarized into four categories: (i) review features, (ii) user features, (iii) group features, and (iv) topological features. In this

Identifying Spam in Reviews, Table 1 Representative features for reviews

Type	Name	Polarity	Description	Source
Behavior	Rank	L	Rank order among all the reviews of product	Jindal and Liu (2008)
	DEV	H	Deviation between rating of the review and the average rating.	Mukherjee et al. (2013a)
	EXT	H	Extremity of rating: 1 for ratings {4,5}, 0 otherwise	Rayana and Akoglu (2015)
	ETF	H	Early time frame: early review can increase impact (see Eq. 2).	Mukherjee et al. (2013a)
	ISR	H	Is singleton? If review is user's sole review, then 1; otherwise 0	Rayana and Akoglu (2015)
Text	LEN	L	Review length in words	Li et al. (2011)
	PP1	L	Ratio of 1st person pronouns ("I," "my," etc)	Li et al. (2011)
	OW	L	Ratio of objective words (by WordNet)	Li et al. (2011)
	RCW	H	Ratio of ALL-capitals words	Li et al. (2011)
	RC	H	Ratio of capital words	Li et al. (2011)
	SW	H	Ratio of subjective words (by WordNet)	Li et al. (2011)
	RES	H	Ratio of exclamation sentences containing "!"	Li et al. (2011)

section, we shall introduce a few universal features in each category.

Review Features

The *metadata* of a review usually includes rating, timestamps, and review text (Heydari et al. 2015; Rayana and Akoglu 2015), which is helpful to define review features. We categorized review features in the literature (Jindal and Liu 2008; Mukherjee et al. 2013a; Rayana and Akoglu 2015; Li et al. 2011) as behavior-based and text-based, respectively. Meanwhile, we use the "polarity" to indicate whether a high (H) or a low (L) value is more suspicious for each feature. Table 1 shows several representative features for reviews. Most of them are self-explanatory, and hence we omit detailed explanation for brevity. Instead, we only introduce one feature: early time frame (ETF).

A lot of evidence has shown that the early reviews can greatly impact the users' opinion on a product, and hence spammers prefer writing early reviews. The ETF is designed to capture this spamming characteristic. Assuming r_u denote a review written by user u for the product p , the ETF is

$$ETF(r_u, p) = \begin{cases} 0, & \text{if } L(a, p) - A(p) > \delta \\ 1 - \frac{L(a, p) - A(p)}{\delta}, & \text{otherwise} \end{cases} \quad (2)$$

where $L(a, p)$ is the posting time of r_u , $A(p)$ is p 's launch date, and δ is a threshold. In (Mukherjee et al. 2013a), δ is suggested to be 7 months for Amazon data.

User Features

Similar to review features, we also list a set of representative features for users in Table 2. Besides behavior and text-based features, there is a new type of features, i.e., social-based, to depict spamming characteristic of users in social networking sites. For instance, some users like entertainment stars might buy a number of zombie fans to amplify her influences on Weibo (Liu et al. 2013), and thus NFF and RFF explained in Table 2 emerge as important features. Furthermore, as reported in (Grier et al. 2010; Lee et al. 2010), users who often release tweets containing URLs and hashtags are very suspicious, which gives birth to features RTU and RTS.

The reviewing bursiness (BST) is used to the time period between the earliest review and the latest review of a user.

$$BST(u) = \begin{cases} 0, & \text{if } L(u) - A(u) > \tau \\ 1 - \frac{L(u) - F(u)}{\tau}, & \text{otherwise} \end{cases} \quad (3)$$

where $L(u) - F(u)$ is the number of days between last and first review of a user u , and τ is a threshold

Identifying Spam in Reviews, Table 2 Representative features for users

Type	Name	Polarity	Description	Source
Behavior	MNR	H	Max number of reviews/tweets 1 day	Mukherjee et al. (2013a)
	PR/NR	H	Ratio of positive (4–5 star)/negative (1–2 star) reviews	Mukherjee et al. (2013b)
	BST	H	Reviewing burstiness as defined in Eq. 3	Mukherjee et al. (2013a)
	BRR	L	Ratio of the reviews in a burst pattern to the total reviews	Fei et al. (2013)
	ARD	H	Average rating deviation of user's reviews	Lim et al. (2010)
	WRD	H	Weighed rating deviation	Lim et al. (2010)
	AVP	L	Ratio of Amazon verified purchase	Fei et al. (2013)
Text	ARL	L	Average review/tweet length in number of words	Mukherjee et al. (2013b)
	ACS	H	Average content similarity among one's reviews	Lim et al. (2010)
	MCS	H	Maximum cosine similarity among all review pairs	Mukherjee et al. (2013b)
Social	ATD	H	Average tweets per day	Lee et al. (2010)
	NFF	L	Number of followers/followees	Liu et al. (2013)
	RFF	H	Ratio of the number of followers/followees	Lee et al. (2010)
	RBF	H	Ratio of bidirectional friends	Lee et al. (2010)
	ROR	L	Ratio of original/reposted tweets	Liu et al. (2013)
	RTU	H	Ratio of tweets containing URL	Lee et al. (2010)
	RTS	H	Ratio of tweets containing special characters (“@,” “#,” etc.)	Grier et al. (2010)

set to be 28 days for Amazon data (Mukherjee et al. 2013a). BST indicates that if all reviews are posted within a very short burst, it is likely to be a spam inflection.

Group Features

To improve the effectiveness of spamming, review spammers might be well organized and implement spamming after a premeditated planning. A few studies have been devoted to detect review spamming groups as well as group anomaly in social media (Mukherjee et al. 2012; Wang et al. 2016; Yu et al. 2014). Intuitively, group spam behavior indicators can be parallel extended based on user features. As features of each review and user in a group have been defined (see Table 1 and 2), some aggregation functions, i.e., $\max(\cdot)$ and $\text{avg}(\cdot)$, can be utilized to compute features of the group. Let us take group early time frame (GETF) as an example. With ETF as shown in Eq. 2, GETF is

$$\text{GETF}(g) = \max_{u \in g, p \in P_g} (\text{ETF}(r_u, p)), \quad (4)$$

where g denotes the group of users and P_g is the set of products rated by all group members. Similarly, many features, such as BST, ARD, WRD, ACS, etc., can be extended for groups.

Topological Features

In social media, the graph can be directly constructed based on social relations. These relations might be *heterogeneous* for representing different activities on the website, such as sending friend requests or messages, viewing another user's profile, being friend/follower/followee of another user, and so on. In online shopping sites, however, the relation between users is usually hidden in the user-product reviewing relation. For example, the relation between users could be created when they frequently rate the same products, e.g., over 20 times. Nevertheless, the graph among users is easily obtained in different spam detection scenarios, which provides the metadata for computing topological features.

The topological features attempt to describe the importance of every node in the graph, and

they are usually derived from the graph analytics methods. Commonly used topological features include (Fayazi et al. 2015; Fakhraei et al. 2015):

- *Degree*. The total degree, in-degree, and out-degree of every node.
- *PageRank*. By running PageRank algorithm, a score for each node can be regarded as a feature. The important nodes receiving more links from other nodes have the higher score.
- *Centrality*. The greater the number of paths in which a node participates, the higher centrality value of this node. There are lots of metrics that can be used as the centrality features (Costa et al. 2007), including k -core, betweenness centrality, eigenvector centrality, Katz centrality, Freeman’s closeness centrality, etc.
- *Triangle Count*. The triangle count of a node is the number of triangles (a complete subgraph of three nodes) in the graph the node participates in.
- *Community*. If using graph clustering algorithms to partition nodes into a number of communities, the community ID can be used as a categorical feature. The most coarse-grained clustering is to find connected components (Fakhraei et al. 2015).

To sum up, the underlying assumption of topological features is that the spamming nodes are more important in the graph, i.e., having higher connectivity, centrality, influence, etc. It is naturally accord with the target of group spam detection, that is, spammers are likely to have strong connections with each other.

Detection Algorithms

According to Eq. 1, the review spam detection problem is to predict whether an object with an unobserved label is spam or not. Since most classifiers could assign a probability to each instance, this classification problem can also be regarded as a ranking problem. That is, this problem is the assignment of a probability to every object for ranking them from the most to the least probable spam. In this section, we

categorize the existing detection algorithms into two classes: (i) supervised and semi-supervised classification, and (ii) unsupervised ranking. When introducing algorithmic details, the target objects to be detected will be no longer distinguished. Instead, we shall select several spam detection frameworks with different principles and algorithms.

Supervised and Semisupervised Methods

With a variety of features, including both user and topological features, the intuitive detection method is to utilize existing classification models, such as Naïve Bayesian, SVM, logistic regression, random forest, and so on. In Fakhraei et al. (2015), sequential k -gram features are further constructed based on a set of graph features, and then many classifiers can be integrated into the detection framework flexibly. Beyond this, many recent studies focus on incorporating the topological information (i.e., pair potentials) with user features (i.e., singleton potentials) into the supervised or semisupervised models.

In Wu et al. (2013), Fayazi et al. (2015), Rayana and Akoglu (2015), and Wu et al. (2015), the network is represented as a pairwise Markov Random Field (MRF), where the joint probability of class labels is written as a product of singleton and pair potentials. Mathematically, if let \mathbf{y} be an assignment of labels to all objects and y_i denote the label of i -th object, the pairwise MRF can be written as:

$$\Pr(\mathbf{y}) \propto \prod_i \phi_i(y_i) \prod_{i,j} \phi_{ij}(y_i, y_j). \quad (5)$$

In Eq. 5, $\phi_i(y_i)$ is called singleton potentials which is determined by a set of user features, while $\phi_{ij}(y_i, y_j)$ is called pairwise potentials which is generated on every edge of the network. The specific definitions of both $\phi_i(y_i)$ and $\phi_{ij}(y_i, y_j)$ are often given based on different assumptions or the requirements of reference models. For example, in Fayazi et al. (2015), $\phi_i(y_i)$ is defined as a Bayesian estimator under the independence assumption of user features, and $\phi_{ij}(y_i, y_j)$ is modeled as an exponential function. However, in Wu et al. (2013), $\phi_i(y_i)$ is represented by using the Logistic

Regression model, and $\varphi_{ij}(v_i, y_j)$ is modeled using the Bernoulli distribution.

To maximize $\Pr(\mathbf{y})$ as shown in Eq. 5, the supervised methods (Wu et al. 2013; Fayazi et al. 2015) only use training data (i.e., all y_i used in Eq. 5 is known), but the semisupervised (Rayana and Akoglu 2015; Wu et al. 2015) utilize both labeled and unlabeled data for model training (i.e., a majority of y_i used in Eq. 5 is unknown and should be estimated during model training).

Unsupervised Methods

Other than the supervised classification, the unsupervised methods target at assigning a ranking score to every object for indicating the likelihood that the object is a spam, without the help of the labeled instances. In what follows, we introduce two kinds of unsupervised models.

Topic Model

The basic idea of topic model is that objects are represented as random mixtures over latent topics, where each topic is characterized by a distribution over features. It is actually a generative probabilistic model. The Author Spamicity Model (ASM) (Mukherjee et al. 2013a) is a typical example of applying the topic model for spam detection. Generally, ASM normalizes continuous user features in $[0,1]$ and models them by a Beta distribution. Then, ASM transforms review features as binary variables and models them by a Bernoulli distribution. ASM further denotes two latent variables as the spamicity of each user and each review, respectively. The objective of ASM is to learn the latent behavior distribution for spam and non-spam clusters along with spamicities of users from the observed features. In other words, the number of latent topics is set as 2. Hence, the Monte Carlo Gibbs sampling is employed for approximate posterior inference, and the Expectation Maximization (EM) is used for estimating parameters.

ASM provides a clear logic of using the topic model to solve the spam detection problem, which is very similar to the clustering analysis. Some other research attempts to use unsupervised clustering for spam detection. In Lee and Zhu (2012), a three-phase detector is designed: it first extracts a subset of effective users by using the matrix

decomposition, invokes K-means to divide the selected users into a number of clusters, and finally employs a metric to judge spamming clusters.

Ranking Model

The earliest detection algorithm using the ranking model is known as PCASelectUsers (Mehta and Nejd1 2009). It uses user-product rating matrix as the unique input, rather than various features. This method transforms user-product rating matrix to a user-user covariance matrix and employs Principal Component Analysis (PCA) to decompose the covariance matrix. Thus, each user is represented as a number of principal components (PCs) with the largest Eigen values, where the sum of squares on PCs is adopted as the ranking score. The underlying assumption of this algorithm is that the rating vectors of spammers are highly similar with each other. With a similar approach, the recent research (Wang et al. 2016) has extended the PCA ranking model to the feature space for identifying spamming groups.

A more general ranking framework called GSRank is presented in Mukherjee et al. (2012). GSRank is designed to spot fake reviewer groups. Based on both group and individual spam features, GSRank models three kinds of relations: group-product, member-product, and group-member, where each relation is represented as a matrix. Meanwhile, GSRank initializes the spamming score for groups, members, and products respectively, forming three vectors. Then, GSRank adopts a PageRank-like procedure to iteratively compute the spamming score of three kinds of objects, where each score vector is modeled as a product of one relation matrix and one score vector. The distinguishing feature of GSRank is to incorporate features into relation matrices and exploit the natural relationships among user, group, and product to derive the spamming score of groups.

Key Applications

Spam reviews have been very prevalent in e-commerce sites and social networking sites, and thus

many commercial websites have organized dedicated anti-fraud teams to filter deceptive reviews (Kc and Mukherjee 2016). Since identifying spam by human experts is very time-consuming, more and more sites begin to develop commercial filter for the automatic identification of spam reviews (e.g., Yelp implemented review filtering a decade ago). However, the existing commercial filters including Yelp's filter are far from perfect. As mentioned earlier, there has been significant amount of work done on spam detection. Clearly, these research findings have a great application prospect on commercial websites for enhancing both effectiveness and efficiency of their commercial antifraud algorithms.

Future Directions

Despite recent advances in spam review detection, there are still a plethora of open issues that need serious and immediate attention. We list several important directions as follows:

- *Correlation analysis on spamming with their social networking.* Existing studies have shown that the spam detection methods could benefit from social network (i.e., topological features). However, it is not clear that how spamming behavior interact with social networking. For example, spammers might be very active on some kinds of social relations yet oscitant on other kinds of relations, and specific malicious users might prefer to use some kinds of relations to commit spam to normal users yet use other kinds of relations to contact with other spammers.
- *Novel detection methods with more heterogeneous data.* There is no doubt that more data can provide more signals for identifying spam. Similar to the aforementioned pairwise MRF model incorporating user features and topological information, we expect an advanced model to integrate more heterogeneous data as a uniform objective function, such as features, network, text, and even trajectory data. Hence, some novel learning framework based on heterogeneous data might be devised.

- *Deployment of advanced detection methods for commercial use.* Many commercial websites need spam filters but the existing commercial filters are still very rough. Therefore, the detection techniques for commercial use should balance the theorization and the practicality. Also, the deployment and the use of some advanced detection techniques on commercial websites will provide more empirical findings and more benchmark data.

Cross-References

- ▶ [Collective Classification](#)
- ▶ [Multirelational Social Networks](#)
- ▶ [Opinion Diffusion and Analysis on Social Networks](#)

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Imputation of Missing Network Data

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Synonyms

[Imputation](#); [Link prediction](#); [Missing data mechanisms](#); [Missing data](#); [Multiple imputation](#); [Non-response](#); [Reconstruction](#)

Glossary

Actor Non-response (Unit Non-response)	Missing all outgoing ties of an actor
Imputation	Substituting missing data by plausible values
MAR	Missing at random
MCAR	Missing completely at random
MNAR	Missing not at random
Multiple Imputation	Repeated stochastic imputation of a dataset to generate multiple completed datasets. These completed datasets are analyzed separately, after which the results of the analysis are

pooled to generate proper estimates of parameters and standard errors

Tie Non-response (Item Non-response) Missing some ties of an actor

Definition

When confronted with missing data, researchers often want to handle the missing observations by substituting plausible values for the missing scores. This practice of filling in missing items is called *imputation* (e.g., Schafer and Graham 2002). Imputation has several advantages: it is more efficient than analyzing complete cases, it gives the opportunity to use information contained in the observed data in predicting the missing scores, and allows analysis using standard techniques and software on a complete(d) dataset that is the same for all following analyses. The idea of imputation “is both seductive and dangerous,” in the words of Dempster and Rubin (1983). “It is seductive because it can lull the user into the pleasurable state of believing that the data are complete after all, and it is dangerous because it lumps together situations where the problem is sufficiently minor that it can be legitimately handled in this way and situations where standard estimators applied to the real and imputed data have substantial biases” (Dempster and Rubin 1983, p. 8).

The shortcomings of imputation are related to bias and uncertainty. Ad hoc imputations can seriously distort data distributions and relationships, and produce biased estimates. Moreover, in subsequent analyses, predicted scores are treated as observed values which lead to overestimating the sample size and underestimation of uncertainty levels. *Multiple imputation* (Rubin 1987) solves the problem of underestimating uncertainty measures (i.e., standard errors). In multiple imputation, each missing value is replaced multiple times (say m) by random draws from the distribution of the missing values given the observed scores. This results in m completed datasets that are identical

except for the imputed values. The m completed datasets are analyzed separately using the same complete-data method, and the m results are combined according to Rubin’s rules for combining estimates and standard errors (Rubin 1987; Schafer and Graham 2002). The final result reflects the extra uncertainty due to missing data by using the differences between the imputations to correct the standard errors.

Multiple imputation is a flexible procedure that retains much of the advantages of single imputation. As an imputation procedure, it separates the missing data handling and the final analysis of the dataset. This simplifies statistical modeling and enables the researcher to expand the data in the imputation model by including a large number of predictors in order to reduce the bias due to systematic differences between responders and non-responders (data *Missing Not at Random*; van Buuren 2012).

Introduction

In many social network studies, missing data constitute a serious problem. Often, popular software packages can only deal with fully observed network data, while others disregard the missing data or treat the missing observations as non-existing. These practices result in (serious) loss of information, leading to decreased statistical power, and may lead to serious bias due to the systematic nature of the missingness (e.g., Schafer and Graham 2002; Graham 2009). Moreover, due to the complex dependencies that exist within networks, missing scores of one actor will influence the local neighborhoods of other actors (directly or indirectly via others). This makes careful treatment of missing network data essential.

Missing data treatment procedures that are common in statistical literature can roughly be classified into four categories: analysis of available data, (re)weighting data, likelihood-based procedures (e.g., the EM algorithm), and imputation. Much is already known about the effects of missingness on (statistical) data analysis and the effectiveness of the various treatment procedures

(e.g., Schafer and Graham 2002; Graham 2009). The effects of missing data treatments on estimating structural properties of social networks are less often studied, although the field is catching up (e.g., Huisman 2009; Koskinen et al. 2010, 2013; Hipp et al. 2015; Wang et al. 2016). In this entry, we investigate in which way imputation can be used to treat missing network data. We translate common imputation strategies to the context of social network data and inspect the effect of imputation on estimating network properties.

Graham (2009) recommends that researchers use missing data procedures from the latter two categories: likelihood-based, or model-based, methods and multiple imputation. He calls these methods the “modern” missing data procedures (p. 555). For social network analysis, “modern” model-based procedures were proposed by Robins et al. (2004), Handcock and Gile (2010), and Koskinen et al. (2010, 2013), who describe model-based approaches (likelihood-based or Bayesian) within the framework of exponential random graph models (ERGMs; see Lusher et al. 2013). In the proposed approaches, values for the missing data are simulated in the course of parameter estimation, and observed statistics are replaced by expected values based on these simulations, in a manner similar to the EM algorithm approach (e.g., Schafer and Graham 2002; Graham 2009). For longitudinal network data, Snijders (2005) proposed a model-based procedure incorporated in stochastic actor-driven models for network evolution. Analogous to the EM algorithm, the model-based procedures can also be used for link prediction and imputation of missing ties.

This entry is concerned with imputation methods. The “modern” imputation procedure that is recommended by Graham (2009) is multiple imputation (Rubin 1987). Although this procedure is generally recommended as the best way to impute, (simple) single imputations can still be useful for specific analyses that do not require hypothesis testing or confidence intervals (Graham 2009). Such analyses are not uncommon for social networks (e.g., blockmodeling). As generating multiple imputations and combining the results of the separate analyses can be a difficult

task, single imputation methods can be useful to treat missing network data (also as a first step to multiple imputations).

Key Points

In this entry, we assume a fixed and known set of actors and a single, binary relation between the actors. The tie variable X_{ij} indicates whether the tie from actor i to j is present ($X_{ij} = 1$) or absent ($X_{ij} = 0$); Žnidaršič et al. 2017 treat missing data in valued networks. The relation can either be directed, from i to j , or undirected, in which case $X_{ij} = X_{ji}$. Additional information on the ties and/or actors may be available in the form of dyadic covariates or actor-attribute variables. In all three types of variables (tie variables, dyadic covariates, and actor attributes) missing values may occur.

We only consider the situation where missing data is caused by non-response (see Kossinet 2006, or Žnidaršič et al. 2012b, for other sources of sampling errors and missing data in the context of social networks) and distinguish two types of non-response: *unit non-response*, where all scores of an actor are missing (ties and attributes), and *item non-response*, where only particular items are missing. When only tie variables (i.e., network data) are concerned, these two types are also called *actor non-response* and *tie non-response*, respectively (Huisman 2009; Žnidaršič et al. 2012a). A special case of item non-response may occur when all outgoing ties of an actor are missing (actor non-response), but attribute information is available, or vice versa. This form of non-response is sometimes called *partial non-response* (de Leeuw et al. 2003). A specific form of partial non-response is common in longitudinal studies, *wave non-response*, which arises when complete network information for an actor is missing at one (or more) measurement moments (Huisman and Steglich 2008; Hipp et al. 2015).

The type of non-response determines the amount of data that is available for each actor. With actor non-response, more information is missing for each actor than with partial or item non-response. An advantage of social network data is that information on the network context

of incompletely observed actors is often available, at least partially, through observed nominations by other actors. This information can be used to analyze (or even “reconstruct”) the network neighborhood of missing actors, and should not be omitted from the analyses. This approach supposes that the observed and missing data are not systematically different and that all necessary information about the missing data can be found in the observed data. In the statistical literature, this situation is known as data that are *Missing at Random* (MAR; e.g., Schafer and Graham 2002). When data are MAR, the probability of missingness is related to the observed data, and not to the missing data. If, in addition, the missingness is not related to the observed data either, the data are called *Missing Completely at Random*. If, on the other hand, the probability of missingness is related to the missing (and therefore unknown) values themselves, the data are *Missing Not at Random* (MNAR). Huisman (2009) and Smith et al. (2017) provide more details on missing data mechanisms for social network data, and Handcock and Gile (2010) give formal definitions.

The “modern” missing data methods of Graham (2009) assume MAR. This means that all information about the missingness is contained in the observed data, and given these data the missing data mechanism is ignorable. In this situation, the causes of missingness do not have to be taken into account (Koskinen et al. 2010). Simple (older) missing data methods only give unbiased results when data are MCAR, which is only realistic when there is no reason to assume that actors differ in their propensity to fill in network questionnaires (Huisman and Steglich 2008).

Historical Background

One of the first studies on the effects of non-response on the structural properties of social networks is the study by Burt (1987). He calls missing data “doubly a curse to survey network analysis” (p. 63), because the complexity of network questionnaires is more likely to generate missing data, and the dependence structure of

the network increases the impact of missing ties. Others followed-up on this study and found, among others, that missing data have a negative effect on network mapping (Borgatti and Molina 2003), underestimate the strength of relationships (Burt 1987), make centrality measures and degree measures unstable (Costenbader and Valente 2003; Kossinets 2006; Borgatti et al. 2006; Huisman 2009), underestimate clustering coefficients (Kossinets 2006; Huisman 2009), and underestimate reciprocity measures (Huisman 2009).

Bias due to non-response generally increases with higher missing data rates. The amount and nature of the bias, however, varies across network measures, features of the network of interest, and nature of the missing data mechanism, as was clearly shown in two large simulation studies by Smith and Moody (2013) and Smith et al. (2017). In these studies, the effect of missing nodes on 22 network measures for 12 empirical networks was investigated, using 12 missing data rates. In the first study, a MAR mechanism for nodes was used. The latter study investigated the effects of four MNAR mechanisms based on centrality. Overall, large, centralized networks are generally more robust to missing-at-random data, and for most network measures, biases are generally larger when more central nodes are missing (MNAR), although some (e.g., degree centrality) are robust to both types of missingness.

Some of the studies show that the extent to which structural properties of the network are affected by missing data also depends on how the available information is used to calculate the measures. For instance, measures based on indegrees are reasonably robust to missing data when the observed incoming ties of non-respondents are included in the analyses (Costenbader and Valente 2003). The same was found for reciprocity measures (Huisman 2009). This is the result of the unique property of social networks that non-participation (or partial participation) does not necessarily mean that the missing actors are not included in the analyses (Borgatti and Molina 2003), that is, when incoming ties of respondents to non-respondents are available. Because of this property, Stork and Richards

(1992) proposed using the information in partially described ties of non-respondents to reconstruct the missing part of the network.

Stork and Richards (1992) explore problems in analyzing incomplete network data due to non-response. They propose a treatment for incomplete data based on reconstruction of the missing ties, and make suggestions for designing network studies that improve response rates and that provide information to make decisions about analysis methods. The impact of non-response on network properties was further explored by Kossinets (2006). He investigated a broader set of missing data sources, including boundary specification problems, non-response, and fixed choice designs. An even broader set of sources of measurement error in network data is discussed by Butts (2003), Žnidaršič et al. (2012a), and Wang et al. (2012).

Imputation of Missing Network Data

Imputation is a general and popular approach to handle missing data, and various imputation procedures are thoroughly studied in the statistical literature. Schafer and Graham (2002) give a general overview by distinguishing four classes of single imputation methods. Before discussing imputation methods for network data, we first present these general classes.

Imputing Unconditional Means A simple (ad hoc) procedure is replacing each missing value with the mean over the observed cases of that item. Although the means of items are preserved, variances and covariances (relations) are often severely biased. Rounding the mean values, in case of binary or categorical data, even adds more error to the imputed values. Although the added variability is random, it is better to keep rounding to a minimum (Graham 2009).

Imputing from Unconditional Distributions The underestimation of variances by imputing means can be corrected by using the observed (empirical) distributions of the items to impute the missing scores. In one class of

procedures, called hot-deck procedures, these distributions are simulated by (randomly) selecting an observed donor case from the same dataset, and replacing the missing values with the observed scores of the donor (e.g., Sande 1982). Although such procedures preserve univariate distributions of variables (i.e., means and variances), relations are still biased.

Imputing Conditional Means Prediction of mean values can be improved using a formal model that accurately captures the association between a missing item and observed items. Often linear (regression) models are used to predict the conditional means of the missing items. These procedures result in more accurate predictions of the missing scores and yield unbiased estimates of means under MAR, but underestimate variances and generally overestimate covariances.

Imputing from Conditional Distributions The biases in variances and covariances found in the previous procedures are largely reduced by using conditional distributions to impute the missing values, conditional on observed variables. The conditional distribution of the missing values is simulated using the imputation models of the previous procedure (imputing conditional means), conditional on the observed independent variables in the model. The missing scores are replaced by draws from this distribution. In the practice of empirical research, this procedure usually amounts to imputing regression predictions with an added error term, randomly drawn from the normal distribution (of which the standard error is estimated in the regression analysis; for this purpose, a t distribution is also often used instead of the normal distribution). Multiple imputation procedures fall in this class of imputations methods (Schafer and Graham 2002; Graham 2009).

Imputation of Missing Ties

Based on the classification of Schafer and Graham (2002), Huisman (2009) presents an overview of simple imputation methods to impute missing ties caused by both actor and tie non-response, which

were previously applied in empirical network research. These methods belong to the first two classes of imputation methods, some of which are also investigated by others (Ouzienko and Obradovic 2011; Žnidaršič et al. 2012a). In this section, simple imputation procedures for missing network data will be presented together with more sophisticated imputation models that are more recently proposed in the literature.

Imputing Unconditional Means

For binary tie variables, the total mean value equals the network density. Rounding the density (using a threshold of 0.5) results in filling in zeros in sparse networks, and ones in dense networks. In the former case, missing ties are treated as absent. This is called *null tie imputation* by Žnidaršič et al. (2012a) and is sometimes even used in dense networks.

Instead of filling in the overall mean value of the tie variable, the mean of the incoming ties of an actor (“average popularity”) or the mean of the outgoing ties of an actor (“average activity”) can be used. The latter method can obviously not be used in the case of actor non-response. The former method results in imputing the modal value of the incoming ties and is called *imputation based on model indegree values* by Žnidaršič et al. (2012a).

Reconstruction

Stork and Richards (1992) suggest reconstructing the missing part of the network by replacing the missing outgoing ties of non-respondents by observed incoming ties to these actors. As a result, that part of the network with ties between respondents and non-respondents becomes symmetric. Additional imputations are required for ties between non-respondents. For these, Huisman and Steglich (2008) and Huisman (2009) use random imputation proportional to observed density (i.e., the probability of a tie is equal to the observed density of the network). Žnidaršič et al. (2012a) propose additional imputations based on modal indegree values.

Note that reconstruction is an imputation procedure when applied to directed networks. For undirected networks it is an available-case method using partially described links (i.e., reported by

only one of the two actors; Stork and Richards 1992), because no new ties are added. The underlying assumption is that the tie between two actors can be measured by the report of only one of the actors, and that respondents and non-respondents do not systematically differ in reporting their relationship. In directed networks, the two tie variables X_{ij} and X_{ji} are allowed to differ (in asymmetric dyads), and reconstruction is an imputation method in which missing ties are replaced with plausible values: the reversed tie within the dyad.

Imputing from Uncondition or Simple Distributions: Hot-Deck Imputation

Hot-deck imputation uses completely observed donor actors to replace all ties of the missing actor (actor non-response), or the missing ties of an incomplete actor (tie non-response). Donor actors can either be randomly selected, or by using observed attribute values or structural properties of the network, or both. Huisman (2009) gives an example of the latter option where actors are matched on indegree and attribute values. Instead of finding only one donor actor (the “best” donor), a set of donors can be selected from which one is randomly chosen. Note that reconstruction, as discussed in the previous section, can be regarded as hot-deck imputation, defining the donor actor as the second actor in the dyad whose incoming tie is not observed.

Imputing from Simple Distributions: Preferential Attachment

Preferential attachment was proposed by Barabasi and Albert (1999) as a model for the growth of networks and was used by Huisman and Steglich (2008) as an imputation model. The model states that the probability that a new tie $X_{ij} = 1$ will emerge between actors i and j is proportional to the current number of neighbors (i.e., indegree) of actor j . This means that the probability that an actor (observed or missing) will link to another is dependent on the connectivity of others. Liben-Nowell and Kleinberg (2007) mention preferential attachment as a method for link prediction. Huisman and Steglich (2008) propose a two-step imputation procedure based on random draws

from outdegree distributions and random draws using preferential attachment probabilities to impute missing data caused by actor non-response. Huisman (2009) investigates this method also for tie non-response.

Imputing Conditional Means: Link Prediction

The simple methods presented above all have in common that they are not model based. Although some depend on (often strong) network properties like reciprocity (i.e., reconstruction) or connectivity (i.e., preferential attachment based on indegrees), they do not use statistical models to relate observed and unobserved scores. Imputation methods that use such models (the conditional methods, in the classification of Schafer and Graham 2002) are the link-prediction methods based on stochastic blockmodels proposed by Guimerà and Sales-Pardo (2009); methods based on latent factor models proposed by Hoff (2009); methods based on ERGMs proposed by Koskinen et al. (2010, 2013) and Handcock and Gile (2010); and methods based on Kronecker graph models proposed by Kim and Leskovec (2011).

Imputing from Conditional Distributions: ERGM

The link-prediction methods mentioned above are often used to estimate distributions conditional on the observed data. Missing values are then replaced by a random draw from these conditional distributions. The exponential random graph model (ERGM; see Lusher et al. 2013) is suitable model to fit on the observed data and impute the missing values using the simulated ERGM distribution that results from the estimation process (Wang et al. 2016; Hipp et al. 2015; Koskinen et al. 2010, 2013). The general assumption underlying the procedure is that the observed structure of the network can be used to recreate the missing information.

For longitudinal network data, Snijders (2005) proposed a model-based procedure incorporated in stochastic actor-driven models for network evolution. In this procedure, missing data at the first observation moment are replaced by zeros (null tie imputation). In further observations, missing

entries are replaced by either earlier observations (*last value carried forward*), or zeros if there is no earlier observation. This procedure is described and investigated by Huisman and Steglich (2008), who call it a *hybrid imputation method* as it only uses the imputed values to simulate the evolution of the network and not for the calculation of the target statistics, preventing a direct effect of the imputed values on the estimation of the model. Moreover, the imputations do not automatically result in a completed dataset. Hipp et al. (2015) investigate an adapted version of this method in combination with stochastic ERGM-imputation of the first observation moment.

Multiple Imputation

When the conditional distribution of the missing values is available, draws from this distribution can be used to replace the missing values more than once in a multiple imputation procedure. Currently, there are two proposed multiple imputation procedures based on ERGMs. Wang et al. (2016) propose the procedure described above, fitting an ERGM on the observed part of the network and simulating networks keeping the observed ties fixed, thus only simulating the missing ties (the same procedure was proposed by Hipp et al. 2015 in a longitudinal setting). However, this procedure is not a proper multiple imputation procedure in terms of Rubin (1987), for it does not take the uncertainty about the estimated model parameters into account. While in improper multiple imputation each imputed dataset is generated with the same parameters, proper imputations use different parameters as if they were estimated from different samples drawn from the same population. Koskinen et al. (2010, 2013) provide such a proper multiple imputation procedure based on Bayesian methods in which parameters are drawn from their posterior distributions. Although their work mainly focuses on Bayesian estimation of ERGMs under missing data (i.e., model-based missing data procedures), it also provides proper multiply imputed datasets.

Both methods have shown reasonably good performances in imputing ties (i.e., both presence and absence of ties). Koskinen et al. (2010)

provide evidence on the ability to reconstruct descriptive network statistics using proper multiple imputation. A yet unaddressed problem is the selection of the imputation model. For now, it remains unclear under which circumstances which parameters (i.e., predictors) should be included in the imputation model. For non-network data, the general recommendation is to include at least all predictors that are used in the analysis model (Graham 2009). It was shown that for MAR-data, analyses give unbiased results and adding predictors in the imputation model will make the missing data mechanism closer to MAR (van Buuren 2012). It is yet unknown whether the same will hold for parameters of network models.

Imputation of Missing Actor Attributes

Missing actor attributes could be regarded as “ordinary” missing data in any non-network data set, and treated separately from the network data. As discussed previously, ample imputation methods are available and are well known and discussed in statistical literature. For example, a number of completed attribute sets can be created with multiple imputation and used in subsequent network analyses, after which the results are pooled. Ouzienko and Obradovic (2011) present some simple imputation methods (e.g., mean imputation), to impute the missing attributes without taking into account the tie variables.

Actor attributes, however, are known to be (often strongly) related to structural properties of the network. Imputation of missing attribute data should therefore be considered within a general framework of imputations together with missing tie variables. As Graham puts it “all variables in the analyses model must be included in the imputation model” (p. 559). Omitting variables from the imputation model amount to assuming that there is no association between the omitted variables and the included variables. This may lead to underestimation of relations, that is, the structural properties of the network. Koskinen et al. (2013) illustrate the added difficulty of missing attributes in a model-based (i.e., ERGM) framework, but practical solutions for the combined imputation

of missing attribute and network data are not yet available.

In a longitudinal setting, the method proposed by Snijders (2005) for stochastic actor-driven models is also applied to dependent behavior variables and explanatory attribute variables. The former are imputed using previous or future observations, or zeros if these are not available, the latter are imputed using means. However, imputations are only used in the simulation phase of the algorithm and have therefore no direct impact on parameter estimates.

Key Applications

Huisman (2009) investigated simple imputation methods (all simple methods mentioned in the previous section) for cross-sectional network data suffering from actor non-response and tie non-response. He examined the effects of the methods on some structural properties of social networks (e.g., reciprocity, clustering, assortativity). A similarly designed study was performed by Žnidaršič et al. (2012b), who investigated the effect of simple imputations on blockmodeling (Žnidaršič et al. 2017 extend the simple methods to valued networks). Both studies arrive at the general same conclusion that the majority of simple imputation procedures can severely bias estimates of network properties. In such cases, a complete-case analysis is to be preferred, except in networks with high reciprocity and low proportions of missingness, where the reconstruction method performs best.

Wang et al. (2016) investigated ERGM-based multiple imputation of friendship network datasets from 14 schools in the In-School Survey of Add Health to handle both actor and tie non-response. They also developed a procedure to validate imputation methods (Held-Out Predictive Evaluation) and concluded that the ERGM-based imputations are doing a better job of imputing data than simple methods do.

Huisman and Steglich (2008) investigated the effect of imputation of missing ties in longitudinal network data suffering from wave non-response.

They compared two simple methods (reconstruction and imputation based on preferential attachment) and the model-based procedure of Snijders (2005), and studied the effects on parameter estimates for actor-driven models for network evolution. They found that the latter, model-based method generally had smaller biases than the simple imputations. This is as expected, given that the simple imputation methods do not take into account the longitudinal aspect of the data, whereas the model-based method is designed to do so.

Hipp et al. (2015) also studied several missing data strategies in longitudinal network data using the Add Health data. They recommend using the procedure proposed by Snijders (2005) with added ERGM-based imputations of the first wave of network observations and stress the point that researchers should carefully consider missing data handling when estimating statistical models for social networks.

Future Directions

It was found that simple imputation methods are unable to capture the structural properties of networks, because relationships are not incorporated in the imputation models and therefore poorly estimated. The ERGM-based imputations seem promising and the results of the first simulation studies show good performances. We want to build on these results and further investigate the performance of ERGM-based imputation, especially multiple imputation procedures. Repeated imputations are needed in “modern” missing data methods to give correct estimates of uncertainty levels needed for inferences. For both cross-sectional and longitudinal network data, different aspects of multiple imputation (e.g., model selection, non-ignorable non-response) need to be explored in Monte Carlo simulation studies.

In this entry, actor attributes played a minor role, both as predictors in imputation models, and as (missing) response variables to be predicted by the imputation models. We would like to explore the possibilities to use attributes in imputation models and methods for the combined imputation

of attributes and network data. As stated above, practical solutions with respect to imputation of actor attributes are topics of ongoing research.

Cross-References

- ▶ [Exponential Random Graph Models](#)
- ▶ [Link Prediction: A Primer](#)
- ▶ [Missing Data](#)
- ▶ [Research Designs for Social Network Analysis](#)
- ▶ [Sampling Effects in Social Network Analysis](#)

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Incentives in Collaborative Applications

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Synonyms

[Cooperation](#); [Game theory](#); [Over-justification](#); [Social interaction](#); [Prisoner's Dilemma](#)

Glossary

Collaborative System	A system that relies on the collaboration of its users when providing a service
Contribution	Any user activity that is or might be of benefit to other users or the system itself

Extrinsic Interest	Interest that is based on an outside stimulation (such as a reward)
Free Rider	A participant who enjoys the contribution of others but fails to contribute or contributes significantly less than others
Intrinsic Interest	Interest that is based on inner satisfaction

Definition

Collaborative systems rely on the cooperation of their users to provide a service. We can distinguish between various types of collaborative systems, such as Q&A services, social networking services, sharing platforms, and crowdsourcing platforms. Despite the fact that each one of these services and platforms has its own unique characteristics, they all rely on the cooperation of their participants and community members in order to operate successfully.

The study of incentives and cooperation is multidisciplinary and is approached through diverse and sometimes contradictory points of view. Social science, economy (game theory), and biology are merely a part of the different approaches which attempt to explain cooperation in platforms that require it.

Game theory approaches assume that users are rational and act in order to maximize their benefit (Kreps 1990). These approaches model systems using well-defined game settings and provide mathematically proven strategies for maximizing self and/or overall benefit. However, it was shown that human decision-making is biased when operating under risk and uncertainty (Kahneman and Tversky 1979). This bias is not captured by game theoretic approaches.

Moreover, different users have diverse reactions to various types of incentives. For example, pro-self-oriented users will respond best to extrinsic incentives that ensure benefit from using the system. Pro-social-oriented users will better cooperate in communities with social ties and trust.

It is important to understand the population at hand and the system objectives in order to design the most appropriate incentive mechanisms for each collaborative system.

Introduction

Collaborative systems rely on the cooperation of their users to provide service in the form of knowledge or information, products, paid or unpaid tasks, etc. Some well-known examples of collaborative systems, protocols, or applications include informative websites, such as Wikipedia or Yahoo! Answers; crowdsourcing platforms, such as Amazon Mechanical Turk; peer-to-peer file sharing protocols, such as BitTorrent; social networking services and microblogging, such as Facebook and Twitter; and media sharing platforms, such as YouTube or Flickr.

Many systems explicitly require contribution and cooperation from their users. For example, one must enable uploading in order to download files in most peer-to-peer file sharing systems. However, contribution to a system is often time and effort consuming. For example, filling questioners in a collaborative recommendation system or reviewing an article in Wikipedia often require a significant time contribution. The non-negligible effort that users contribute in many collaborative systems leads toward an inevitable question: what drives users to cooperate in collaborative systems, and, in fact, why do users cooperate at all?

There are various reasons for users to cooperate. In some cases, cooperation might be beneficial for a user in the future. For example, a user is willing to cooperate with others now in the hopes that others will cooperate with her later on. Even though cooperating now may not be beneficial to this user, the expected benefit she is to receive makes it in her best interest to cooperate. For example, a user sharing a file fragment in a peer-to-peer system will rely on the fact that other users share their files with whoever shares with them.

This consideration of benefit (also referred to as utility) is the foundation of incentive mechanisms that assume that users behave rationally,

i.e., are motivated by maximizing their utility. Game theory approaches analyze such incentive mechanisms by modeling the systems with well-defined game settings. These approaches will be briefly reviewed in the section “[Game Theory](#).”

Despite proliferation of game theory in the academic community, human participants rarely act rationally. A Nobel Prize winning paper (Kahneman and Tversky 1979) exemplified decision-making behavior under risk and uncertainty which is significantly different from the optimal decisions when only the expected utility is considered. This entry and other works in the domain of social sciences and human behavior will be reviewed in section “[Social Studies](#).”

Once both disciplines are presented, we will compare the different types of incentives that each approach suggests and continue on to describe the incentive mechanisms relevant to collaborative on-demand applications.

Lastly, we discuss free riders – a population of users that use the contribution of others but fail to contribute for themselves.

Classifying Collaborative Systems

Collaborative systems utilize the knowledge, the time, and the effort of their participants to create value and share that value among their users. Such systems were thoroughly studied and classified according to nine major dimensions (Doan et al. 2011). We will refer to the three dimensions that are most relevant for designing incentives mechanisms: users’ contribution, distribution of effort, and roles.

Users’ Contribution

The first dimension refers to a users’ contribution to the system. Contribution can be manifested in different ways according to the nature of the different collaborative applications. In Wikipedia, a contributing user is one who creates and edits the Wikipedia pages. On Facebook, a contributing user is one who shares their own information, such as photographs, videos, and text (referred to as status).

To put things in order, Doan et al. defined five ways to contribute:

- Evaluation** Contributing users provide their perspective on a topic by reviewing, ranking, or tagging. An example of evaluation would be a user’s feedback to a recommender system
- Sharing** Contributing users provide content rather than perspective by sharing media items or knowledge, either on demand by their own initiative. An example can be found on Q&A web pages like Yahoo! Answers as well as in sharing platforms like YouTube and Flickr
- Networking** Refers to users’ social activities and interactions. Examples can be found in user’s activities on Facebook, LinkedIn, and other social networks in the form of messages, status posts, pictures, etc
- Building artifacts** Contributing users collaborate to build a product. This contribution could refer to a textual knowledge base, such as Wikipedia, or collaborative software developments like Linux and Apache
- Task execution** The contributor performs a task or a set of tasks defined by other users or by the system’s owners. Amazon Mechanical Turk exemplifies this contribution method, where some users define tasks and others perform them for monetary or alternative compensation

Timing and Relevance of Contribution

From the point of view of incentive mechanisms, collaborative systems are particularly interesting where contribution is provided upon request from other system users. In Q&A websites, like Yahoo!

Answers, participants may share textual information as a response to previously asked questions.

However, the timing of a response can play a significant role in the satisfaction of the querying user. Answers on Q&A websites might be helpful for a long period of time. Take, for example, a query about a vacation planned for 1 year ahead. Replies to this query will stay relevant until time of the vacation. In other cases, however, sharing might be limited in its relevance. In the example of peer-to-peer file sharing, if a peer wishes to download a file segment, she needs other peers who are currently available to upload it. While uploading the file when the requiring peer is disconnected might be relevant to other peers, it is no longer relevant to the peer who requested it in the first place.

Distribution of Effort

Effort in collaborative systems can be distributed among users and among system owners (Doan et al. 2011). A recommender system requires some participation from its users (a rank, an opinion), while most of the effort is imposed on the system itself (combining the ranks and providing recommendations). On the other hand, most of the effort in writing, reviewing, and merging the Wikipedia pages is invested by its users.

In systems where the effort is mainly distributed among the users, cooperation is critical to the proliferation of the system. Imagine, for instance, a Wikipedia with no authors or a Yahoo! Answers website with no responders. In fact, without this cooperation, such systems could not exist.

User Roles

Doan et al. (2011) describe four different roles users may have in collaborative systems:

Slaves	Refers to users who solve together a common problem by dividing effort among them.
Perspective providers	Refers to users who provide their own thoughts and perspective on various topics, such as book reviews.
Content providers	Refers to users who share self-generated content, for example,

photos on Flickr or movies on YouTube.

Component providers	Refers to users who serve as parts of a bigger system (participants in a social network).
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A single user can play multiple roles in one collaborative system. The different roles depend on the systems' design, architecture, and purpose. Evaluating users will generally play the role of perspective providers, sharing users are commonly content providers, and users who contribute by networking will commonly play the role of component providers. However, users who participate in systems that build an artifact may be of differing roles according to the collaborative system's structure. All Wikipedia writers, for example, are content providers, but reviewers play the role of perspective providers as well.

Why Do People Cooperate?

Psychology of Cooperation

The motives of cooperation begin in a reward system present in our brain. This reward system can be modulated by a cognitive system of extrinsic rewards and by a social cognition system based on trust (Declerck et al. 2011; Walter et al. 2005). The reward system foresees an expected payoff and guides behavior accordingly. The cognition system interprets the mental state of other people and is sensitive to trust signals in order to avoid betrayal. Together these two systems collaborate in the effort of decision-making and, particularly, in cooperation.

Furthermore, the oxytocin hormone, a trust-associated hormone present in the brain, was found to increase cooperation in games that include social information and decrease cooperation in games where social information was lacking (Declerck et al. 2010). This gives us a clue to interpret different cooperation behaviors in different settings. While one game setting may encourage intrinsic cooperation, another may cause the opposite effect.

Social Studies

We already mentioned the irrationality of human behavior. A rational decision would take into account the expected utility of each action a person performs and select the action that guarantees the highest payoff to the person. However, people's behavior was not in accordance to the expected utility strategy when presented with decisions that they were to take under risk and uncertainty (Kahneman and Tversky 1979). Take, for example, a decision to choose between a lottery where there is a 50% to win \$1000 prize or an assured amount of \$450. According to expected utility, one should choose the lottery with the highest expected utility of \$500 and not the assured prize of \$450; however, people decided to choose the assured prize over an expected one. We can therefore conclude that expected utility is not the only factor that influences human decision-making.

In order to better understand the factors which influence decision-making and, in particular, the decision to cooperate, we shall review studies investigating pro-social behavior and the incentives affecting it. Benabou and Tirole (2006) reviewed the set of motives that shape people's social conduct. They found three motivations that affect social decisions: intrinsic (out of inner interest), extrinsic (dependent on an outside interest such as a reward), and reputational.

These results further explain the expected outcomes of the following incentive mechanisms in a social environment:

Reward and punishment

- Presence of extrinsic rewards crowd out voluntary cooperation. Authors point out that the reason is due to the spoil of reputation, where a good deed is no longer perceived as something a person performed out of good will but out of extrinsic interest to earn the reward. This was supported in Fehr and Falk (2002), where monetary incentives were shown to reduce the performance of agents and their compliance with rules.

Publicity, praise, and shame

- Generally encourages pro-social behavior. In some cases, good actions are suspected

of reputation considerations, thus making people refrain public praising and other "image" rewards.

Social and personal norms

- People act according to social norms in their surroundings. A human's desire is to avoid being driven out of the rest of the community. As such, a community supporting contributions with high intrinsic altruism will cause individuals to contribute and cooperate.

Welfare and competition

- Authors warn of a holier-than-thou competition between contributing agents. Such competition can motivate agents to commit high visibility contributions that will eventually reduce social welfare (e.g., hosting an expansive fund raiser instead of a modest one).

Further research strengthens the social effect of society and norms (Fischbacher and Gächter 2010). Researchers conducted an experiment on sharing in the public environment. Results indicate that participants' belief about the contribution of others directly influences their own contribution.

In the aspect of reward and punishment, we witness the manifestation of a well-documented side effect called the over-justification effect. This effect was presented by Deci in 1975 and contained two hypotheses.

Deci's First Hypothesis (Over-Justification Effect)

When one is offered a reward based on the execution of an activity, there will be a decrease in the intrinsic interest one had in the activity. Furthermore, a previous intrinsic interest that might have been associated with the activity will transform into an extrinsic interest. In other words, the reason to perform a task will transform into reward compensation rather than interest and enjoyment.

Deci's Second Hypothesis

When one is offered a reward based on competence to fulfill an activity, the intrinsic interest for the activity is increased. This is explained by the increase in the perception of self, having a greater competence.

Deci's hypotheses were put to the test in a set of experiments (Enzle and Ross 1978), and the results indicate that subjects who received high rewards based on execution showed less intrinsic interest than subjects who received the same reward unexpectedly. Furthermore, subjects who received rewards based on competence showed a high intrinsic interest. Enzle and Ross (1978) conclude that a high value reward should make subjects feel more competent in an activity in order to increase their intrinsic interest.

Game Theory

Not all proposed solutions for cooperation believe human players are irrational. Peer-to-peer protocol clients, for example, often implement a sharing strategy that benefits their human user.

Cooperation in collaborative systems is often compared to different game theory settings, parts of which are reviewed in this section. Comparing the cooperation to well-defined games allows for analyzing and defining strategies, according to Nash equilibrium, minimax, Pareto optimality, and other game theory measures of beneficial strategies. In addition, analyzing these strategies enables the definition of incentives which agents need to provide in order to increase overall cooperation (Akçay and Roughgarden 2011).

Noncooperative Game

A noncooperative game is a game where the participants make decisions independently, without collaboration or communication with other participants (Nash 1951); thus, any cooperation is dictated by the player's own will and benefit.

Normal Form Games

In a finite game where players have knowledge of the payoffs and strategies of other players and all players share this information, a game is often represented as a payoff matrix. The matrix states the expected payoff of each player when performing a particular action.

An example of a payoff matrix of two players is illustrated in Table 1. Both row players and column players are playing simultaneously by moving to the right (R) or to the left (L). The payoff a player receives depends on the action of

Incentives in Collaborative Applications, Table 1 Normal form game example

	R	L
R	1,1	0,0
L	0,0	1,1

the opponent. If both players choose the same side, both receive a payoff of 1. On the other hand, choosing two different directions will result in a payoff of 0 to both players.

Prisoner's Dilemma

Prisoner's dilemma is a game that is often used to model cooperation in collaborative applications. We will briefly describe the game and its variations.

Prisoner's dilemma received its name from a story about two prisoners being caught by the police. Unfortunately for the police, they did not have enough evidence to charge the criminals for their crimes so they decided to interrogate the prisoners in two separate rooms in the hope that at least one of them would supply evidence against the other. The terms presented to both prisoners were as follows:

- If one criminal is to supply evidence on the other (defect), while the other remains silent (cooperates), the silent criminal is to receive 10 years in prison while the first one goes free.
- If both criminals supply evidence on each other (defect), both will split the penalty of 10 years, i.e., each criminal will spend 5 years in prison.
- If both criminals decide to remain silent (cooperate), the police will charge both of them with minor crimes, causing each criminal to spend a year in prison.

Table 2 (left) represents the normal form of the prisoner's dilemma game. The payoff values of every action (cooperate or defect) in the matrix can vary as long as the following equation holds: $b < d < a < c$. An example to a payoff matrix in the prisoner's dilemma is presented in Table 2 (right).

Iterated Prisoner's Dilemma

The iterated prisoner's dilemma is a game of prisoners dilemma played repeatedly with the same

Incentives in Collaborative Applications, Table 2 Prisoner’s dilemma payoff

Normal form representation			Payoff example		
	C	D		C	D
C	a,a	b,c	C	3,3	0,5
D	c,b	d,d	D	5,0	1,1

opponent. We noticed that in a repeated game, one remembers the actions of the other player and is able to “punish” defecting behavior. For example, a punishing strategy may be to defect three iterations in a row after the opponent player defected once.

Random Matching Game

The random matching game version of prisoners’ dilemma is an iterated game where opponents are randomly matched in every iteration. We noticed that “punishing” is no longer applicable as it was in the iterated prisoners dilemma because opponents alter through every iteration.

The Effects of Incentives

Different types of incentives may have diverse and even opposite effects on different types of participants. It is therefore important to understand the crowd we are trying to motivate and to apply the right incentives in order to encourage cooperation.

Volunteer Work

Studies have evaluated the motivation behind voluntary work. Volunteering is important to collaborative applications that do not grant extrinsic rewards upon cooperation. In fact, rewards in volunteering environments were found to decrease volunteering work due to the over-justification effect. Monetary rewards were shown to decrease intrinsic motivation of volunteers (Frey and Goette 1999) and sometimes even to harm their image and reputation (Carpenter and Myers 2010).

In fact, when examining volunteer work, the presence of peers and social norms appeared to have a great impact. When volunteering in a social

environment, the absence of excuses to stop voluntary work did not only keep volunteers in their environment but also did not change their efficiency at the voluntary job (Linardi and McConnell 2011).

Nevertheless, even monetary rewards may result in different effects depending on the way they are presented. According to a study by Lacetera and Macis (2010), the majority of blood donors reported to have stopped the donation if given a cash reward for it; however, same donors were not reluctant to receive a voucher worth the exact same amount.

Social Ties

Social ties were vastly researched in working environments. Implicit incentives, such as social relations, were shown to increase voluntary cooperation at work (Gächter et al. 2010). Good social ties in the working environment were shown to increase altruism between peers (Dur and Sol 2010) and increase efficiency to the point that managers were recommended to encourage social ties and friendship among workers in order to increase overall performance (Bandiera et al. 2010). Monetary incentives, in contrast, were shown to crowd out voluntary cooperation (Fehr and Gächter 2000).

Comparing economic incentives to social norms reveals that social norms are derived from the aspiration to be socially efficient. On one hand, when workers were presented with individual incentives, the effect of social norms decreased to the point of irrelevance. The same effect was caused by peer competition. On the other hand, group incentives were shown to increase worker’s efficiency (Huck et al. 2010).

Strong social ties were shown to benefit stock market exchanges as well. Brokers with strong social ties were able to disseminate and absorb more knowledge. Furthermore, social returns on investments tend to affect broker’s position more than private benefits (Fritsch and Kauffeld-Monz 2010).

Extrinsic and Intrinsic Rewards

Not all extrinsic rewards crowd out cooperation. A point system technique in collaborative recommender systems was shown to increase

cooperation among system users (Melamed et al. 2007). Studies examining opportunism (i.e., the willingness to be persuaded to cooperate by incentives) found punishments to be efficient as long as one cannot get away with defection (Hilbe and Sigmund 2010).

An experiment on the crowdsourcing platform Amazon Mechanical Turk shows some interesting insights on the perception of workers on monetary rewards (Mason and Watts 2010). Crowdsourcing workers who were paid more money for a job increased the quantity of the work, but not the quality. This occurred due to a perception of workers that their work is more valuable, thus reducing the effort put into it.

In general, people with a pro-self-value orientation tend to respond better to extrinsic incentives, while people with a pro-social value orientation tend to respond better to trust and social ties (Boone et al. 2010). In order to fit the right incentives scheme to the right collaborative application, one must study the user population and apply incentives that crowd in cooperation and crowd out free riding.

Free Riders

It is not surprising that some users would rather enjoy the contributions of others while not contributing themselves. These users are typically called free riders.

Free riders can be found throughout all popular collaborative systems today. In Q&A engines, free riders are easily recognized as users who ask questions but do not answer the questions of others. Peer-to-peer file sharing protocols distinguish between users who upload file segments (in addition to downloading) and users who mainly download.

The prevalence of free riders has different effects on different collaborative systems. On one hand, most peer-to-peer file sharing protocols discourage free riding by using extrinsic rewards and punishments. On the other hand, a vast majority of Wikipedia users are free riders! The number of contributors is very small compared to the number of users who take a look at the Wikipedia pages once in a while.

In general, a lack of collaboration introduces significant difficulties to the system, as the number of users who are able to contribute becomes smaller. For example, if a user is asked a question on a very specific topic, the number of participants who are able to respond is limited. In a system with a big number of free riders, the chances of the question being answered are small. However, a general question, oriented to a large number of potential responders, has a higher chance of receiving a reply. Therefore, every system will usually apply different measures to deal with free riders, respective to the damage inflicted by their existence.

Reducing Free Riding

The benefits of free riding are easy to comprehend. However, one must address the moral issues involved in enjoying one's contribution while refusing to contribute to others. A study examining moral judgment of free riders found that one's free riding behavior depends on the free riding of others (Cubitt et al. 2011).

Society-based solutions that deal with free riders contain coordinated punishments (Boyd et al. 2010) in environments where the cost of an individual punishment exceeds the individual gain of cooperation. Punishments jointly coordinated by contributing peers were shown to reduce free riding when the punishments are rare.

To implement a coordinated punishment, participants must distinguish the cooperating users from the defecting ones. In order to maintain public knowledge on peer contributions, several suggestions were made in order to design a public reputation system where noncooperative users are punished by other peers until they start cooperating and increase their reputation (Blanc et al. 2005; Tseng and Chen 2011). In other cases, a priority system which benefits cooperating users increased the motivation to cooperate and decreased the motivation to free ride (Carlsson and Eager 2008).

Unfortunately, social punishment is not applicable in systems where pseudonyms are changed easily. An entry fee to acquire a pseudonym (Friedman and Resnick 2001) could come as a

solution to free rider users who aim “to get away with it.” If the cost of changing a pseudonym is bigger than contributing and maintaining a reputation, it is no longer beneficial to ride for free. Another solution is to treat all new users as one (Feldman et al. 2004). If new users are treated according to the actions of new users before them, then they must gain their own reputation and contribution history in order to enjoy the contribution of others. Changing pseudonyms is now worthwhile only if the number of new free riders is significantly small (i.e., most of new users cooperate).

Summary

Collaborative systems are diverse in their architecture, purpose, and user population. In order to provide the best incentives and encourage the cooperation of users, one must understand the population at hand and its motivations.

In this entry we covered several disciplines that investigate incentives in collaborative systems. Collaboration theory and experiments compare different approaches toward incentives and define the types of incentives to use within different environments to motivate different types of users. In general, pro-self-oriented participants respond best to extrinsic incentives that ensure benefits from using the system, and pro-social-oriented participants will better cooperate in communities with social ties and trust.

When dealing with free riders, the collaborative system must understand the damage inflicted by a lack of cooperation and whether this damage is critical to its survival. Minor damages will not justify extreme measures and sometimes might even reduce contribution (imagine Wikipedia enforcing its users to write a page before reading one).

Cross-References

- ▶ [Collaboration Patterns in Software Developer Network](#)
- ▶ [Collective Intelligence for Crowdsourcing and Community Q&A](#)

- ▶ [Collective Intelligence: Overview](#)
- ▶ [Competition Within and Between Communities in Social Networks](#)
- ▶ [Crowdsourcing](#)
- ▶ [Crowdsourcing and Social Networks](#)
- ▶ [Game Theory and Social Networks](#)
- ▶ [Mobile- and Context-Aware Applications of Social Networks](#)
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- ▶ [Role Discovery](#)
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- ▶ [User Behavior in Online Social Networks: Influencing Factors](#)
- ▶ [Virtual Team](#)
- ▶ [Wikipedia Collaborative Networks](#)
- ▶ [Wikipedia Knowledge Community Modeling](#)

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Incremental Computation

► [Stream Querying and Reasoning on Social Data](#)

Independent Component Analysis

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Synonyms

[Blind source separation](#); [Causal analysis](#); [Non-Gaussianity](#)

Glossary

ICA	Independent component analysis
BSS	Blind source separation
pdf	Probability density function
cdf	Cumulative distribution function
EEG Signal	Electroencephalogram signal

Definition

Independent component analysis (ICA) (Hyvarinen et al. 2001; Stone 2004) extracts statistically independent variables from a set of measured variables, where each measured variable is affected by a number of underlying physical causes. Extracting such variables is desirable because independent variables are usually generated by different physical processes. Thus, by extracting independent variables, ICA can effectively extract the underlying physical causes for a given set of measured variables.

Introduction

Most measured quantities are actually mixtures of other quantities. Typical examples are: (a) sound signals in a room with several speakers; (b) an electroencephalogram (EEG) signal, which contains contributions from many different brain regions; and (c) a person's height, which is determined by contributions from many different genetic and environmental factors.

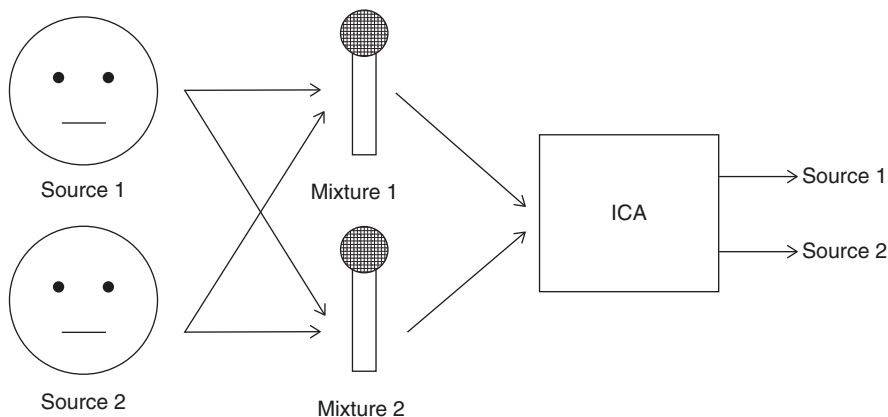
It is often the case that measured quantities, whether these involve height, EEG signals, or even IQ, consist of components of different nature. Under certain conditions, the signals underlying measured quantities can be recovered by using of independent component analysis

(ICA). ICA is one of the blind source separation (BSS) methods. The success of ICA depends on one key assumption that independent variables or signals are generated by different underlying physical processes. If two signals are independent, then the value of one signal cannot be used to predict anything about the corresponding value of the other signal. As it is not usually possible to measure the output of a single physical process, it follows that most measured signals must be mixtures of independent signals. Given such a set of measured signals (i.e., mixtures), ICA works by finding a transformation of those mixtures, which produces independent signal components, on the assumption that each of these independent component signals is associated with a different physical process. The measured signals are known as signal mixtures, and the required independent signals are known as source signals.

ICA Description

Cocktail Party Example (Applying ICA to Speech Data)

Suppose we have two speakers in a room with two microphones, as depicted in Fig. 1. Voice signals are considered to be independent from each other, because they are generated by two unrelated physical processes (i.e., by two different people). If we know that the voices are unrelated, then one key



Independent Component Analysis, Fig. 1 Simple example of ICA: a room with two speakers (source 1 and 2) and two microphones (mixture 1 and 2); the aim is to divide mixture signal in two independent source signals

strategy is to feed microphone outputs to the algorithm which can separate mixtures of sound signals and extract independent signals from these mixtures. The property of being independent is of fundamental importance.

While it is true that two voice signals are unrelated, this informal notion can be captured in terms of statistical independence, which is often truncated to independence. If two or more signals are statistically independent of each other, then the value of one signal provides no information regarding the value of the other signals.

The different distance of each source (i.e., person) from a microphone ensures that each source contributes a different amount to the microphones output.

ICA Premises

Non-Gaussianity

The central limit theorem ensures that a signal mixture that is the sum of almost any signals yields a bell-shaped, normal, or Gaussian histogram. In contrast, the histogram of a typical source signal has a non-Gaussian structure. If sources could be interpreted as Gaussian random variables, then symmetry of histogram source and mixture signals make it impossible to use ICA with appropriate result. The assumption of “non-Gaussianity” of source signals is the necessary criteria for ICA.

Independence

Whereas source signals are independent, their signal mixtures are not. This is because each source signal contributes to every mixture, and the mixtures cannot, therefore, be independent. Thus, the second criteria for ICA algorithm is an assumption about independence of source signals; in other words, ICA finds source signals which are maximally independent between each other.

Mathematical Formulation

A speech source signal $s^{(t)}$ is represented as $s^{(t)} = (s_1^{(t)}, s_2^{(t)}, \dots, s_n^{(t)})$, where $s_k^{(t)}$ is a signal (e.g., amplitude of signal) from speaker k at time t

and n is a number of input signals (number of speakers).

The microphones' output is a column of linear combinations (or mixtures) $x^{(t)}$ that consists of a weighted sum of the source signals $x^{(t)} = A_1s_1^{(t)} + A_2s_2^{(t)} + \dots + A_ns_n^{(t)}$, where A_k , $k = 1 \dots N$, are columns with unknown mixing coefficients and N is a number of microphones. For our example with two people and two microphones (see Fig. 1), the model can be represented as a system of linear equations:

$$\begin{aligned} x_1^{(t)} &= a_{11}s_1^{(t)} + a_{12}s_2^{(t)} \\ x_2^{(t)} &= a_{21}s_1^{(t)} + a_{22}s_2^{(t)} \end{aligned}$$

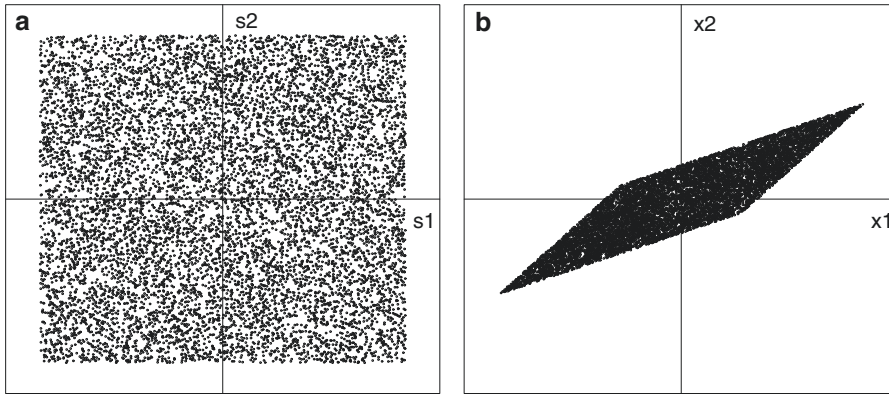
or using vector notation

$$x^{(t)} = As^{(t)},$$

where

$$x^{(t)} = \begin{pmatrix} x_1^{(t)} \\ x_2^{(t)} \end{pmatrix}, s^{(t)} = \begin{pmatrix} s_1^{(t)} \\ s_2^{(t)} \end{pmatrix}, A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

By definition, x is a set of mixtures, s is a set of signals, and A is a mixing matrix. Generating mixtures from source signals in this linear manner ensures that each source signal can be recovered by a linearly recombining signal mixtures. The precise nature of this recombination is determined by an unmixing matrix $W^* = A^{-1}$ such that $s = W^*x$, if A were known. However, as we are ultimately concerned with finding W^* when A is not known, we cannot, therefore, use A^{-1} to estimate W^* . For arbitrary elements the unmixing matrix is suboptimal and is denoted W . In this case, the signals extracted by W are not necessarily source signals, and are denoted $y = Wx$. Thus, the problem solved by ICA, and by all other BSS methods, consists of estimating elements (unmixing coefficients) for the unmixing matrix W . Figure 2 shows the plots for original signals and microphones'; output in case if original signals are distributed uniformly. Basically, ICA helps to find linear transformation which recover graph Fig. 2a from the graph Fig. 2b.



Independent Component Analysis, Fig. 2 (a) Uniformly distributed source signals s_1 and s_2 ; (b) non-uniformly distributed mixtures x_1 and x_2

Machine Learning Formulation

The data is represented by design matrix $X = \begin{pmatrix} x_1 \\ \dots \\ x_N \end{pmatrix}$, where each column X_i represents signal at time i . In the cocktail party example, each row $x_j, j = 1 \dots N$, of matrix X can be a sequence of amplitudes for j -th microphone output. The task is to transform the design matrix X to maximally independent set of sources $S = \begin{pmatrix} s_1 \\ \dots \\ s_n \end{pmatrix}$ using linear transformation $W: S = WX$.

In practice, it is extremely difficult to measure the independence of a set of extracted signals unless we have some general knowledge about those signals. In fact, the observations above suggest that we do often have some knowledge of the source signals. Specifically, we know that they are non-Gaussian, and that they are independent. This knowledge can be specified in terms of a formal model, and we can then extract signals that conform to this model. More specifically, we can search for an unmixing matrix that maximizes the agreement between the model and the signals extracted by that unmixing matrix.

ICA Implementation

As noted above, mixtures of source signals are almost always Gaussian, and it is fairly safe to assume that non-Gaussian signals must, therefore, be source signals. The amount of ‘‘Gaussianness’’

of a signal can be specified in terms of its histogram, which is an approximation to a probability density function (pdf).

For original source s we denote the probability density function (pdf) as $p_s(s)$. In case if we know pdf $p_s(s)$ for the original sources the density $p_x(x)$ of microphone’s output x is defined as follows:

$$p_x(x) = p_s(Wx) \cdot |W|.$$

We illustrate this formula for one source signal and one microphone. Let us assume that $p_s(s) = I\{0 \leq s \leq 1\}$, i.e., s is uniform on $[0,1]$ and $x = 2s$. In this example, $A = 2, W = \frac{1}{2}$. Then $p_x(x) = p_s(Wx) \cdot |W| = p_s(s) \cdot \frac{1}{2} = I\{0 \leq x \leq 2\}$, i.e., x is uniform on $[0, 2]$.

As we know the source signals are independent, we need to incorporate this knowledge into our model. The degree of mutual independence between signals can be specified in terms of their joint pdf $p(s)$. Crucially, if signals are mutually independent, then the joint pdf $p(s)$ of s can be expressed as the product of the pdfs $p_s(s_i), i = 1 \dots n$.

We can consider the probability of obtaining the observed mixtures x in the context of such a model, where this probability is known as the likelihood of the mixtures. We can then pose the question: given that the source signals have a joint pdf $p(s)$, which particular mixing matrix A (and, therefore, which unmixing matrix $W = A^{-1}$) is most likely to have generated the observed signal mixtures x ? In other words, if the likelihood of

obtaining the observed mixtures (from some unknown source signals with joint pdf $p(s)$) were to vary with A , then which particular A would maximize this likelihood? ICA algorithm is based on the assumption that if the model joint pdf $p(s)$ and the model parameters A are correct, then a high probability (i.e., likelihood) should be obtained for the mixtures x that were actually observed. Conversely, if A is far from the correct parameter values, then a low probability of the observed mixtures would be expected. We will assume that all source signals have the same pdf $p_s(s)$. This may not seem much to go on, but it turns out to be perfectly adequate for extracting source signals from signal mixtures.

ICA Algorithm

Our objective is to find an unmixing matrix W that yields extracted signals $y = Wx$, which have a joint pdf as similar as possible to the model joint pdf $p(s)$ of the unknown source signals s . This model incorporates the assumptions that source signals are non-Gaussian and independent.

One way to choose the probability density function $p_s(s)$ is to define cumulative distributive function (cdf) $F(s) = P(S < s)$, where S is a random variable. According to general probability theory $F'(s) = p_s(s)$. Possible choices for cdf are sigmoid function $F(s) = \frac{1}{1+e^{-s}}$ or Kurtosis's leptokurtic function. We summarize ICA algorithm as follows:

Step 1 Assume that $p(s) = \prod_{i=1}^n p_s(s_i)$ as s_i , $i = 1 \dots n$, are independent. Then the probability density function for outputs x

$$p(x) = \left[\prod_{i=1}^n p_s(W_i^T x) \right] \cdot |W|,$$

where W_i^T is row i of matrix W .

Step 2 Choose $p_s(s_i)$.

Step 3 Given training set $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$, where $x^{(t)}$ is a column vector of mixtures at time t , we write log-likelihood function on our parameters W :

$$l(W) = \sum_{t=1}^m \log \left(\prod_{i=1}^n p_s(W_i^T x^{(t)}) \right) \cdot |W|.$$

Step 4 Apply stochastic gradient ascent $W := W + \alpha \nabla_W l(W)$, where $\nabla_W l(W)$ is a gradient of $l(W)$ (vector of partial derivatives with respect to unknown parameters W).

Step 5 Estimated source signals are calculated as follows: $s^{(t)} = Wx^{(t)}$.

ICA, Principal Component Analysis, and Factor Analysis

ICA is related to conventional methods for analyzing large data sets such as principal component analysis (PCA) and factor analysis (FA). Whereas ICA finds a set of source signals that are mutually independent, PCA and FA find a set of signals that are mutually decorrelated (consequently, neither PCA nor FA could extract speech signals, e.g.). The “forward” assumption that signals from different physical processes are uncorrelated still holds, but the “reverse” assumption that uncorrelated signals are from different physical processes does not. This is because lack of correlation is a weaker property than independence. In summary, independence implies a lack of correlation, but a lack of correlation does not imply independence.

Key Applications

ICA has been applied to separation of different speech signals (Bell and Sejnowski 1995), analysis of EEG data (Makeig et al. 1997), functional magnetic resonance imaging (fMRI) data (McKeown et al. 1998), image processing (Bell and Sejnowski 1997), and as a model of biological image processing Van Hateren and Van der Schaaf (1998).

Cross-References

- ▶ [Data Mining](#)
- ▶ [Eigenvalues: Singular Value Decomposition](#)

- ▶ [Matrix Decomposition](#)
- ▶ [Principal Component Analysis](#)
- ▶ [Probabilistic Analysis](#)
- ▶ [Theory of Probability: Basics and Fundamentals](#)

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Industrial Marketing

- ▶ [Business-to-Business Marketing](#)

Inferring Social Ties

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Synonyms

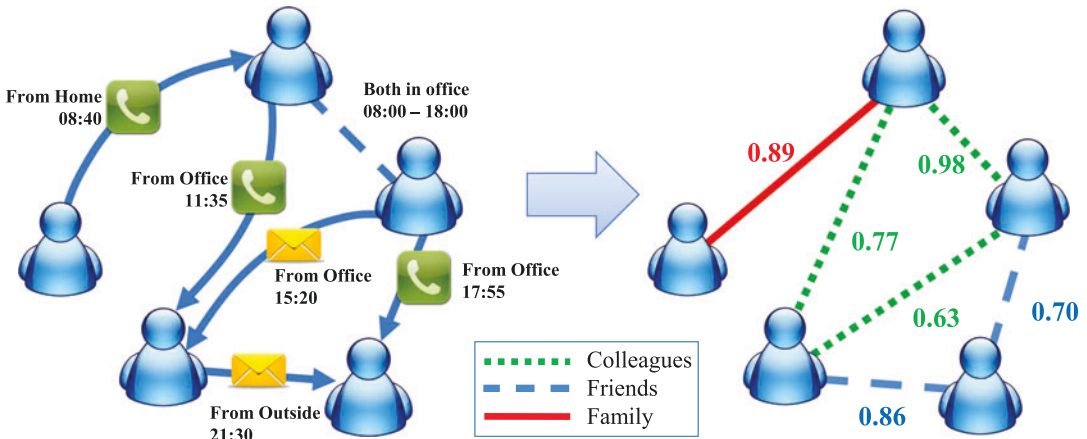
[Link prediction](#); [Relationship mining](#); [Social relationships](#)

Glossary

Active learning	Active learning refers to a learning task which allows an algorithm to interactively query the user (or some other information source) to obtain the desired outputs at new data points. For inferring social ties, it tries to maximally enhance the inferring model by actively acquiring the labels of some unknown relationships
Influence maximization	Influence maximization refers to the problem of finding a small subset of nodes (seed nodes) in a social network that could maximize the spread of influence
Social tie	In sociology, social tie is defined as information-carrying connections between people. It generally comes in three varieties: strong, weak, or absent
Supervised learning	Supervised learning is a machine learning task, aiming to learn a function from the labeled training data. For inferring social ties, it aims to learn a function from the labeled relationships, so as to infer the type of unknown relationships
Unsupervised learning	Unsupervised learning attempts to find hidden structure in unlabeled data. For inferring social ties, it aims to find patterns that could distinguish different types of social relationships

Definition

In online social networks, most relationships are lack-of-meaning labels (e.g., “colleague” and “intimate friends”), simply because users do not take the time to label them. Statistics show that



Inferring Social Ties, Fig. 1 An example of inferring social ties in a mobile communication network. The left figure is the input of the task, and the right figure is the output of the task of inferring social ties

only 16% of mobile phone users in Europe have created custom contact groups (Roth et al. 2010; Grob et al. 2009) and less than 23% connections on LinkedIn have been labeled.

The goal of inferring social ties is to automatically recognize the type of social relationships. Awareness of the types of social relationships can benefit many applications. For example, if we could have extracted friendships between users from the mobile communication network, then we can leverage the friendships for a “word-of-mouth” promotion of a new product.

Figure 1 gives an example of relationship mining in mobile calling network. The left figure is the input of the problem: a mobile social network, which consists of users, calls and messages between users, users’ location logs, etc. The objective is to infer the type of the relationships in the network. In the right figure, the users who are family members are connected with red-colored lines, friends are connected with blue-colored dash lines, and colleagues are connected with green-colored dotted lines. The probability associated with each relationship represents our confidence on the detected relationship types.

To formally define the problem of inferring social ties, let us start with some basic definitions. A social network can be represented as $G = (V, E)$, where V is a set of $|V| = N$ users and $E \subset V \times V$ is a set of $|E| = M$ relationships between users. The objective of our work is to learn a model that can

effectively infer the type of social relationships between two users. More precisely, we first define the output of our problem, namely, relationship semantics.

Definition 1 (Relationship semantics) Relationship semantics is a triple (e_{ij}, r_{ij}, p_{ij}) , where $e_{ij} \in E$ is a social relationship, $(r_{ij} \in \mathcal{Y})$ is a label associated with the relationship, \mathcal{Y} is the set of all the labels, and p_{ij} is the probability (confidence) obtained by an algorithm for inferring relationship type.

Social relationships might be undirected in some networks (e.g., friendships discovered from the mobile communication network) or directed in other networks (e.g., advisor–advisee relationships in the publication network). To be consistent, we define all social relationships as directed relationships. In addition, relationships may be static (e.g., the family-member relationship) or dynamic over time (e.g., colleague relationship). Here, we mainly introduce the problem using the static case.

To infer relationship semantics, we could consider different factors such as user-specific information, link-specific information, and global constraints. For example, to discover advisor–advisee relationships from a publication network, we can consider how many papers were coauthored by two authors, how many papers in total an author has published, and when the first

paper was published by each author. Besides, there may exist some labeled relationships. Formally, we can define the input of our problem, a partially labeled network.

Definition 2 (Partially labeled network) A partially labeled network is an augmented social network denoted as $G = (V, E^L, E^U, R^L, W)$, where E^L is a set of labeled relationships and E^U is a set of unlabeled relationships with $E^L \cup E^U = E$; R^L is a set of labels corresponding to the relationships in E^L ; and W is an attribute matrix associated with users in V where each row corresponds to a user, each column an attribute, and an element w_{ij} the value of the j th attribute of user v_i .

Based on the above concepts, we can define the problem of inferring social ties. Given a partially labeled network, the goal is to detect the types (labels) of all unknown relationships in the network. More precisely,

Problem 1 (Inferring social ties) Given a partially labeled network $G = (V, E^L, E^U, R^L, W)$, the objective is to learn a predictive function

$$f : G = (V, E^L, E^U, R^L, W) \rightarrow R$$

Therefore, the problem is how to find a function f that can leverage both the labeled relationships and the unlabeled relationships to infer the unknown relationships.

Historical Background

Inferring social ties is an important problem in social network analysis. One research branch is to predict and recommend unknown links in social networks. Liben-Nowell and Kleinberg (2007) study the problem of inferring new interactions among users given a snapshot of a social network. They develop several unsupervised approaches to deal with this problem based on measures for analyzing the “proximity” of nodes in a network. The principle is mainly based on similarity of either content or structure between users. Backstrom and Leskovec (2011) propose a supervised random walk algorithm to estimate the

strength of social links. Leskovec et al. (2010) employ a logistic regression model to predict positive and negative links in online social networks, where the positive links indicate the relationships such as friendship, while negative links indicate opposition. However, these works consider only the existence of social relationships and do not consider the types of the relationships.

There are also several works on mining the relationship semantics. Diehl et al. (2007) try to identify the manager–subordinate relationships by learning a ranking function. They define a ranking objective function and cast the relationship identification as a relationship ranking problem. Menon and Elkan (2010) propose a log-linear matrix model for dyadic prediction. They use matrix factorization to derive latent features and incorporate the latent features for predicting the label of user relationships. Wang et al. (2010) propose a probabilistic model for mining the advisor–advisee relationships from the publication network. The proposed model is referred to as time-constrained probabilistic factor graph model (TFGM), which supports both supervised and unsupervised learning. Eagle et al. (2009) present several patterns discovered in mobile phone data and try to use these patterns to infer the friendship network. Tang and Liu (2009) develop a classification framework for categorizing the type of social connections in social media. However, these methods mainly focus on a specific domain, while our model is general and can be applied to different domains. Moreover, these methods also do not explicitly consider the correlation information between different relationships.

Recently, Hopcroft et al. (2011) explore the problem of reciprocal relationship prediction. They propose a learning framework to formulate the problem of reciprocal relationship prediction into a graphical model and evaluate the proposed method on a Twitter data set. The framework is demonstrated to be very effective, i.e., it is possible to accurately infer 90% of reciprocal relationships in a dynamic network. Tang et al. (2012) further propose a general framework for classifying the type of social relationships by learning across heterogeneous networks. The idea is to

use social theories (e.g., social balance theory, social status theory, structural hole theory, two-step flow theory, and strong/weak tie) as the bridge to connect different social networks. Social theory-based features are defined and incorporated into a triad-based factor graph model to infer the type of social relationships in different networks.

Another related, but different, research topic is relational learning (Califf and Mooney 1999; Getoor and Taskar 2007). Relational learning focuses on the classification problem when objects or entities are presented in relations and the goal is to categorize each object by considering both entities and relations. A number of supervised methods for link prediction in relational data have also been developed (Taskar et al. 2003; Popescul and Ungar 2003).

Supervised Learning to Infer Social Ties

For inferring the type of social relationships, we could have several basic intuitions. First, the user-specific or link-specific attributes will contain implicit information about the relationships. For example, two users who make a number of calls in working hours might be colleagues, while two users who frequently contact with each other in the evening are more likely to be family members or intimate friends. Second, relationships among different users may have a correlation. For example, in the mobile network, if user v_i makes a call to user v_j immediately after calling user v_k , then user v_i may have a similar relationship (family member or colleague) with user v_j and user v_k . Third, we also need to consider some global constraints such as common knowledge or user-specific constraints.

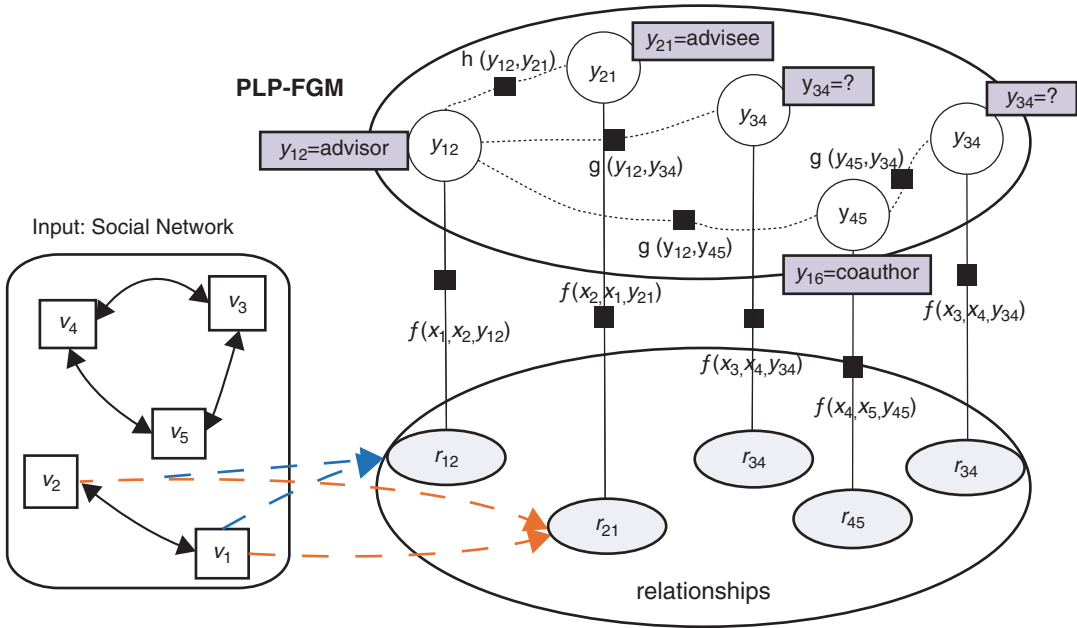
Based on the intuitions above, Tang et al. (2011) propose a Partially-Labeled Pairwise Factor Graph Model (PLP-FGM). It allows us to take all the factors mentioned above into account to better infer the social relationships. Typically, there are two ways to model the social tie inferring problem. The first way is to model each user as a node and for each node to estimate the probability distribution of different relationships. The

resultant graphical model thus consists of N variable nodes. Each node contains a $(d \times |\mathcal{Y}|)$ matrix to represent the probability distributions of different relationships between the user and her/his neighbors, where d is the number of neighbors of the node. This model is intuitive, but it suffers from some limitations. For example, it is difficult to model the correlations between two relationships, and its computational complexity is high. An alternative way is to model each relationship as a node in the graphical model, and the relationship mining task becomes how to predict the semantic label for each relationship node in the model. This model contains M nodes ($2M$ when the input social network is undirected). This model is able to incorporate different correlations between relationships such as the above intuitions.

Figure 2 shows the graphical representation of the PLP-FGM. Each relationship (v_{i_1}, v_{i_2}) or $e_{i_1 i_2}$ in the partially labeled network G is mapped to a relationship node r_i in PLP-FGM. We denote the set of relationship labels as $Y = \{y_1, y_2, \dots, y_M\}$. The relationships in G are partially labeled; thus, all nodes in PLP-FGM can be divided into two subsets Y^L and Y^U , corresponding to the labeled and unlabeled relationships, respectively. The relationships in the input are modeled by relationship nodes in PLP-FGM. Corresponding to the three intuitions, we define the following three factors.

- Attribute factor: $f(y_i, x_i)$ represents the posterior probability of the relationship y_i given the attribute vector x_i .
- Correlation factor: $g(y_i, G(y_i))$ denotes the correlation between the relationships, where $G(y_i)$ is the set of correlated relationships to y_i .
- Constraint factor: $h(y_i, H(y_i))$ reflects the constraints between relationships, where $H(y_i)$ is the set of relationships constrained on y_i .

Given this, one can define a log-likelihood objective function incorporating all the factor functions. By learning the unknown parameters in the objective function, we obtain the social tie inferring model, which can be further applied to infer newly unknown social ties.



Inferring Social Ties, Fig. 2 Graphical representation of the PLP-FGM model

Unsupervised Learning to Infer Social Ties

In some networks, acquiring labeled relationships is very expensive, which makes it infeasible to perform the supervised learning. The unsupervised learning method tries to get around of this and directly infers the type of relationships without labeled relationships. Such a method is usually task oriented.

Wang et al. (2010) propose a two-stage framework for inferring advisor–advisee relationships in the coauthor network. The main idea is to leverage a time-constrained probabilistic factor graph model to decompose the joint probability of the unknown advisor of every author. The time-related information associated to the hidden social role is captured via factor functions, which form the basic components of the factor graph model. By maximizing the joint probability of the factor graph, one can infer the relationship and compute a ranking score for each relationship on the candidate graph.

More specifically, at the first stage of the framework, commonsense knowledge is defined for

recognizing interesting semantic relationships. Here the authors try to make a few general assumptions based on the commonsense knowledge about advisor–advisee relationships.

- The first assumption is to reflect the following fact for general consideration of advising relationship. At each time t during the publication history of an author x , x is either being advised or not being advised. Once x starts to advise another author, it will not be advised again.
- Another assumption indicates that for a given pair of advisor and advisee, the advisor always has a longer publication history than the advisee.

Based on the two assumptions, the framework processes the task in the following two stages:

Stage 1: Preprocessing – The purpose of preprocessing is to generate the candidate graph H' and reduce the search space while keeping the real advisor not excluded from the candidate pool in most cases. First, one needs to generate according to the coauthor information

a homogeneous author network G' by processing the papers in the network one by one. For each paper p_i , we can construct an edge between every pair of its authors.

Then, a filtering process is performed to remove unlikely relations of advisor–advisee. For each edge e_{ij} on G' , a_i and a_j have collaboration. To decide whether a_j is a_i 's potential advisor, the following conditions are checked. First, the second assumption is checked. Only if a_j started to publish earlier than a_i , the possibility is considered. Second, some heuristic rules are applied, which are based on the prior intuitive knowledge about advisor–advisee relations. For more detailed definitions of those rules, please refer to Wang et al. (2010).

Stage 2: The factor graph model – From the candidate graph H' we know the potential advisors of each author and the likelihood based on local information. By modeling the network as a whole, we can incorporate both structural information and temporal constraint and better analyze the relationship among individual links.

By learning the factor graph model, we can find a configuration of the latent variables for each node in the candidate graph H' that maximize the objective function. For learning the model, one can consider the sum-product and the junction tree algorithms (Wang et al. 2010).

Active Learning to Infer Social Ties

Another problem is how to learn the social tie inferring function f effectively. In many situations, labeled data is limited and expensive. The question is, how to design a strategy to actively learn the model with minimal labeling cost? Formally,

Problem 2 (Active social tie inference) Given a partially labeled network $G = (V, E^L, E^U, R^L, W)$ and a labeling budget b (number of user interactions), our objective is to select a subset of unknown relationships $A \subset E^U$ within the constraint of b to label, so that the performance of

predictive function f can be maximally improved.

Formally, for actively selecting relationships to query the user, we define a quality function $Q(A)$, which measures the expected improvement of the prediction performance by labeling relationships in set A . The problem can be then defined as an optimization problem of $Q(A)$, i.e.,

$$A^* = \operatorname{argmax}_{A \subset Y^U} Q(A), \quad |A| = b, b > 0.$$

To quantify $Q(A)$, one could consider how a selected node can influence the others. For example, correction of a centered relationship may trigger a spread of the correction, thus helping infer correlated relationships.

The quality function $Q(A)$ can be defined in different forms. Without any constraints, optimizing the quality function $Q(A)$ needs to enumerate all possible subsets $A \subset Y^U$, which is obviously NP-hard. Let us start with two baseline greedy algorithms.

Maximum Uncertainty (MU) – A most common selection strategy for active learning is to select the most uncertain relationships. The uncertainty of an unlabeled relationship y_i is measured by the entropy $H(y_i) = -\sum_{y \in Y} p(y_i = y) \log p(y_i = y)$. Based on this intuition, we can define the quality function as

$$Q_{MU}(A) = H(A) \quad (1)$$

where $H(A) = \sum_{y_i \in A} H(y_i)$.

Information Density (ID) – A drawback of the maximum uncertainty strategy is its tendency to choose outliers. We can consider another strategy, information density, proposed in Settles and Craven (2008). The idea is to choose the most representative nodes in Y^U , which are supposed to be the most informative ones. Based on this intuition, we measure the informativeness of a node by its cosine similarity to all other unlabeled nodes in the sense of the attributes attached to a node. Formally, we define the quality function as

$$Q_{ID}(A) = \sum_{i \in A} H(y_i) \times \left[\frac{1}{|Y^U|} \sum_{j \in Y^U} \text{sim}(\mathbf{x}_i, \mathbf{x}_j) \right] \quad (2)$$

where $\text{sim}(\mathbf{x}_i, \mathbf{x}_j) = \frac{\mathbf{x}_i \cdot \mathbf{x}_j}{\|\mathbf{x}_i\| \times \|\mathbf{x}_j\|}$. Note that we again employ the entropy of a relationship node $H(y_i)$ to leverage the “base” informativeness.

All the strategies mentioned above do not consider the network structure information. To deal with this problem, Zhuang et al. (2012) have developed two new methods, i.e., Influence-Maximization Selection model (IMS) and Belief-Maximization Selection model (BMS), for actively learning to inferring social ties.

Influence-Maximization Selection (IMS) – As relationships could be correlated, the most influential nodes are more likely to help improve the overall performance of the model. Existing work has studied several influence propagation models, including the linear threshold model (LTM) (Kempe et al. 2003). The LTM sets a threshold value $varepsilon_i$ for each node, and weights $b_{i,j}$ for its connected edges, satisfying $\sum_{j \in NB(i)} b_{i,j} \leq 1$. In each time stamp, if $\sum_{j \in NB(i) \wedge \text{activated}(j)} b_{i,j} \geq \varepsilon_i$, then the node i will be activated. We use the PLP-FGM model (cf. section “Supervised Learning to Infer Social Ties”) as the example and develop a variation of the LTM by incorporating a score for each node reflecting the strength of the influence spreading in our model. The propagation process is described as:

- The graph is the same as the PLP-FGM model. In addition, a relationship node as “activated” when its label y_i is determined. The initial activated set of nodes is Y^L . We can assign a threshold $\varepsilon_i = \sum_{y \in \mathcal{Y}} |p(y_i = y | G, Y^L) - \frac{1}{|\mathcal{Y}|}|$ for each node. Thus, a node with higher uncertainty will be easier to be activated.
- When a node i is activated, it spreads its gained score increment ($g_i - \varepsilon_i$) to its neighbor nodes $j \in NB(i)$ with a weight $b_{i,j}$, i.e., $g_j \leftarrow g_j + b_{i,j} (g_i - \varepsilon_i)$. The gained score increment reflects the improvement of confidence brought by

user labeling; therefore, the influence by labeling an uncertain relationship will be greater than labeling a more certain relationship. To simplify the problem, we set weight $b_{i,j} = 1/|NB(j)|$.

- If a node is labeled by the user, we set it as activated and assign its gained score as 1. The gained score for other nodes is set to 0 at the beginning. Once an inactivated node k gains a score that is larger than the threshold, i.e., $g_k > \varepsilon_i$, it will become activated and spreads its gained score similarly. Note that the activated node only spreads its gained score once and remains its status.

The quality function $Q_{IMS}(A)$ is defined as the total number of activated nodes after the propagation process. Finding the set A that maximizes the quality function $Q_{IMS}(A)$ with the IMS model is again NP-hard. Finally, one can use a greedy strategy to approximate the solution.

Belief-Maximization Selection (BMS) – To quantify the influence of one node on the others, we employ the belief of each node obtained by loopy belief propagation in our model. We define a heuristic by removing the effect of attributes from the belief score, denoted by $\mathcal{B}(y_i | G, Y^L)$. More precisely, from a graphical model such as the abovementioned PLP-FGM model, by normalizing the belief of one relationship node, one could obtain the belief marginal probability $p_{\mathcal{B}}(y_i | G, Y^L)$, which estimates the marginal probability distribution of a relationship node where the information of its attribute vector is absent.

A basic intuition is the belief of a relationship node is monotonically increasing with respect to the number of relationship nodes of the same type, i.e., $\mathcal{B}(y_i = y | G, Y^L)$ is monotonically increasing with respect to the number of relationships with label y . Without loss of generality, let us first consider the binary relationship mining problem, i.e., there are only two possible labels of relationships ($\mathcal{Y} = \{0, 1\}$). In the binary setting, we further consider the active selection for each type separately. This is because when mixing the different types of relationships together, it cannot be guaranteed to have a closed-form solution. Thus,

when users provide only positive feedback, our objective is to find a set of positive nodes. Accordingly, we define the quality function of the positive-oriented BMS strategy as

$$Q_{\text{BMS}^+}(A) = \sum_{y_i \in Y_{(1)}^U} p_{\mathcal{B}}(y_i = 1 | G, Y^L \cup A) \quad (3)$$

where $Y_{(1)}^U = \{y_i | y_i \in Y^U \wedge \mathcal{B}(y_i = 1 | G, Y^L) \geq \mathcal{B}(y_i = 0 | G, Y^L)\}$.

Symmetrically, if the users provide only negative feedback, we can adopt a negative-oriented BMS strategy, with the following quality function:

$$Q_{\text{BMS}^-}(A) = 7 \sum_{y_i \in Y_{(0)}^U} p_{\mathcal{B}}(y_i = 0 | G, Y^L \cup A) \quad (4)$$

The optimization of both quality functions $Q_{\text{BMS}^+}(A)$ and $Q_{\text{BMS}^-}(A)$ is NP-hard. However, as both quality functions are submodular, a solution with an approximation ratio of $(1 - 1/e)$ can be obtained using a greedy algorithm: at each time, it selects the relationship which is expected to provide the maximum marginal increase of the quality function. Notice that we treat the examining relationship node y_i as if it is positive labeled when optimizing $Q_{\text{BMS}^+}(A)$, or negative labeled for $Q_{\text{BMS}^-}(A)$, since the active learning algorithm is label unaware in the selection stage. In order to leverage the risk that a selected relationship is not labeled as expected, we employ a weighting factor $p(y_i | G, Y^L)$ to reflect how likely the relationship would be labeled as positive(negative). Further, to prevent making an imbalance selection, one can use $Q_{\text{BMS}^+}(A)$ to choose $b/2$ nodes (where b is the number of relationships we expect to query the user each time) and then use $Q_{\text{BMS}^-}(A)$ for the rest.

Five Challenges for Inferring Social Ties

The general problem of inferring social ties represents an interesting research direction in social network analysis. There are still many challenges and also potential future directions on this topic. Here we list of five major challenges.

- *Big network.* As social networks increasingly becoming larger, it is important to study how to incrementally learn the inferring model, so that we can dynamically feed new data to the model or involve user interactions into the learning process.
- *Globality versus locality.* Most existing works focus on studying social ties in the entire network. However, as most users and their behaviors are influenced by friends in their local circles, it would be interesting to study the problem from the locality perspective, for example, inferring personal social circles (McAuley and Leskovec 2012; Zhang et al. 2013).
- *Social theories.* How to seamlessly incorporate social theories into the inferring model? Although Hopcroft et al. (2011) and Tang et al. (2012) propose using social balance, social status, structural hole theories to define features for help infer social ties. However, it is still unclear how the strength of social connections (strong/weak tie) correlates with the type of social ties.
- *Dynamic evolution.* Some social ties are stable, for example, the family relationships, while some other social ties will change over time, for example, colleagues and even friendships. It is important to capture the dynamic pattern and infer the changes of social ties.
- *Applications.* It has many real applications based on the results of social tie analysis. For example, we can use the inferred social ties to help information recommendation in the social network. According to the social influence theory, a user's connections with different social ties would have very different influence on her/his behaviors from different aspects.

Cross-References

- ▶ [Collective Classification](#)
- ▶ [Link Prediction: A Primer](#)
- ▶ [Social Influence](#)
- ▶ [Social Network Analysis](#)

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Infiltration

- ▶ [Socialbots](#)

Influence Diffusion in a Social Network

- ▶ [Mathematical Model for Propagation of Influence in a Social Network](#)

Influence Maximization

- ▶ [Influence Maximization Model](#)
- ▶ [Influence Propagation in Social Networks with Positive and Negative Relationships](#)

Influence Maximization Model

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Synonyms

[Influence maximization](#); [Social influence propagation](#); [Viral marketing](#); [Word of mouth](#)

Glossary

Active nodes	The nodes that adopt the piece of information propagated
Diffusion	The spread of information, idea, or product in social networks
Influence spread	The expected number of active nodes when the process of information diffusion terminates
Seed nodes	The nodes that are the initial disseminators of an information

Definition

According to the opinion of Aristotle, human beings are social animals. Specifically, in social networks, people often make decisions (e.g., repost a tweet) under the influence of their friends. By utilizing such “word-of-mouth” effect, influence maximization aims to trigger a large cascade of influence spread in a social network by targeting on only a small set of individuals. Technically and more specifically, given a diffusion model, which specifies the dynamics of influence spread, each influence maximization model figures out a way to select a set of nodes such that if the selected nodes are the initial disseminators of an information (e.g., adopting a product), the expected spread of this information in the social network is maximized.

Introduction

Recent years have witnessed the advance of social networks. Many social networking sites (e.g., Facebook and Twitter) emerge and attract billions of users. People publish and share different kinds of information on these sites, which makes them become important platforms for the spread of information, idea, or product. A key factor that leads to this phenomenon is the “word-of-mouth” effect. In social networks, users often have an influence on their neighbors. As a result, we can trigger a large cascade of (product) adoption spreading in a social network by targeting on only a small number of individuals (persuade

them to adopt the product). To explore and exploit such an effect, influence maximization was proposed by the researchers from the area of computer science. Due to its important application in viral marketing and other interesting areas, influence maximization attracts a lot of attentions from both industry and academia. In the past years, different kinds of influence diffusion models were proposed to describe the dynamics of influence spreading and lots of algorithms were designed to solve the optimization problem of influence maximization.

Key Points

In this work, we focus on introducing the major ideas and technical solutions of influence maximization problem. Different kinds of influence diffusion models are introduced as well, since these models are the basis for influence maximization.

Historical Background

The study of social influence can be traced back to early social science research (Bass 1969; Wasserman and Faust 1994). Previously, researchers had to manually collect the data (e.g., social investigation), which makes their work limited to a small scale. Nevertheless, these works laid a good foundation for the development of the analysis of social influence propagation, for example, Granovetter (1978) introduced the well-known linear threshold model in 1978. In recent years, with the rise of the Internet and the advances of computing performance, research on social influence begins a rapid growth. In 2002, Domingos and Richardson (2001) studied the problem of leveraging the influence between social users for viral marketing. A year later, Kempe et al. (2003) formulated the problem as the well-known influence maximization problem. In their paper, the two widely used influence diffusion models, the independent cascade (IC) model and the linear threshold (LT) model, were first introduced in the current form. Meanwhile, Kempe et al. proved that influence

maximization is NP-hard in both two models and proposed a greedy algorithm to approximate the solution. Along this line, many efforts have been devoted to explore the process of influence diffusion (information propagation) in social networks, and these works can be generally grouped into three categories. First, in terms of social influence diffusion, traditional IC model and LT model were extended to better describe the propagation process in different kinds of scenarios (Kempe et al. 2005; Borodin et al. 2010). Second, how to solve influence maximization in a given influence diffusion model is always a hot topic and attracts lots of attention (Chen et al. 2013; Borgs et al. 2014). Last, as social influence has become an important tool of social behavior analysis, a growing number of influence based applications begin to emerge (Wu et al. 2015; Zu et al. 2016).

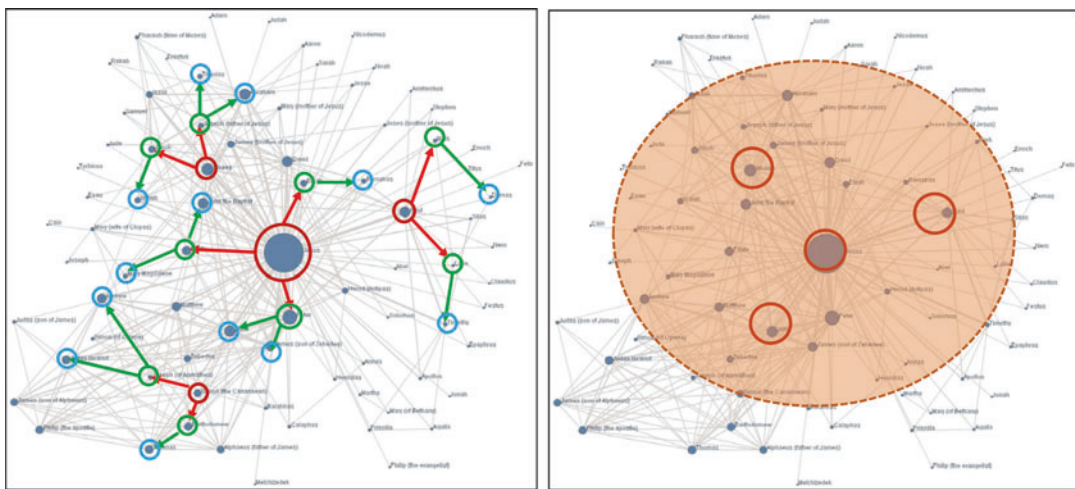
Scientific Fundamentals

Consider a social network $G = (V, E)$, where V is the set of nodes and $E \subseteq V \times V$ is the set of edges connecting pairs of nodes. A node $v \in V$ represents an individual in the social network and an edge (u, v) represents the relationship between individual u and v . Influence diffusion models

formally describe the process of influence spread in social networks, and therefore, they can output the estimated social influence of each node/node set (i.e., social users, as shown in Fig. 1a). Given an influence diffusion model, influence maximization aims to maximize the influence spread by elaborately selecting a small set of initial disseminators (as shown in Fig. 1b). In the following, we first introduce some widely used diffusion models, and then show the major ideas and technical solutions of influence maximization.

Diffusion Models

Generally speaking, diffusion models describe the process of information propagation in social networks with some predefined rules. Among them, stochastic diffusion models are the most widely used ones. In stochastic diffusion models, there are two possible states, active and inactive, for each node $v \in V$. Intuitively, an active node can be viewed as adopting the propagating information while the inactive state of a node means it has not adopted the information. Also, there are a set of nodes called seed nodes in these models, which are active at the initial time moment. They can be viewed as the source of information propagation, and stochastic diffusion models specify the randomized process of generating active sets at any



(a) Influence Diffusion.

(b) Influence Maximization.

Influence Maximization Model, Fig. 1 A toy illustration for the process of social influence diffusion and the solution of influence maximization

time moment given the seed nodes (Chen et al. 2013). According to different settings, stochastic diffusion models can be classified to several classes. For example, a stochastic diffusion model is called progressive if a node stays active ever since it is activated. In contrast, models in which a node may switch back and forth between active and inactive are called nonprogressive models.

To the best of our knowledge, IC model (Goldenberg et al. 2001) and LT model (Granovetter 1978) are two of the most widely used stochastic diffusion models in the literature. We explain the major ideas of stochastic diffusion models by taking the example of them. Let S_t be the set of the active nodes in step/time t ($t = 0, 1, \dots$). Here, S_0 is the set of seed nodes. In the IC model, each edge $(u, v) \in E$ associates with a propagation probability $p_{u,v}$ (usually, predefined), which is the probability that v is activated by u after u is activated. Then, the randomized process unfolds as follows. At step $t + 1$, each node v in S_t has only one chance to activate each inactive out neighbor u with the probability $p_{v,u}$. The process stops when no more nodes can be activated. In the LT model, correspondingly, each edge $(u, v) \in E$ associates with an influence weight $b_{(u,v)}$ (e.g., 0.1, which is also prelearned), indicating the importance of u influencing v . Then, the randomized process unfolds as follows. Initially, each node v picks a threshold θ_v in range $[0, 1]$ uniformly at random. At step $t > 0$, an inactive node v is activated if $\sum_{w \in N(v) \cap (\cup_{i < t} S_i)} B_{w,v} \geq \theta_v$.

The process also stops when no more nodes can be activated. It is worth noting that, in both IC and LT model, once a node becomes active, it stays active. We call such diffusion models as progressive models. Due to their importance, many variations of the IC and LT model have appeared in literature (Kempe et al. 2005; Rodriguez et al. 2011; Borodin et al. 2010; Budak et al. 2011), and these variations extend the IC model and the LT model to accommodate various application scenarios.

In addition to stochastic diffusion models, there are also other diffusion models for different purposes. For instance, to efficiently approximate the influence, Yang et al. (2012) simplified the IC

model by solving a linear system. Furthermore, Xiang et al. (2013) proposed a linear social influence model which formulates the influence with a linear system, and they showed the linear social influence model is a good approximation of the IC model and revealed the connection between the model and PageRank. To directly find influential nodes from raw data, Goyal et al. (2011a) proposed credit distribution model. In the model, an influenced node distributes influence credits to its predecessors and ancestors in the action trace.

Influence Maximization

Let $\delta(S)$ denote the influence spread of a set of nodes S under a specific influence diffusion model, which is the expected number of final activated nodes if S is selected as the seed nodes. Now, we can formally define influence maximization problem as follows. In a social network G , given a diffusion model \mathcal{M} and a budget k , select a set of seed nodes S^* such that

$$S^* = \arg \max_S \delta(S), \quad s.t. S \subseteq V, |S| = k.$$

Kempe et al. (2003) first studied this problem in the IC model and LT model. They proved that influence maximization problem is NP-hard under both IC model and LT model. Fortunately, they also pointed out that the influence spread function $\delta(\cdot)$ is submodular, and to solve the problem, a simple but powerful greedy algorithm was proposed. In each iteration, the algorithm selects a node v and adds it into the current candidate set S , such that the selected node v provides the largest marginal contribution to influence spread δ with respect to S . Due to the submodularity of the influence spread function, the greedy algorithm can approximate the optimal solution with a factor of $1 - 1/e$. However, one main drawback of this algorithm is computing the influence of a given set of seed nodes relies on a time-consuming Monte Carlo simulation method. In (Chen et al. 2010a, b), Chen et al. proved that computing the influence of a given set of seed nodes is #P-hard in both IC model and LT model. Thus, Monte Carlo simulation is necessary for the computation of influence. To address the efficiency issue,

Leskovec et al. (2007) presented a “Lazy Evaluation” strategy which takes advantage of the sub-modular property of the influence spread function to reduce the number of evaluations on the influence spread of nodes. The key idea behind lazy evaluation is to avoid evaluation when it is not necessary. Specifically, denote by $\delta(v|S)$ the marginal gain of node v given set S , that is, $\delta(v|S) = \delta(S \cup \{v\}) - \delta(S)$. Suppose in the i -th iteration, the greedy algorithm evaluated $\delta(u|S)$ for some $u \in V \setminus S$. If in an earlier iteration when the candidate set is \hat{S} , the greedy algorithm has evaluated $\delta(v|\hat{S})$ for some $v \in V \setminus \hat{S}$ and $\delta(v|\hat{S}) \leq \delta(u|S)$, then by sub-modularity we have $\delta(v|S) \leq \delta(v|\hat{S}) \leq \delta(u|S)$. It follows that there is no need to evaluate $\delta(v|S)$ in the i -th iteration. In this way, lazy evaluation strategy significantly reduces the total number of Monte Carlo simulations. Goyal et al. (2011b) proposed the CELF++ algorithm to further enhance the lazy evaluation strategy for influence maximization problem. However, even with the lazy evaluation strategy that provides hundreds of times of improvement to the greedy algorithm, the time cost is still too high for practical applications. To deal with this problem, a series of heuristic algorithms (Chen et al. 2010a, b, 2014; Goyal et al. 2011c; Jung et al. 2012; Liu et al. 2014b) have been proposed. For instance, Chen et al. (2010a) approximated the influence propagation using local arborescence structures of each node, which leads to a quick estimation of influence spread. Later, Chen et al. (2010b) applied the same idea in the LT model and designed the LDAG algorithm, which restricts influence diffusion to node v on a local directed acyclic graph of v . Liu et al. (2014b) developed a quantitative metric, named Group-PageRank, which is actually the upper bound of the social influence based on a linear social influence model. They plugged Group-PageRank into the greedy algorithm to efficiently find the seed nodes with maximal influence spread. All of these algorithms have one thing in common: compromising the approximation guarantee to achieve high efficiency.

Recently, the efficiency-effectiveness dilemma has been solved by the polling-based algorithm (Borgs et al. 2014), which can efficiently return a solution with provable approximation guarantee.

The algorithm includes two steps. In the first step, it estimates the influence spread through sampling. In the second step, it finds an approximation solution for maximizing the estimation. To estimate the influence spread, instead of simulating the process of information diffusion that starts from the seed nodes, the algorithm uniformly selects a node v at random and runs a simulation of information spread that starts from v along the reverse direction. Such an operation is named “poll” by the authors. If we can bound the estimation error, then the solution also enjoys an approximation guarantee for the influence maximization problem. Along this line, Tang et al. (2014b, 2015) reduced the sample complexity and improved the efficiency. Later, Nguyen et al. (2016) further reduced the time complexity with a different bounding technique. Experimental results showed that these randomized algorithms have outperformed the heuristic algorithms (Tang et al. 2015). In addition to IC model and LT model, influence maximization in other diffusion models have been explored as well. For instance, Du et al. (2013) studied influence maximization in the continuous time IC model and proposed an efficient sketch based algorithm. Borodin et al. (2010) studied influence maximization under the competitive threshold model and showed the original greedy algorithm cannot be applied.

Key Applications

The study of social influence propagation has found applications in many fields, such as viral marketing, the spread of trust, expert finding, social media analysis, social recommendation. Among these applications, viral marketing is the most important application of social influence. In fact, influence maximization was proposed as a mathematical prototype of viral marketing. In the following, we first introduce applications in viral marketing. Then, we show some other interesting applications based on social influence analysis.

Viral Marketing

Due to the “word-of-mouth” effect, social network is an ideal platform for viral marketing.

The key idea behind viral marketing is that by targeting on only a small number of individuals (persuade them to adopt the product), we can trigger a large cascade of (product) adoption spreading in a social network. Kempe et al. (2003) proposed its mathematical prototype, the well-known influence maximization problem, which we have discussed in the previous sections. Although influence maximization has its root in viral marketing, it is still too simple for many real-life scenarios. To fill this gap, researchers have proposed different kind of variations of influence maximization. For instance, Chen et al. (2012) considered the time-delay factor in information diffusion and solved the influence maximization problem with respect to time constraint. Wang et al. (2016) took spatial factor into consideration when they dealt with influence maximization in geo-social network. Tang et al. (2014a) tried to maximize the influence and the diversity of the influenced crowd simultaneously. The original setting of influence maximization assumes that a node is either a seed or not. In other words, either we offer a user the free product (i.e., select her as the seed node) or not. Yang et al. (2016) relaxed this assumption and investigated the question about what discounts we should offer to social users so that the product adoption is maximized. Another implicit assumption of influence maximization is that the active state of a node means she adopts the product. Bhagat et al. (2012) pointed out that product adoption should be distinguished from influence spread, since influence spread is essentially used as proxy for product adoption. Similarly, Wang et al. (2015) argued that information awareness is not the same as the information propagation and proposed a new concept “information coverage,” which captures the values of nodes that are inactive but aware of the propagating information. In the case of multi-items diffusion, Lu et al. (2015) studied comparative influence maximization problem which covers the full spectrum of item interactions from competition to complementarity.

Influence-Based Applications

Since people are often influenced by their friends in social networks, social influence is an important

factor that we need to consider when we try to analyze different kind of social behaviors. Therefore, in addition to viral marketing, there are other interesting influence-based applications. For example, to predict product adoption rate in social networks, Wu et al. (2015) introduced a social user decision function which leverages various factors, including neighbor influence, crowd wisdom, etc. To analyze the behaviors of taxi drivers, Xu et al. (2012b) verified the existence of the latent vehicle-to-vehicle network and revealed how social influence propagation affects the prediction of taxi drivers’ future behaviors. For better social marketing, Liu et al. (2014a) combined recommendation techniques and social influence tools. Meanwhile, Ma et al. (2015) utilized social influence to identify interested but hesitant users. In the area of social media analysis, Xu et al. (2012a) annotated media content through social influence analysis. Similarly, Wang et al. (2013) applied social influence maximization method on words network for text summarization.

Future Directions

In the past years, social influence has been extensively studied and much progress has been achieved. However, to have a more comprehensive understanding of social influence, there are also some important issues should be addressed, and these issues lead to the future research directions.

Parameter Learning

Some key parameters exist in most diffusion models, such as propagation probability in the IC model and influence weight in the LT model (i.e., $b_{(u,v)}$). These parameters have a great influence on the performance of the corresponding models (He and Kempe 2016). To assure the effectiveness of the diffusion models, we need to learn these parameters from real-world data instead of just assigning some empirical values. Until now, however, how to learn model parameters remains a big challenge. One reason is that the sample complexity of such learning tasks is very high, since we need to learn influence strength

(propagation probability or influence weight) of each node pair. Another important reason is that there is too much noise in real-world data. It is hard to isolate the contribution of influence from other factors (e.g., homophily) in the diffusion of information in social networks. Therefore, there is still a long way to go before we can tackle this issue.

Effectiveness Validation

Influence maximization models output target customers for viral marketing. Then a natural question arises: how do we know if the marketing strategy works? In other words, are the selected customers really worth to invest on? As we mentioned above, it is hard to recognize the contribution of influence in the diffusion of information propagation, which means that we actually do not have any ground truth to validate the effectiveness of the strategy. A possible way to solve this problem is conducting randomized controlled experiments in real social network sites (Chen 2015). However, how to cooperate with social network service providers and deploy the testing system is still a problem we have to face.

Dynamic Network

Until now, most of works on social influence assume that social network is static. However, in real world, social networks evolve over time. Thus, to better describe the process of information diffusion, we should consider the dynamics of information propagation and network structure at the same time. In the aspect of diffusion models, we need to improve existing models so that the change of network structure can be detected and processed in real time. As for influence maximization and other applications, online algorithms should be developed so that the results can be updated incrementally.

Cross-References

- ▶ [Influence Propagation in Social Networks with Positive and Negative Relationships](#)
- ▶ [Mathematical Model for Propagation of Influence in a Social Network](#)
- ▶ [Social Influence Analysis](#)

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Influence Networks

- ▶ [Policy Networks: History](#)

Influence of Behaviors

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Influence Propagation in Social Networks with Positive and Negative Relationships

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Synonyms

[Friendship and antagonistic relationships](#); [Influence maximization](#); [Influence propagation](#); [Signed social networks](#)

Glossary

Signed network A social network in which social links are labeled with positive or negative signs

Balanced triangle	A triangular network pattern with an odd number of positive links
Unbalanced triangle	A triangular network pattern with an odd number of negative links
Monolithic relationship	Social relationships that can take a single type, e.g., friendship
Propagation cost	Cost that is incurred by social or physical entities such as propagation delay, social tie strength, or the impact of propagating ideas

Definition

Online social networks exhibit a wide range of relationship types, including friendship and antagonism. As such, the type of relationship between two persons, whether positive or negative, impacts how they are influenced by one another. Signed social networks are defined as networks in which individuals can be linked with positive or negative relationships. Influence diffusion and propagation models in signed social networks have been considered in two major line of problems. The first group of problems is based on influence diffusion and maximization in signed networks, among which we review the voter model, epidemic models, independent cascade model, and game theoretic models. These models focus on the spread behavior of opinions in a signed social network or on identifying the key nodes that can trigger a propagation behavior leading to maximum influence spread. The second group of problems is focused on propagation strategies for influencing a target node by taking into account key network metrics such as propagation costs. The propagation cost is due to both social and physical factors such as propagation delay, strength of social ties, or the impact factor of the propagating idea.

Introduction

Social networks have become a major domain for information dissemination, with the proliferation

of smart devices and portable computers (Easley and Kleinberg 2010). Individuals in online social communities are often linked with relationships ranging from friendship to antagonism. Conventional social network analysis, on the other hand, is often focused on *monolithic* relationships that treat all relations as *friendly*. The need for incorporating different relationship types in social network analysis has recently been emphasized in various studies (Brzozowski et al. 2008; Kunegis et al. 2009; Hogg et al. 2008; Lampe et al. 2007; Anchuri and Magdon-Ismael 2012).

Influence diffusion and propagation characteristics in online social networks has been extensively studied for networks with monolithic relationships (Domingos and Richardson 2001; Kempe et al. 2003; Kimura and Saito 2006; Chen et al. 2009, 2010; Goyal et al. 2010), primarily focusing on how to identify the key users that maximize the spread of influence through the use of probabilistic methods or optimization approaches as well as effective heuristics. For networks that consist of both positive and negative relationships, it has been showed in Li et al. (2013a, 2015a), Chen et al. (2011) and Guler et al. (2015) that taking the relationship type into consideration can lead to significant changes in the diffusion patterns for the spread of opposing ideas.

Influence propagation models in social networks with positive and negative relationships are often based on the principle of *homophily* (McPherson et al. 2001), which states that persons are more likely to agree with others who are similar to them and oppose to the ideas that come from others that are dissimilar (Brzozowski et al. 2008). For instance, consider a voting process between two candidates, candidate X and candidate Y , representing two opposite political views. Suppose that *Alice* supports candidate X and tells Bob that she supports candidate X . If Bob perceives Alice as a like-minded friend, he is more inclined to support candidate X , whereas it is more likely that he will support candidate Y if he and Alice have antagonistic world views. This phenomenon can impact the diffusion of ideas, opinions, or products in a social network, leading

to influence patterns that are based on the type of relationship that exists between the interacting parties. In particular, a person is more likely to favor an idea if that idea is promoted by a like-minded friend who shares similar interests. In contrast, an idea supported by an individual with an antagonistic world view is more likely to cause the person to pause and even resist the idea. If, on the other hand, an individual with an opposite world view is against an idea, going against the neighbor leads to a positive disposition towards the original idea. This intuition fits well with various historical observations, including, for instance, the formation of the European alliances before World War I (Langer 1977; Schmitt 1924). In essence, relationship structures can have significant impact on the influence patterns in a social network.

In the sequel, major influence propagation models are reviewed for social networks with positive and negative relationship types. Initially, we review the network models for influence diffusion and maximization, including the voter model, epidemic models, game-theoretic models, and the independent cascade model (Li et al. 2013a, b, 2014, 2015b; Chen et al. 2011; Shafaei and Jalili 2014). These models are focused on the spread behavior of opposing ideas in a network with positive or negative relationships, or the identification of a small subset of nodes that can trigger a propagation behavior in the network so that maximum number of nodes are influenced eventually. Next, we discuss the targeted influence propagation model, which finds the optimal propagation policies to influence a target node in the network positively, by taking into account key network metrics such as the total propagation cost or the number of negatively influenced users (Guler et al. 2014a, b, 2015). These models are based on the intuition that parties in social communities have a tendency to take a side *in favor of* or *against* an opinion, candidate, or product by taking into account the information that they have access to. Accordingly, a judicious strategy for choosing the propagation path for an idea or a product is impactful in influencing a target node positively or negatively.

Key Points

Understanding influence diffusion and propagation mechanisms in social networks is a major step towards understanding how phenomena spreads in human societies. Influence diffusion has been well studied for networks with monolithic relationships, by treating all social relationships as friendship. Human societies, on the other hand, often exhibit various types of social relations, including both friendship and antagonism. The type of relationship between two individuals has a key role in influence propagation, that is, whether the latter develops a positive or a negative opinion about a product, candidate, or an idea favored by the former. To this end, one needs to take into account the type of relationships, positive or negative, in the social network while studying diffusion policies. Therefore, influence diffusion in networks with positive and negative relationships is distinct from networks with monolithic relationships, which is demonstrated by the differences in the optimal diffusion and propagation strategies between the two types of networks.

Historical Background

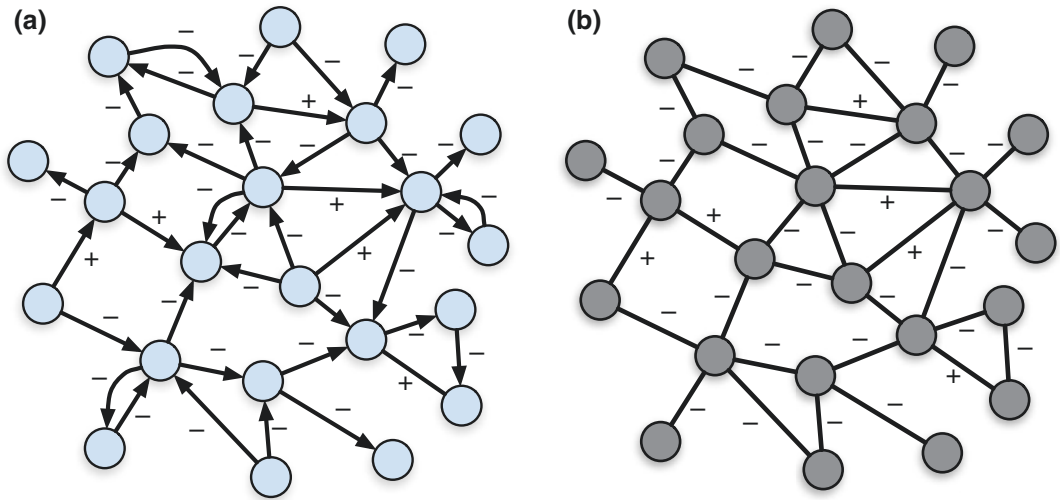
The impact of social relations on influence spread and information dissemination have been investigated in various studies (Brzozowski et al. 2008; Kunegis et al. 2009; Hogg et al. 2008; Lampe et al. 2007; Anchuri and Magdon-Ismael 2012; Domingos and Richardson 2001; Kempe et al. 2003; Watts and Strogatz 1998; Girvan and Newman 2002). Relationship types in online social networks often have a complex structure, ranging from like-minded friends to ideological foes. By focusing on purely monolithic relationships, however, conventional social network analysis often treats all relations as friendship relations. Identifying positive and negative relationship types in social networks dates back to balance and status theories in social psychology (Heider 1946; Cartwright and Harary 1956; Harary and Kabell 1980), to provide a graph-theoretic description

of balanced structures in organizational networks. Signed links have been incorporated to represent positive and negative relationships in a social network (Leskovec et al. 2010a) and to explore the evolution of user behavior patterns. A machine-learning framework is utilized in Leskovec et al. (2010b) to predict positive and negative links in social networks. Detecting the community structures in a social network with positive and negative relationships has been considered in (Anchuri and Magdon-Ismael 2012). Another direction in signed network analysis has been to turn an unsigned social network into a signed one by predicting the type of a social tie, positive or negative, between the parties in the social network (Yang et al. 2012; Agrawal et al. 2013; Papaoikonomou et al. 2014).

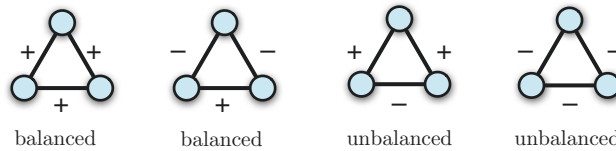
The impact of relationship types on the propagation behavior has also been utilized to investigate the diffusion of opposing ideas and to identify the important nodes in the network that can trigger a propagation behavior which achieves the maximum spread of influence (Li et al. 2013a, 2015a; Chen et al. 2011). Various propagation models are proposed to investigate the diffusion behavior in networks with positive and negative relationships, including the voter model (Li et al. 2013a, 2015a), independent cascade model (Chen et al. 2011; Li et al. 2014; Srivastava et al. 2015), game-theoretic and linear threshold models (Shafaei and Jalili 2014), and epidemic models (Li et al. 2013b; Fan et al. 2012). The influence propagation behavior in networks with positive and negative relationships has also been investigated for identifying the optimal influence propagation policies for influencing a target node positively and by taking into account key network metrics such as propagation costs (Guler et al. 2014a, b, 2015).

Signed Social Networks

A signed social network is a social network in which the relationships between users are defined as either positive or negative.



Influence Propagation in Social Networks with Positive and Negative Relationships, Fig. 1 Signed social network models: (a) directed, (b) undirected



Influence Propagation in Social Networks with Positive and Negative Relationships, Fig. 2 Balanced and unbalanced triangle structures

Definition 1 (Signed social network) Let $G = (V, E)$ be a directed graph representing a social network with $|V|$ nodes, where the vertex set V and the edge set E represent the persons and social relationships, respectively. A directed edge (i, j) from node i to node j exists if $(i, j) \in E$. The edge (i, j) is labeled with the sign “+” if user i holds a positive opinion about user j , e.g., user i trusts user j or perceives user j as a friend. If user i holds a negative opinion about user j , e.g., user i distrusts user j or perceives user j as a foe, then the edge (i, j) is labeled with a “-” sign.

A signed social network with directed links is illustrated in Fig. 1a. If the relationships between the individuals are mutual, i.e., the edge label of (i, j) is equal to that of (j, i) , then, the social network can be represented with an undirected graph as in Fig. 1b.

The social links in signed networks often evolve with time. The steady state behavior of the link structures often demonstrates common patterns. These common patterns are described by the structural balance theory (Heider 1946; Cartwright and Harary 1956; Harary and Kabell 1980), which distinguishes between balanced and unbalanced social structures demonstrated in Fig. 2 for a network of three nodes. Structural balance theory states that the social links in a signed network evolve to a balanced structure because unbalanced structures cause tension between the parties who eventually change the type of their social relationships and form balanced structures.

Influence Diffusion and Maximization

Diffusion characteristics of information has been studied broadly in the context of social networks

with monolithic relationships, when parties are connected to each other via friendship relations (Domingos and Richardson 2001; Kempe et al. 2003; Kimura and Saito 2006; Chen et al. 2009, 2010; Goyal et al. 2010). The diffusion process for signed networks, i.e., when parties can engage in multiple types of relationships, has received growing interest with the observation that the type of relationship between two parties, positive or negative, has great impact on the influence spread behavior (Li et al. 2013a). Influence diffusion models for signed networks are primarily based on extending the diffusion models for networks with monolithic relationships to take into account the impact of positive and negative relationships.

In a social network, influence maximization is the study of identifying a small subset of nodes in the network that will initiate a propagation pattern leading to the maximum spread of influence (Kempe et al. 2003). An important application area of this model is recommender systems and viral marketing, in which companies provide samples of their products to a small number of promoters, with the expectation that they will influence their friends who will then influence their friends. As such, companies try to identify the best group of promoters that will trigger a cascade structure that will reach the maximum number of users. In the remainder of this section we review the major influence diffusion and maximization models in the context of signed social networks.

Independent Cascade Model

Originally proposed for unsigned social networks in Kempe et al. (2003), the independent cascade model allows users to actively make attempts to influence their neighbors. In this model, each node in the network can be in one of two states, *active* or *inactive*, noting that all nodes in the network are in the *inactive* state in the beginning. In a sense, the active state corresponds to a user adopting an opinion or a product, whereas an inactive state corresponds to a user who has not adopted it. A selected subset of k nodes are then influenced so that they are in the *active* state. In the next step, each user in the active state makes a single attempt

to influence each one of its inactive neighbors. Denoting i as the active node and j as the inactive neighbor, then node i succeeds in influencing node j with probability $p(i, j)$. A common practice is to assume $p(i, j) = \alpha$ for all i, j for some constant $0 < \alpha < 1$. If node i succeeds in influencing node j , then node j takes the *active* state starting from the next time step. Once a node makes an attempt to influence a neighbor, whether it succeeds or fails, it can take no actions to influence the same neighbor in the subsequent time steps. Whenever a node is successfully influenced by a neighbor, no other neighbors can try to influence it. The propagation terminates once no more activation attempts can take place in the network.

The conventional independent cascade model is focused on the propagation of positive opinions and a social network in which all social links correspond to friendship relations. The propagation of positive opinions corresponds to the scenario where each active user is positively influenced by an opinion or a product and consequently tries to influence others in a positive manner. Consequently, no negative opinions emerge or propagate in the network. Chen et al. (2011) consider the scenario when the nodes can develop and propagate negative opinions. For instance, a user may have a bad experience with a product given to her for promotion, and subsequently tells her friends about her experience, influencing them with negative opinions about the product. Similar to the original independent cascade model, the propagation model in Chen et al. (2011) allow the selection and activation of a group of k nodes in the network, for instance, by providing free samples of a product or a service. Each activated node is influenced positively by the product as a result of a good experience with probability q , whereas with probability $1 - q$, the node is influenced negatively due to a bad experience. Accordingly, the parameter q is termed the *quality factor*. Each active node, whether positively or negatively influenced, makes an attempt to activate each of its inactive neighbors. Node i succeeds in activating node j with probability $p(i, j)$. If node i succeeds in activating node j , the type of influence adopted by node j depends on the type of influence of node i . In particular, if

node i is a positively influenced node, then node j is influenced positively with probability q , or negatively with probability $1 - q$. On the other hand, if node i is a negatively influenced node, then node j is also influenced negatively. In that sense, being activated corresponds to buying a product, whereas the type of influence depends on the experience the individual has with the product. The case when $q = 1$ reduces to the conventional independent cascade problem.

This model incorporates the concept of *negativity bias* from social psychology, as negative opinions in the model dominate the positive ones. The goal is to maximize the expected number of positively influenced nodes, which is termed as the *positive influence spread*. The influence maximization problem is formally defined as follows. Consider a social network represented by a directed graph G with a set of nodes V , a quality factor q , and activation probabilities $p(i, j)$ for $i, j \in V$. Let $\sigma(U, q)$ denote the expected number of positively influenced nodes when the set of seed nodes that are activated positively in the beginning is $U \subseteq V$. Then, the influence maximization problem is defined as finding the optimal seed set U^* such that

$$U^* \in \arg \max_{U \subseteq V, |U|=k} \sigma(U, q) \quad (1)$$

The influence maximization problem in (1) is shown to exhibit properties such as monotonicity and submodularity, allowing the use of greedy approximation algorithms as in the original independent cascade problem (Kempe et al. 2003). The social network considered in this model consists solely of friendship links.

The independent cascade process for the propagation of opposing ideas has been extended in Li et al. (2014) to take into account the positive and negative relationships. Accordingly, the new model is called *polarity-related independent cascade* (IC-P). The polarity is associated with the type of relationship exhibited between the parties in the social network. In that sense, each social link is labeled with a positive or negative sign. The sign of the link from node i to node j is

represented by a function $f(i, j) \in \{+, -\}$. In this model, $f(i, j) = +$ indicates that node j perceives node i as a *friend*, or *trusts* node i . On the other hand, $f(i, j) = -$ indicates that node j perceives node i as a *foe*, or *distrusts* node i . The state of a node i is denoted by $S(i)$, where

$$S(i) = \begin{cases} 1 & \text{if } i \text{ is active and positively influenced} \\ -1 & \text{if } i \text{ is active and negatively influenced} \\ 0 & \text{if } i \text{ is inactive} \end{cases} \quad (2)$$

Initially, a small set of nodes are selected and activated with positive influence, while the remaining nodes in the network are inactive. In other words, denoting the set of all nodes in the network by V and the initial set of activated nodes by U ,

$$S(i) = \begin{cases} 1 & \text{for all } i \in U \\ 0 & \text{for all } i \in V \setminus U \end{cases} \quad (3)$$

In the next time step, each active node makes a single attempt to activate each one of its inactive neighbors. The probability that node i succeeds in activating node j is given by $p(i, j)$ where $0 \leq p(i, j) \leq 1$. If node i succeeds in activating node j , then node j is influenced with a polarity given by

$$S(j) = S(i) \times f(i, j). \quad (4)$$

In that sense, Eq. 4 implies that if node i is a positively influenced node, then node j is also influenced positively if it trusts node i , whereas node j is influenced negatively if it distrusts node i . If instead node i is a negatively influenced node, then node j is influenced negatively if it trusts node i and positively if it distrusts node i . Once node i makes an attempt to activate node j , it can make no further attempts to activate the same node in the consequent time steps. The state of a node cannot be altered once it is activated positively or negatively. Different from the original independent cascade model, Li et al. (2014) allow each node to be activated at most once in a given time step, whereas in the original independent cascade model, multiple active neighbors can

make attempts to influence the same node in a single time step.

Given the influence diffusion model, Li et al. (2014) propose two influence maximization problems, corresponding to the maximization of positive and negative influence, respectively. The positive influence maximization (PIM) problem is to find a subset U^* of size k from the set of all nodes V that maximizes the expected number of positively influenced nodes,

$$U^* = \arg \max_{U \subseteq V, |U|=k} \sigma_+(U) \quad (5)$$

where the function $\sigma_+(U)$ identifies the expected number of positively influenced nodes when the initial set of nodes activated with positive influence, i.e., the set of seed nodes, is U .

The negative influence maximization (NIM) problem, on the other hand, is to find a subset U^* of size k that maximizes the expected number of negatively influenced nodes,

$$U^* = \arg \max_{U \subseteq V, |U|=k} \sigma_-(U) \quad (6)$$

where $\sigma_-(U)$ is expected number of negatively influenced nodes when the initial set of nodes activated with positive influence is U . As the initial set of seed nodes in both Eqs. 5 and 6 are activated with positive influence, negative influences that emerge in the diffusion process are due to the relationship structure of the signed social network. It is shown in Li et al. (2014) that both influence maximization problems from Eqs. 5 and 6 satisfy monotonicity and submodularity properties. As a result, similar to the original independent cascade problem, a greedy hill-climbing algorithm can be utilized to find an approximate solution to Eqs. 5 and 6 that approximates the optimal solution within a factor of $(1 - \frac{1}{e})$ (Kempe et al. 2003).

The influence diffusion scheme considered in Srivastava et al. (2015) extends the independent cascade model from Li et al. (2014) by allowing the seed nodes, i.e., nodes that are initially activated in the network, to be influenced positively or negatively. The influence maximization

problem is to find a set of k initial nodes and their influence types, positive or negative, to maximize the expected number of nodes that are influenced, without loss of generality, positively. Srivastava et al. (2015) show that the new problem, termed the *Signed Network Influence Maximization* (SNIMax) problem, is NP-hard. It is also proved via a counter-example that, unlike the previous independent cascade models (Kempe et al. 2003; Li et al. 2014), the new influence diffusion model does not satisfy the monotonicity property. Accordingly, the greedy algorithm from Kempe et al. (2003) cannot guarantee the approximation of the optimal solution to within a factor of $(1 - \frac{1}{e})$. Instead, Srivastava et al. (2015) propose a novel heuristic algorithm to find the set of k initial nodes and their influence types, positive or negative. The algorithm starts with an empty seed set and for each $t = 1, \dots, k$, incrementally adds node i with an influence type $c \in \{\text{positive, negative}\}$ to the seed set if activating i with the influence type c maximizes the expected number of new positive influences in the next time step. The independent cascade model is extended in Li et al. (2015b) by allowing each node to be influenced multiple times by his/her friends and by taking into account the impact of structural balance during influence propagation.

Voter Model

The classical voter model considers a set of nodes that are connected to each other with monolithic relationships, i.e., each directed edge denotes a positive relation. At each step, each user selects one of its outgoing neighbors at random, and adopts the opinion of the selected neighbor. In that sense, user opinions are not fixed in the voter model, and can change in time.

This model is extended in Li et al. (2013a, 2015a) to the context of signed networks, to study how positive and negative relationships affect the spread of two competing opinions. As in the classical voter model, this model allows, at each step, every user to adopt an opinion based on interacting with a random outgoing neighbor. Unlike the classical model, the type of opinion adopted depends on the type of relationship

between the user and its neighbor. Letting X and Y denote two competing opinions, a user will adopt opinion X if the selected outgoing neighbor supports X and the two users have a positive relationship between them. In contrast, the user will adopt opinion Y if the neighbor supports X but the two users have a negative relationship.

The relationship structure for the social network is represented by a weighted adjacency matrix A , with the element A_{ij} in row i and column j corresponding to the directed edge from node i to node j . The individual entries of A may be positive, corresponding to a positive relationship, or negative, corresponding to a negative relationship. If there is no social contact from node i to node j , either positive or negative, then $A_{ij} = 0$. The absolute value $|A_{ij}|$ of each entry represents the strength of the social tie from node i to node j . Each user holds one of two opposite opinions $\{X, Y\}$. In the proposed voter model for signed networks, each user selects an outgoing neighbor uniformly at random, with probability of choosing neighbor j being $p(i, j)$ where

$$p(i, j) = \frac{|A_{ij}|}{\sum_l |A_{il}|} \quad (7)$$

which is proportional to the weight of the strength of the social tie from node i to node j . Node i then adopts the opinion of user j if

$$A_{ij} > 0 \quad (8)$$

whereas if

$$A_{ij} < 0 \quad (9)$$

then node i adopts the opinion other than the opinion of user j . This propagation model is explored for both short-term and long-term diffusion dynamics. Short-term diffusion dynamics investigate the distribution of nodes in the network influenced by opinions X and Y at a future step $t > 0$. Long-term diffusion dynamics investigate whether or not the distribution of nodes influenced by opinions X and Y converge. If they do so, then the goal is to identify the steady state

distribution of the two opinions. For the short-term diffusion patterns, an exact formula is derived for the distribution of opinions at each propagation step. For the long-term diffusion patterns, the steady-state distribution of opinions is obtained in closed-form.

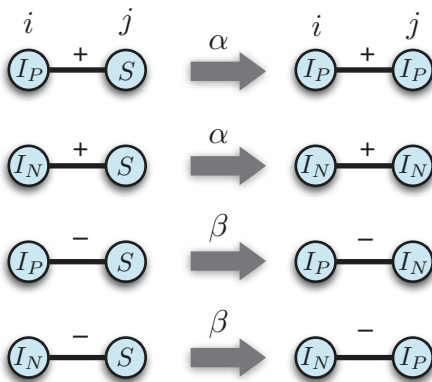
The diffusion model is then utilized to study influence maximization in a signed social network. In this framework, a small number of nodes, i.e., no more than k nodes, can be initialized with the opinion X , and all the remaining nodes are assumed to be following opinion Y . The goal is to find the best set of initial nodes to maximize the expected number of nodes who have adopted opinion X in the short and long terms. To do so, the Signed Voter model Influence Maximization (SVIM) algorithm is introduced, which is based on computing the influence contributions of the individual nodes to find the subset with the highest contribution.

Epidemic Models

Diffusion characteristics of social phenomena often have similarities to the spreading behavior in epidemics. As a result, several propagation models have been introduced to model influence diffusion in a signed network as an epidemic (Li et al. 2013b; Fan et al. 2012). In essence, epidemic models combine the diffusion principles of epidemics with the propagation characteristics of signed networks, which are based on the fact that opinions spread differently through positive and negative links, as people tend to adopt the opinions of their friends while opposing to the opinions of their foes.

The diffusion of two opposing ideas in a signed network is modeled as an epidemic in (Li et al. 2013b). The diffusion process is based on the susceptible-infected-recovered (SIR) epidemic model (Kermack and McKendrick 1927). The SIR epidemic model assigns to each node in the network one of three states: *susceptible* if a node is healthy but is susceptible to infection, *infected* if the node is infected by a disease and may spread it to the neighboring susceptible nodes, and *recovered* or *removed* if a node that was once infected has recovered from the infection due to immunization and is no longer susceptible to the disease.

In order to model influence propagation in a signed network as an epidemic, Li et al. (2013b) assume that a node can be in one of five states: susceptible with neutral opinion (S), infected by positive opinion (I_P), infected by negative opinion (I_N), recovered with positive opinion (R_P), and recovered with negative opinion (R_N). State S indicates that a node holds no opinion about the topic but is susceptible to be influenced by a positive or negative opinion in the future. States I_P and I_N imply that the node currently holds a positive or negative opinion about the topic, respectively, and may influence neighboring nodes. States R_P and R_N imply that the node holds a positive or negative opinion about the topic, respectively, but will no longer attempt to influence the neighboring nodes. Propagation of influence depends on the relationship between the interacting parties. Consider two nodes, node i and node j , with a positive relationship and assume that node i is in state I_P or I_N whereas node j is in state S . Then, node j adopts the same opinion as node i with probability α . On the other hand, if the relationship between the two nodes is negative, then, with probability β , node j adopts the opinion opposite to that of the opinion of node i . The characteristics of the influence process is illustrated in Fig. 3. Node i cannot influence node j if node j is already in one of I_P , I_N , R_P , or R_N states. Lastly, a node in state I_P or I_N switches to state R_P or R_N , respectively, with probability ρ .



Influence Propagation in Social Networks with Positive and Negative Relationships, Fig. 3 Influence characteristics for the epidemic model

The time evolution of the density of the nodes in each state is described by a set of coupled non-linear differential equations,

$$\begin{aligned} \frac{ds(t)}{dt} &= -(\alpha\theta + \beta(1 - \theta))Ks(t)i_P(t) \\ &\quad - (\alpha\theta + \beta(1 - \theta))Ks(t)i_N(t) \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{di_P(t)}{dt} &= \alpha\theta Ks(t)i_P(t) \\ &\quad + \beta(1 - \theta)Ks(t)i_N(t) - \rho i_P(t) \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{di_N(t)}{dt} &= \beta(1 - \theta)Ks(t)i_P(t) \\ &\quad + \alpha\theta Ks(t)i_N(t) - \rho i_N(t) \end{aligned} \quad (12)$$

$$\frac{dr_P(t)}{dt} = \rho i_P(t) \quad (13)$$

$$\frac{dr_N(t)}{dt} = \rho i_N(t) \quad (14)$$

where $s(t)$, $i_P(t)$, $i_N(t)$, $r_P(t)$, and $r_N(t)$ represent the density of the nodes in states S , I_P , I_N , R_P , and R_N , respectively, such that

$$s(t) + i_P(t) + i_N(t) + r_P(t) + r_N(t) = 1 \quad (15)$$

and θ is the probability of positive relationships, whereas K is the average number of neighbors of any node in the network.

The set of Eqs. 10, 11, 12, 13 and 14 are solved at an early stage of the diffusion process when $i_P(t) \ll 1$ and $i_N(t) \ll 1$ with the initial conditions

$$\begin{aligned} s(t) &= 1 - \varepsilon_P - \varepsilon_N \approx 1, \\ i_P(0) &= \varepsilon_P \cong 0, \\ i_N(0) &= \varepsilon_N \cong 0, \\ r_P(0) &= r_N(0) = 0 \end{aligned} \quad (16)$$

via linear approximation, leading to the following densities of positively or negatively influenced nodes in the network,

$$i_P(t) = \frac{(\varepsilon_P + \varepsilon_N)e^{\mu t} + (\varepsilon_P - \varepsilon_N)e^{\lambda t}}{2} \quad (17)$$

$$i_N(t) = \frac{(\varepsilon_P + \varepsilon_N)e^{\mu t} + (\varepsilon_N - \varepsilon_P)e^{\lambda t}}{2} \quad (18)$$

where $\mu = (\alpha\theta + \beta(1 - \theta)K - \rho)$ and $\lambda = (\alpha\theta - \beta(1 - \theta))K - \rho$, leading to the following definition of critical rates.

Given a signed network with $\varepsilon_P \approx 0$ and $\varepsilon_N \approx 0$, rates (α^c, β^c) are called *opinion spreading critical rates* such that, if $\alpha \leq \alpha^c$ and $\beta \leq \beta^c$, then $r_P(\infty) \approx 0$ and $r_N(\infty) \approx 0$, whereas if $\alpha > \alpha^c$ and $\beta > \beta^c$, then $r_P(\infty) > 0$ and $r_N(\infty) > 0$. The critical rates are characterized for the following three network structures. The first one is a network with positive relationships only, i.e., $\theta = 1$. In this case, the critical rates are shown to satisfy $\alpha^c = \frac{\rho}{K}$ and $\beta^c = 1$. The second one is the case when the network has solely negative relationships, i.e., $\theta = 0$, for which the critical rates are shown to satisfy $\alpha^c = 1$ and $\beta^c = \frac{\rho}{K}$. The third one is a network with $0 < \theta < 1$. In this case, the critical rates satisfy $\alpha = \frac{\rho}{\theta K}$ and $\beta = -\frac{\theta \alpha}{(1-\theta)} + \frac{\rho}{(1-\theta)K}$. Simulation results for the opinion spreading patterns demonstrate that the numerical results agree with the theoretical analysis.

Game Theoretic Models

Influence propagation in a signed network is modeled in Shafaei and Jalili (2014) by utilizing a network coordination game. By considering two behaviors X and Y , each pair of nodes is associated with a payoff matrix with respect to the behaviors the parties can take and the sign of the relationship between the two nodes. The payoff matrix in Fig. 4a corresponds to the case when the two nodes have a positive relationship, whereas the

payoff matrix in Fig. 4b corresponds to the case when the two nodes have a negative relationship. The payoff for each person is then calculated by taking into account the number of neighbors that have adopted each behavior. To do so, \mathcal{X}_i^+ and \mathcal{X}_i^- is defined as the set of neighbors with positive and negative relations with node i who have taken behavior X , respectively. Similarly, \mathcal{Y}_i^+ and \mathcal{Y}_i^- is defined as the set of neighbors with positive and negative relations with node i who have taken behavior Y , respectively. Then, the payoff received from taking behavior X is given as

$$f_i(X) = |\mathcal{X}_i^+|x_1 + |\mathcal{X}_i^-|x_2 \quad (19)$$

whereas the payoff received from taking behavior Y is given as

$$f_i(Y) = |\mathcal{Y}_i^+|y_1 + |\mathcal{Y}_i^-|y_2 \quad (20)$$

for node i . In Eq. 19, $x_1 > 0$ and $x_2 < 0$ are defined as the payoffs two friends or foes receive when they both choose behavior X , respectively, whereas $y_1 > 0$ and $y_2 < 0$ in Eq. 20 are the payoffs received when two friends or foes choose behavior Y , respectively. Node i then takes the behavior that maximizes its payoff. Accordingly, node i takes behavior X whenever,

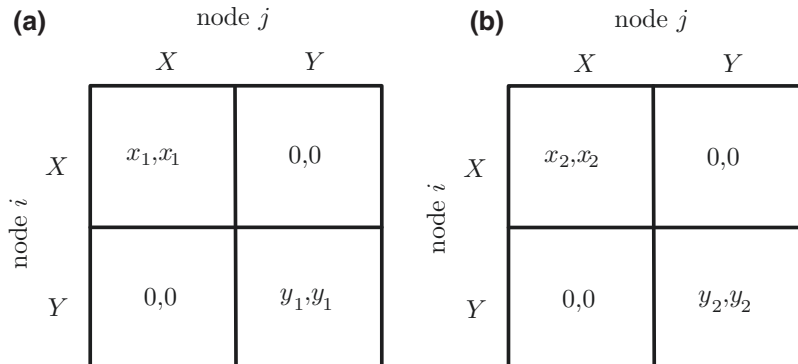
$$|\mathcal{X}_i^+|x_1 + |\mathcal{X}_i^-|x_2 > |\mathcal{Y}_i^+|y_1 + |\mathcal{Y}_i^-|y_2 \quad (21)$$

and takes behavior Y otherwise.

The propagation model considers a network in which all nodes have initially adopted behavior Y .

Influence Propagation in Social Networks with Positive and Negative Relationships,

Fig. 4 Payoff matrix for a pair of nodes with a (a) positive relationship, (b) negative relationship



A given number of randomly selected nodes are then switched to behavior X . At each subsequent step, each node in the network evaluates its payoff and selects the optimal behavior, X or Y , by using Eq. 21. Propagation continues until no changes occur in the network, i.e., steady state is reached. Numerical evaluations are performed to identify the relation between the number of nodes that has adopted behavior X at the end of propagation, termed as the cascade depth, and the community structure of the signed network. To do so, the community structure of the signed network is modeled by leveraging structural balance theory. The numerical results show that the cascade depth and the extent of influence diffusion is closely tied to the community structure in the signed network, in that the cascade depth decreases as the social network becomes more closely tied. In contrast, the cascade depth increases as the network becomes more homogeneous.

In case the entries of the payoff matrix is a fixed constant, then the model reduces to the linear threshold model (Kempe et al. 2003), in which each user adopts a behavior if the number of neighbors who have already adopted the behavior is above a given threshold.

Targeted Influence Propagation

The targeted influence propagation model addresses the optimal strategies for influencing a target node (Li et al. 2011, 2015c; Srinivasan et al. 2014) as well as how to optimize key network metrics such as propagation costs (Guler et al. 2014a, b, 2015). For instance, suppose that an online voting process is taking place between two candidates, candidate X and candidate Y , who possess opposite world views. An online recommender is suggesting one of the two candidates to a set of users. Assume that *Alice* is such a user with two neighbors, *Bob* and *Eve*. Bob has a similar world view with Alice, whereas Eve has an antagonistic world view. The recommender can make suggestions such as “Bob likes candidate Y , do you want to support candidate Y , too?” Suppose that the recommender knows that both Bob and Eve support candidate Y . The recommender

can suggest either “Bob likes candidate Y , do you want to support candidate Y , too?” or “Eve likes candidate Y , do you want to support candidate Y , too?” to Alice. Since Bob has similar interests, Alice is likely to support candidate Y if the recommender follows the former statement. If, instead, the recommender follows the latter statement, Alice is more likely to have a negative opinion about candidate Y as she considers Eve as an ideological foe. As such, the recommender has to decide what type of statement to follow while making a recommendation. Therefore, social relationship structures can have significant impact on the optimal recommendation strategies tailored towards the network interests.

For cases when it is unfeasible to influence a target node directly, one may utilize an indirect strategy by identifying a group of nodes that can propagate the information to the target node resulting in the target node being influenced positively. Real world networks often incur propagation costs for the transmission of various types of information that originates from both social and physical aspects of communication. As a result, designing the optimal propagation path to influence a target node necessitates taking into account the network propagation costs. As such, Guler et al. (2014a, b, 2015) consider the problem of identifying the optimal propagation strategy to influence a target node in favor of a product or an item, by taking into account the network propagation costs.

Guler et al. (2015) study the problem of minimizing the end-to-end propagation cost incurred through the network for influencing a target person in favor of a given idea. In this model, every social link incurs a propagation cost, due to a combination of various social and physical phenomena such as the interaction frequency between the users, network propagation delay, the strength of social ties, or the impact of the propagating idea. A single node, termed the source node, is activated initially by an external event such as an article in the news or an advertisement. The information is then passed by the source node to one of its neighbors, leading to the neighbor being positively or negatively influenced. The information propagation continues until it is received at the

target node. This model can also be applied to a recommender system, in which a recommender makes suggestions to a group of online users, by utilizing the preferences of other users. In this case, a person is more likely to be influenced positively if a product has been favored by a friend in the past, whereas the person is more likely to oppose buying the product if it has been favored by a foe. This model applies to networks in which no direct link exists to directly influence the target node or the direct link is very costly, such as a public figure known by a large population. In the sequel, we review the propagation models designed for targeted influence.

Minimum-Cost Influence Propagation

Consider an acyclic directed signed network. The coordinates of node $i \in V$ are denoted by the tuple (i_x, i_y) . A node is represented by its index and its coordinates interchangeably. Every edge $(i, j) \in E$ is labeled with a sign $s_{i,j} \in \{-1, 1\}$ according to the type of relationship, positive or negative, between parties i and j . Denote by i_o and i_d the source and destination nodes, respectively. From node i to a neighbor node j there exists a propagation cost $d_{i,j} \geq 0$. The sign of the relationship type, positive or negative, between node i and its neighbor j is represented by $s_{i,j}$. The set of all possible paths from the source to the destination is given by \mathcal{P} . The optimal propagation strategy to positively influence the target node, i.e., the destination, with minimum expected end-to-end cost can be determined from,

$$\begin{aligned} \min_{P \in \mathcal{P}} \quad & \sum_{i,j:(i,j) \in P} d_{i,j} \\ \text{s.t.} \quad & \prod_{i,j:(i,j) \in P} s_{i,j} = +1 \end{aligned} \tag{22}$$

where the objective function is for the total cost of path P and the multiplicative constraint is to ensure that the destination is influenced *positively*. The node indices are labeled in a way that for every edge $(u,v) \in E, u \leq v$ (Dreyfus and Law 1977).

The total cost of the optimal path from node i to the destination is quantified by the optimal value function $S(i,z)$, where $z \in \{0,1\}$ is a parity variable such that $z = 0$ indicates that the signs from node i to the destination node have a product that is equal to $+1$, and $z = 1$ indicates that the signs from node i to the destination node have a product that is equal to -1 . The optimal decision taken at i is given by the optimal policy function $\pi(i)$ that identifies the index of the next node. Algorithm 1 provides the backward induction algorithm that solves the dynamic program in Eq. 22.

If the network includes cycles, a modified Dijkstra-like algorithm can be used to solve Eq. 22. To do so, positive and negative temporary labels $\pi_+(i)$ and $\pi_-(i)$ can be defined for each node $i \in V$. Next, $\pi'_+(i)$ and $\pi'_-(i)$ are defined as permanent positive and negative labels for each $i \in V$. The sets N_+ and N_- are introduced for the nodes that are assigned permanent positive/negative labels, respectively. The steps of the algorithm is given in Algorithm 2, which has time

Algorithm 1 Minimum-Cost Targeted Influence Propagation

1. Assign $s_{i,j}$ and $d_{i,j}$ for every $(i, j) \in E$.
 2. Boundary conditions: set $S(i_d, 0) = 0, S(i_d, 1) = \infty$, infinite cost to any direction with no edge.
 3. Start from the destination node i_d and evaluate the value functions at each node i from

$$S(i, 0) = \min_{j:(i,j) \in E} \{d_{i,j} + \delta(s_{i,j} - 1)S(j, 0) + \delta(s_{i,j} + 1)S(j, 1)\}$$

$$S(i, 1) = \min_{j:(i,j) \in E} \{d_{i,j} + \delta(s_{i,j} - 1)S(j, 1) + \delta(s_{i,j} + 1)S(j, 0)\}.$$
 where $\delta(x) = 1$ if $x = 0$ and $\delta(x) = 0$ otherwise.
 5. Upon reaching the source node i_o , determine the minimum end-to-end cost $S(i_o, 0)$.
 6. Find the optimal decision $\pi(i), \forall i \in V$.
 7. Identify the optimal path from $\pi(i)$.
-

Algorithm 2 Minimum-Cost Targeted Influence Propagation in a Cyclic Graph

1. Define the sets $N_+ = \{i_o\}$, $N_- = \{i_o\}$.
2. Assign the permanent labels of the source node i_o as $\pi'_+(i_o) = 0$ and $\pi'_-(i_o) = \infty$.
3. Initialize temporary labels of remaining nodes $i \in V$ via:

$$\pi_+(i) = \begin{cases} d_{i_o,i} & \text{if } s_{i_o,i} = +1 \\ \infty & \text{o.w.} \end{cases}$$

$$\pi_-(i) = \begin{cases} d_{i_o,i} & \text{if } s_{i_o,i} = -1 \\ \infty & \text{o.w.} \end{cases}$$

where an infinite cost is used if there exists no edge between nodes i_o and i .

4. Find a node $j \in V$ that satisfies:

$$\pi(j) = \min_{i \in V - N_+, j \in V - N_-} \{\pi_+(i), \pi_-(j)\}$$
5. **if** $\pi(j) = \pi_+(j)$

$$\pi'_+(j) = \pi(j) \text{ and } N_+ = N_+ \cup \{j\}$$
6. **else**

$$\pi'_-(j) = \pi(j) \text{ and } N_- = N_- \cup \{j\}$$
7. **if** $N_+ \cap N_- = V$
STOP
- else**
8. Evaluate and update temporary labels $\forall (j, i) \in E$:
9. **if** $s_{j,i} = +1$
10. **if** $(\pi(j) = \pi_+(j)) \wedge (i \in N - N_+)$

$$\pi_+(i) = \min(\pi_+(i), \pi'_+(j) + d_{j,i})$$
11. **else if** $(\pi(j) = \pi_-(j)) \wedge (i \in N - N_-)$

$$\pi_-(i) = \min(\pi_-(i), \pi'_-(j) + d_{j,i})$$
13. **else**
14. **if** $(\pi(j) = \pi_+(j)) \wedge (i \in N - N_-)$

$$\pi_-(i) = \min(\pi_-(i), \pi'_+(j) + d_{j,i})$$
15. **else if** $(\pi(j) = \pi_-(j)) \wedge (i \in N - N_+)$

$$\pi_+(i) = \min(\pi_+(i), \pi'_-(j) + d_{j,i})$$
16. Go back to Step 4.

complexity $O((|E| + |V|)\log|V|)$ as for the conventional Dijkstra’s algorithm.

Propagation with Message Deterioration and Ignorance

Known as the “Telephone” effect (Blackmore 2000), a message propagating in a social network gets *distorted* when it is repeated, changing its content and quality over time. A person may choose to ignore a received message based on how old or distorted the message is as well as the strength of the social relationships between the interacting parties. In this case, the

recommender can introduce a special promotion, with an additional cost, to *refresh* the impact of the message and draw the interest of the individual.

The impact of message deterioration and ignorance can be modeled by taking into account message freshness and the possibility that nodes may ignore one another. For message freshness, k is defined as the age of a message, measured by the number of hops the message has traveled since the last activation. Each activation sets the message age to 1. An activation is necessary in case a node ignores its neighbor. The cost of activating node i is \bar{c}_i . The cost of activating node i when the

Algorithm 3 Minimum-Cost Targeted Influence Propagation with Message Deterioration

1. Initialize the variables $s_{i,j}$ and $d_{i,j}(k_{i,j})$ for all $k_{i,j} = 1, \dots, K$ and $(i, j) \in E$.
2. Set the boundary conditions using $S(i_d, k, 0) = 0, S(i_d, k, 1) = \infty, k = 1, \dots, K$.
3. Starting from the destination node i_d , evaluate $S(i, k, z)$ at every i, k, z by

$$S(i, k, z) = \min_j E[d_{i,j}(k)] + p_{i,j}(k)(\bar{c}_j + \delta(s_{i,j} - 1)S(j, 1, z) + \delta(s_{i,j} + 1)S(j, 1, \bar{z}))$$

$$+ (1 - p_{i,j}(k)) \min \{ \delta(s_{i,j} - 1)S(j, k + 1, z) + \delta(s_{i,j} + 1)S(j, k + 1, \bar{z}),$$

$$\delta(s_{i,j} - 1)S(j, 1, z) + \delta(s_{i,j} + 1)S(j, 1, \bar{z}) + c_j \}.$$
5. Evaluate the minimum cost $S(i_o, 1, 0)$ upon reaching the source node i_o .
6. Starting from the source, identify the optimal decisions for each node.
7. Using the optimal decisions, determine the optimal propagation path.

message is not ignored, i.e., solely to reset the message age to 1 and increase its impact, is c_i . Whenever the maximum message age, denoted by K , is exceeded, the subsequent node on the path has to be activated. The cost of propagating a message of age $k_{i,j}$ from node i to node j is represented by the random variable $d_{i,j}(k_{i,j})$.

Node j ignores a message of age $k_{i,j}$ that is propagated from node i with probability $p_{i,j}(k_{i,j})$. As the age of the message increases, nodes become more likely to ignore it. The optimal propagation strategy for minimizing the expected cost can be obtained from

$$\min_{P \in \mathcal{P}} \sum_{a_{i,j}, (i,j) \in P} \{ E[d_{i,j}(k_{i,j})] + c_j \delta(a_{i,j} - 1) (1 - p_{i,j}(k_{i,j})) + \bar{c}_j p_{i,j}(k_{i,j}) \}$$

$$\text{s.t. } \prod_{(i,j) \in P} s_{i,j} = 1,$$

$$a_{i,j} \in \{0, 1\}, \forall (i,j) \in P,$$

$$k_{i,j} \in \{1, 2, \dots, K\}, \forall (i,j) \in P,$$

$$k_{j,w} = (k_{i,j} + 1) \delta(a_{i,j}), \forall (i,j), (j,w) \in P,$$

$$k_{i_o,j} = 1, \forall (i_o,j) \in P$$
(23)

where $(a_{i,j})$ is the activation sequence such that $a_{i,j} = 1$ if node i chooses to activate node j and $a_{i,j} = 0$ otherwise. By letting $S(i, k, z)$ denote the value function at node i with message age $k \in \{1, 2, \dots, K\}$ and disparity $z \in \{0, 1\}$, the backward induction algorithm from Algorithm 1 can be utilized to solve Eq. 23 for acyclic networks.

Influence Propagation with Limited Negative Influence

In real-life scenarios, it is often preferred to avoid a propagation path in which a large number of intermediate nodes are negatively influenced. Accordingly, the optimal propagation strategy to influence a target node positively while limiting the number of negatively influenced intermediate nodes can be obtained from,

$$\min_{P \in \mathcal{P}} \sum_{i,j:(i,j) \in P} d_{i,j}$$

$$\text{s.t. } \prod_{i,j:(i,j) \in P} s_{i,j} = +1$$

$$\left| \left\{ i : \prod_{i',j' \in P_i} s_{i',j'} = -1 \right\} \right| \leq Q$$
(24)

where Q is the maximum number of negatively influenced intermediate nodes allowed, and P_i is a path from node i_o to node i such that if $(i', j') \in P_i$, then $(i', j') \in P$. The value function of the minimum-cost even-parity path between the source node i_o and node i is given by $S(i, q, 0)$ when no more than q intermediate nodes are influenced negatively. The minimum cost for the odd-parity path between i_o and i is $S(i, q, 1)$ when no more than q intermediate users are influenced negatively. Then, Eq. 24 can be solved for acyclic directed networks via the forward induction dynamic program from Algorithm 4.

Algorithm 4 Forward Induction Dynamic Programming with Limited Number of Negatively Influenced Nodes

1. For each edge, assign the corresponding sign and link cost.
 2. Evaluate the boundary conditions using $S(i_o, q, 0) = 0, S(i_o, q, 1) = \infty, \forall q \in \{0, 1, \dots, Q\}$.
 3. Determine the value functions for each node i from

$$S(i, q, 0) = \min_{j:(j,i) \in E} \{d_{j,i} + \delta(s_{j,i} - 1)S(j, q, 0) + \delta(s_{j,i} + 1)S(j, q - 1, 1)\}$$

$$S(i, q, 1) = \min_{j:(j,i) \in E} \{d_{j,i} + \delta(s_{j,i} - 1)S(j, q - 1, 1) + \delta(s_{j,i} + 1)S(j, q, 0)\}.$$
 where i_o is the starting node.
 5. Evaluate the minimum end-to-end cost $S(i_d, Q, 0)$ for the destination node i_d .
 6. Identify the optimal propagation path using the optimal decisions.
-

Algorithm 5 Minimizing the Number of Negative Influences

1. Initialize the positive and negative relationships in the social network.
 2. Evaluate the boundary conditions using $S(i_o, n, 0) = 0, S(i_o, n, 1) = \infty, n = 0, 1, \dots, N$.
 3. Determine the value functions at every node i using

$$S(i, n, 0) = \min_{j:(i,j) \in E} \{\delta(s_{i,j} - 1)S(j, n - 1, 0) + \delta(s_{i,j} + 1)(S(j, n - 1, 1) + 1)\}$$

$$S(i, n, 1) = \min_{j:(i,j) \in E} \{\delta(s_{i,j} - 1)(S(j, n - 1, 1) + 1) + \delta(s_{i,j} + 1)S(j, n - 1, 0)\}.$$
 starting from the source i_o .
 5. Evaluate $S(i_d, N, 0)$ to find the minimum number of negatively influenced nodes on the path.
 6. Identify the optimal propagation path using the optimal decisions.
-

Propagation with Minimum Negative Influence

While influencing a target node positively, one may seek to find the policy that minimizes the total number of negatively influenced users on the path, i.e.,

$$\begin{aligned} \min_{P \in \mathcal{P}} & \left\{ i : \prod_{i',j':(i',j') \in P_i} s_{i',j'} = -1 \right\} \\ \text{s.t.} & \prod_{i,j:(i,j) \in P} s_{i,j} = +1, \\ & |P| \leq N. \end{aligned} \tag{25}$$

where N is the maximum number of hops allowed before reaching the destination.

The forward induction dynamic program presented in Algorithm 5 can be utilized to solve Eq. 25 for acyclic directed networks. For networks that contain cycles, the Dijkstra-like algorithm

from Algorithm 6 can be utilized for solving Eq. 25. The temporary labels of each node are now updated at each step to identify the minimum number of nodes from the source to the corresponding node that are influenced negatively.

Numerical Results

The propagation patterns and optimal strategies are investigated for a synthetic small-scale directed acyclic grid network in Guler et al. (2014a, b, 2015). Large-scale simulations are performed using the online Epinions dataset (Leskovec et al. 2010a). Epinions is a consumer review website in which users can identify other users as friends or foes. These labels are then used to construct a signed social graph, which has 131828 nodes and 841372 edges. Throughout the numerical evaluations, source and destination nodes are selected randomly. The algorithms are applied to each source-destination pair to find the

Algorithm 6 Minimizing the Number of Negative Influences for Cyclic Graphs

1. Define the sets $N_+ = \{i_o\}$, $N_- = \{i_o\}$.
2. Set permanent labels of the source node i_o as $\pi'_+(i_o) = 0$ and $\pi'_-(i_o) = \infty$.
3. For the remaining nodes $i \in V$, determine the temporary labels from:

$$\pi_+(i) = \begin{cases} 0 & \text{if } s_{i_o,i} = +1 \\ \infty & \text{o.w.} \end{cases}$$

$$\pi_-(i) = \begin{cases} 1 & \text{if } s_{i_o,i} = -1 \\ \infty & \text{o.w.} \end{cases}$$

such that in the case of no edge between i_o and i , use an infinite cost.

4. Identify a node $j \in V$ that satisfies:

$$\pi(j) = \min_{i \in V - N_+, j \in V - N_-} \{\pi_+(i), \pi_-(j)\}$$

5. **if** $\pi(j) = \pi_+(j)$

$$\pi'_+(j) = \pi(j) \text{ and } N_+ = N_+ \cup \{j\}$$

6. **else**

$$\pi'_-(j) = \pi(j) \text{ and } N_- = N_- \cup \{j\}$$

7. **if** $N_+ \cap N_- = V$

STOP

else

8. Evaluate and update temporary labels $\forall (j, i) \in E$:

9. **if** $s_{j,i} = +1$

10. **if** $(\pi(j) = \pi_+(j)) \wedge (i \in N - N_+)$

$$\pi_+(i) = \min(\pi_+(i), \pi'_+(j))$$

11. **else if** $(\pi(j) = \pi_-(j)) \wedge (i \in N - N_-)$

$$\pi_-(i) = \min(\pi_-(i), \pi'_-(j) + 1)$$

13. **else**

14. **if** $(\pi(j) = \pi_+(j)) \wedge (i \in N - N_-)$

$$\pi_-(i) = \min(\pi_-(i), \pi'_+(j) + 1)$$

15. **else if** $(\pi(j) = \pi_-(j)) \wedge (i \in N - N_+)$

$$\pi_+(i) = \min(\pi_+(i), \pi'_-(j))$$

16. Go back to Step 4.

optimal propagation strategy such that, when started from the source node, the destination is influenced positively when the propagation terminates.

The Dijkstra-like algorithm from Algorithm 2 is compared with a naïve myopic approach for finding a low-cost positive path. For the implementation of Algorithm 2, the cost between node i and node j is denoted by $\kappa|i - j|$ with a weight parameter $\kappa = 0.1$. The myopic algorithm, termed as *shortest DFS*, is a depth first search algorithm that traverses the graph starting from the source node trying to reach the destination. The algorithm selects the successor of each node with the

lowest cost, repeated recursively until reaching the destination node. In case no path is identified from a node to the destination, a successor with a higher cost is picked by the algorithm.

Table 1 includes the evaluations for Algorithm 2, whereas Table 2 presents the results of the *shortest DFS* (myopic) algorithm. From comparing the results in Tables 1 and 2, we observe that the average cost of the paths found by the *shortest DFS* algorithm is five times the cost of the paths found by Algorithm 2. From the evaluations performed for 100 sources and 100 destinations, one can observe that, among the 10000 possible source-destination

Influence Propagation in Social Networks with Positive and Negative Relationships, Table 1 Implementation results of Algorithm 2 for the Epinions dataset

Number of source nodes	Number of destination nodes	Number of paths identified	Path length (average)	Path length (median)	Path cost (average)	Path cost (median)
100	100	8830	54.450	40.0	3436.569	2488.4
500	500	218499	55.148	42.0	3419.907	2363.7
10000	10000	78029370	47.024	30.0	5027.842	4145.1

Influence Propagation in Social Networks with Positive and Negative Relationships, Table 2 Implementation results of the myopic algorithm (*shortest DFS*) for the Epinions dataset

Number of source nodes	Number of destination nodes	Number of paths identified	Path length (average)	Path length (median)	Path cost (average)	Path cost (median)
100	100	1041	660.727	638.0	17604.642	16875.4
500	500	27309	726.949	727.0	19134.178	18425.9

Influence Propagation in Social Networks with Positive and Negative Relationships, Table 3 Implementation results of Algorithm 6 for the Epinions dataset

Number of source nodes	Number of destination nodes	Number of paths identified	Path length (average)	Path length (median)	Negatively influenced nodes (average)	Negatively influenced nodes (median)
100	100	8830	4.023	4.0	0.096	0.0
500	500	218499	4.097	4.0	0.057	0.0
10000	10000	78029370	4.646	5.0	0.123	0.0

pairs, the destination cannot be reached from the source in 8957 cases for the *shortest DFS* algorithm. Though some of these pairs may in fact be unreachable due to the network structure, i.e., the source and the destination may be disconnected, with the Dijkstra-like algorithm only a number of 1168 cases the destination is not reachable from the source. As a result, only a tenth of the paths that were discovered by Algorithm 2 were also discovered by the *shortest DFS* algorithm. For 500 sources and 500 destinations, the number of times for which a destination is not reachable from the source with the Dijkstra-type algorithm is 31460, whereas the same number is 222650 for the *shortest DFS* algorithm. Accordingly, the *shortest DFS* algorithm has found only a tenth of the paths identified by Algorithm 2. Moreover, for 10000 nodes, the *shortest DFS* algorithm has not been able to terminate within the maximum allowed computing time.

The numerical results for Algorithm 2 is given in Table 3, from which it can be observed that even for

very large number of source and destination pairs, no nodes are influenced negatively in at least half of the paths. Moreover, less than five hops existed on average in each path. Hence, one can find a relatively short path from one node to another dominated by positive influences and friendship relations, and almost every node can influence a target node *positively* within a small number of hops.

Key Applications

Influence diffusion and maximization has found wide applications in the design of recommender systems and word-of-mouth marketing in online social communities. A recommender can influence the maximum number of people by initially advertising its product to a judiciously selected small number of seed users who can trigger a widespread influence diffusion pattern. As such, influence maximization algorithms can identify which individuals trigger the largest influence spread.

Epidemic models can identify the critical opinion diffusion rates in the network which can be utilized to determine whether a new opinion or trend will die out or continue to spread in the long run.

Applications of targeted influence propagation in signed networks consist of situations that arise out of differing ideas, interpretation of situations, acts, groups, events, or activities that are propagated by users of social media. Examples of these scenarios include promoting an online product or a candidate in a voting process, situations involving radical events such as terrorism, or rivalries between sports teams. The biases held by the online social media users are often publicly available, i.e., can be observed, through the online posts they share or pages they like. In most online social media applications, messages shared from a large number of persons go through a filtering process before appearing on the *newsfeed* or being suggested by the recommender. The recommender, who is in charge of the filtering process, can prioritize the posts, acts, or choices of certain users. It is therefore important to understand the possible strategies that may be adopted by the recommender and its impact on the social community.

Future Directions

Future directions include the design of multilayer influence propagation schemes for networks by taking into account multiple relationship types, by extending the signed network scenario with only positive and negative relationships to multiple types of relationships. Another future direction is the development and analysis of practical modern social network applications that take into account the degree of positivity and negativity of the relationship type based on threshold techniques, and developing inference strategies for identifying influence patterns in social communities.

Cross-References

- ▶ [Influence Maximization Model](#)
- ▶ [Mathematical Model for Propagation of Influence in a Social Network](#)

- ▶ [Opinion Diffusion and Analysis on Social Networks](#)
- ▶ [Signed Graphs](#)
- ▶ [Social Influence Analysis](#)

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Informal Learning

- ▶ [Learning Networks](#)

Informal Network

- ▶ [Intraorganizational Networks](#)

Informal Organization

- ▶ [Intraorganizational Networks](#)

Information

- ▶ [Social Phishing](#)

Information Extraction

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Information Filtering

- ▶ [Recommender Systems Based on Linked Open Data](#)
- ▶ [Recommender Systems Using Social Network Analysis: Challenges and Future Trends](#)
- ▶ [Spatiotemporal Proximity and Social Distance](#)

Information Filtering Systems

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Information Fusion

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Innovator Networks

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Synonyms

[Interorganizational collaborations](#); [Inventor networks](#); [R&D \(Research and Development\) collaborations](#)

Glossary

Architectural control Refers to innovation in high-technology systems and measures how the control over

the architecture of the final product is concentrated within the hand of one (or few) agents (Prybeck et al. 1991).

A concentrated architectural control can be found, for instance, in a high-tech industry where a dominant standard interface incorporates proprietary elements, such as telecommunications networks. Relationships in an Innovation System span across very different kinds of agents, ranging from firms to scientists. In addition, each agent is endowed with specific features and a unique knowledge base.

An analytical framework aimed at understanding how innovation is produced in a complex system of interacting agents. The IS approach, introduced by Lundvall (1985), is now especially used to study innovation at national or regional level.

Applies to product systems and indicates the degree to which components and/or processes are independent and innovation activities can be performed by separate agents. Separable innovation systems are associated with a higher community activity (Baldwin and Clark 2000).

In a rapidly changing technology environment where knowledge often has a tacit component and is strongly distributed over agents, collaborations become a central component of the innovation strategy (Tushman and Rosenkopf 1992). Moreover, collaborations mitigate uncertainty about the direction

Heterogeneity

Innovation system (IS)

Separability of innovation

Technological dynamism and uncertainty



of technological change. Innovators share risks when they collaborate which allows for more flexibility and more investments in future opportunities as compared to an isolated state

Definition

From the perspective of innovation economics evolving institutions, innovating entrepreneurs, technological change, and creative destruction are the driving force of economic growth (Schumpeter 1942). To mitigate the uncertainty involved in the creation of new processes, products, or business models, innovation exhibits an intrinsic collaborative nature. Innovator networks form through formal and informal collaborations between different agents, including firms, institutions, universities, state agencies, inventors, and other stake-holders of the innovation system. Being embedded in a network enables these agents to coordinate innovative efforts, as well as to pool and jointly create knowledge (Kratzer et al. 2009; Raab and Kenis 2009).

Introduction

To cope with the variety of agents in innovator networks, their analysis can be abstracted in a network approach where nodes represent the innovating entities and links represent their collaborations. A large body of literature in this field has focused on collaborating firms (Allen 1983) as the fundamental units in creating innovations, which is in line with recurrent theoretical arguments such as Schumpeter's idea of innovation as a recombination process, or the resource-based view of the firm. Firm-related data sources, such as databases on strategic alliances, offer the possibility to construct large and often longitudinal networks, allowing extensive empirical studies. Hence, innovator networks in this article refer mainly to networks of collaborating firms.

Key Points

Collaboration between innovators is not a new phenomenon; however, the 1980s and 1990s witnessed an unprecedented growth of strategic alliances aimed at research and development (R&D) activities (Hagedoorn 2002). This has been investigated by two different streams of empirical literature (see Ebers 1997; Veugelers 1998; Walker 2005, for a more extensive overview).

A body of work has studied the salient features of empirically observed collaboration networks (see e.g., Fleming et al. 2007; Powell et al. 1996; Roijakkers and Hagedoorn 2006). These studies have found that collaboration networks exhibit a small-world topology characterized by short path lengths and high clustering. In addition, these networks tend to be highly heterogeneous and centralized, although there exist some differences across industries (Powell et al. 2005; Rosenkopf and Schilling 2007), as we show below. The study by Tomasello et al. (2014) further investigates the drivers behind the formation of interfirm R&D alliances and presents a model to reproduce the observed "small-worldliness" of R&D networks.

Another body of work has studied the network position of firms in relation to their performance and the role of link density in knowledge diffusion. It is of interest whether dense interconnections are more conducive than weak bridging ties between separate communities (Granovetter 1983). Indeed, clusters of densely connected firms foster collaboration efforts by generating trust, punishment of opportunistic behaviors, and common practices (as shown by Ahuja 2000; Walker et al. 1997). Conversely, by creating a structural hole in the network, firms have access to different sources of knowledge spillovers, economizing on the costs of direct collaborations (Burt 1992). Other works (Gulati and Gargiulo 1999; Rosenkopf and Padula 2008) have analyzed the mutual feedback between a firm's position in the network and its knowledge base. As it has been found by Cohen and Levinthal (1990) and Lazer and Friedman (2007), two agents should not be too similar nor too different in their knowledge bases in order to engage in a collaboration.

Historical Background

Following the wave of empirical research, various theoretical models have explored the dynamics of collaboration networks and their impact on innovation. This literature on network formation is basically divided in two strands (Schweitzer et al. 2009). In the dynamic random network approach, mainly developed by mathematicians and physicists, networks are formed either through a purely stochastic process or through some other statistical algorithms (see e.g., Ehrhardt et al. 2006). In the strategic network approach, mainly developed by economists, strategic interaction decides about the link formation: agents may follow different strategies (see e.g., Jackson and Wolinsky 1996; König et al. 2011) to decide about – and interact with – their counterparts; therefore, this approach is also called “games on networks.” While the random network approach gives insights into *how* networks form, the strategic network approach tries to explain *why* networks form.

In the “games on networks,” the network is usually static and taken as given, and the focus is on how the network structure impacts on outcomes and individual decisions. In particular, some works (Ballester et al. 2006; Goyal and Joshi 2003) show that the centrality of an agent in the network predicts its innovation efforts and outcomes.

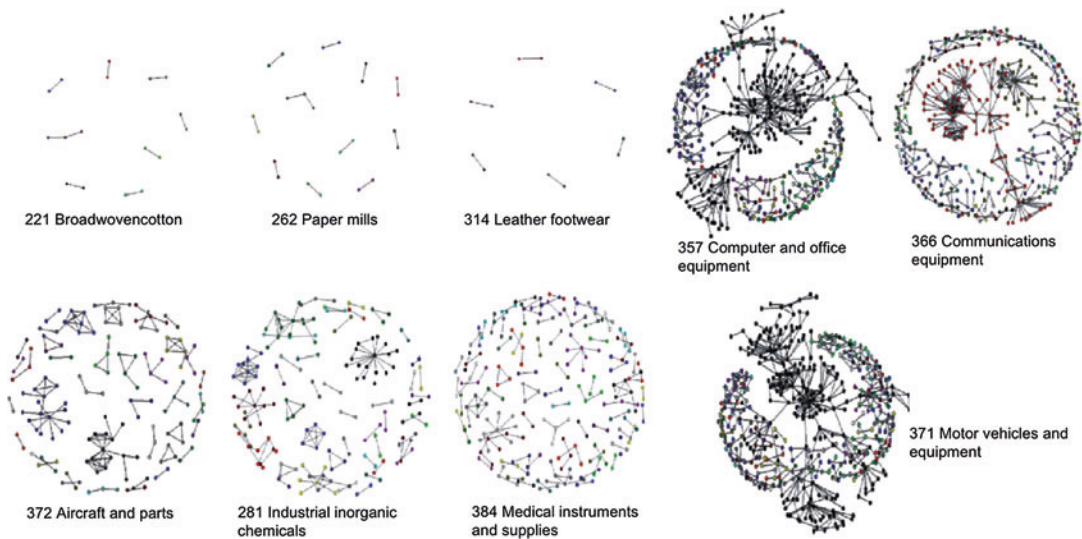
Other works combine a dynamic approach with games on networks. For instance, König et al. (2008) and (2012) examine the theoretical efficiency of a given network in terms of total profits maximization, showing that the most efficient network structure depends on collaboration costs. When the marginal cost is low, the efficient collaboration network is fully connected, while a high marginal cost implies a sparse efficient network, with a core-periphery structure. In another work combining strategic agents’ decisions with a dynamical network evolution (Tomasello et al. 2015, 2016b), the effect of R&D alliances on the firms’ technological positions is studied through an agent-based model. The study uses real patent data for a precise quantification of every firm’s knowledge position, and shows that effective policies for an optimized collaboration network would promote shorter R&D alliances and higher interfirm

knowledge exchange rates (e.g., by including rewards for quick co-patenting by allied firms).

Illustrative Examples

We present here two illustrative examples of innovation networks, from the empirical literature. In the first example (Rosenkopf and Schilling 2007), the comparison of alliance networks across industries highlights how technology relates to network structures. The alliance network for 32 industrial sectors has been analyzed in terms of size, connectivity, centralization, small-world properties, and other indicators. As shown in Fig. 1, the networks exhibit different structures across industries, depending on their technological features. Technological dynamism and separability of innovation are positively related to the number of firms participating in alliances (the size of the network) and to the average number of alliances formed by each firm (the average degree). The concentration of architectural control is instead correlated to the asymmetry in the degree distribution (number of alliances per firm) and to the appearance of small-world architectures in the network (high clustering and short path lengths).

The second work (Tomasello et al. 2016a) extends the investigation of R&D networks to the temporal dimension, by employing a longitudinal dataset (from 1986 to 2009) of alliance formation in several manufacturing sectors. The study has found that most network properties are not only invariant across sectors (as shown in Fig. 2) but also independent of the scale of aggregation at which they are observed (i.e., in the aggregated global R&D network versus the individual sectoral R&D networks). Remarkably, many properties of R&D networks are characterized by a peculiar rise-and-fall dynamics with a peak in the mid-1990s, driven by mechanisms of *accumulative advantage*, *structural homophily*, and *multiconnectivity* (see Powell et al. 2005). In particular, the *multiconnectivity* hypothesis states that partners allowing a firm to reach many other firms through multiple independent paths in the network are the most attractive alliance partners. The study has found that the change from the



Innovator Networks, Fig. 1 The structure of the collaboration networks in nine distinct industrial sectors. Some sectors (industrial codes 221, 262, and 314) exhibit *dis-connected* networks, consisting mainly of pairs of allied firms with no bridging ties. Other sectors (codes 372, 281, and 384) display networks of moderate size, defined

hybrids, with many separate clusters of nodes, but no main component dominating the graph. The last sectors (codes 357, 366, and 371) show large *spider-web* networks, consisting of a main component and several peripheral components (See Rosenkopf and Schilling (2007) for more details)

“rise” to the “fall” phase is indeed associated to a structural break in the importance of multi-connectivity as driving mechanism behind the strategic choice of alliance partners.

Key Applications

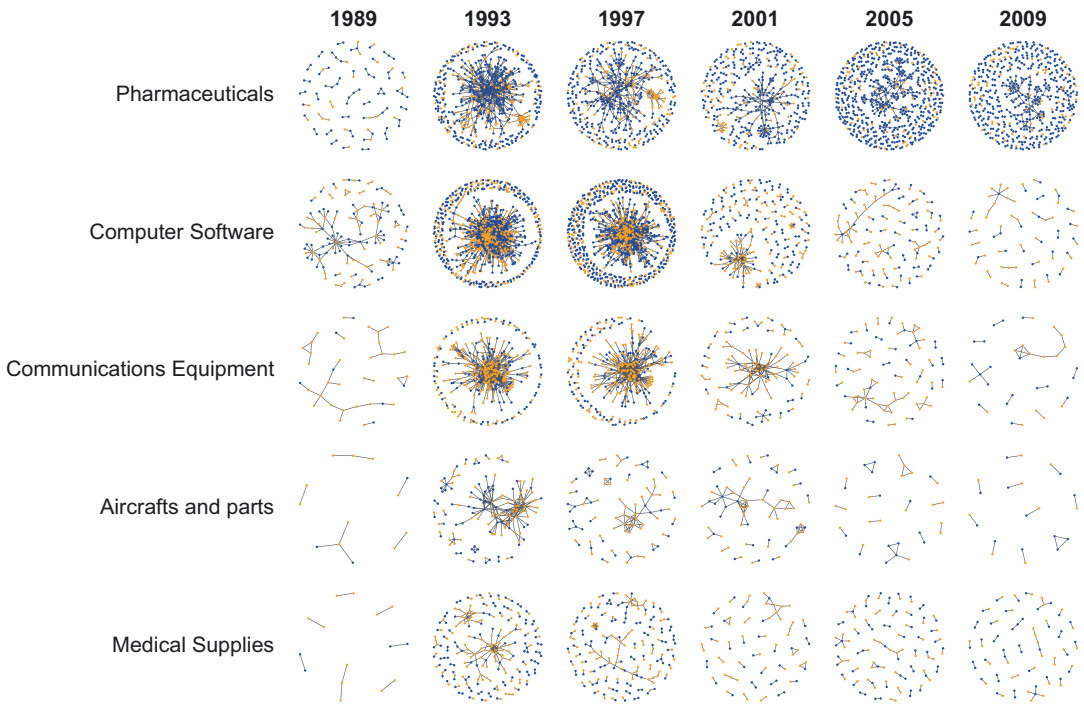
One prominent application in the field of innovator networks is the use of agent-based models to not only reproduce the characteristics of the observed networks but also to predict their formation and evolution, and to possibly optimize some indicator of actual knowledge production and diffusion. In this respect, the study by Tomasello et al. (2014) develops an agent-based model of strategic link formation, to explain the emergence of such structures observed in real collaboration networks. Similarly to the previous illustrative example, the study is inspired to the four fundamental link creation mechanisms identified by Powell et al. (2005) – *accumulative advantage*, *homophily*, *follow-the-trend*, and *multiconnectivity* – and to the stylized facts reported in Rosenkopf and

Padula (2008), showing the presence of distinct clusters (or communities) in a real R&D network. By incorporating a set of appropriate link formation rules into an agent-based model, Tomasello et al. (2014) are able to reproduce the emergence of network clusters (see Fig. 3), as well as other additional network indicators, including the distributions of degree, local clustering, path length, and size of the network components.

Finally, by estimating the link probabilities towards newcomers and incumbent firms from the data, the study has found that the alliance formation process is dominated by network endogenous mechanisms. In other words, the existing network structures (i.e., social capital) are more important than the firms’ own characteristics (i.e., technological and commercial capital) in selecting new R&D partners.

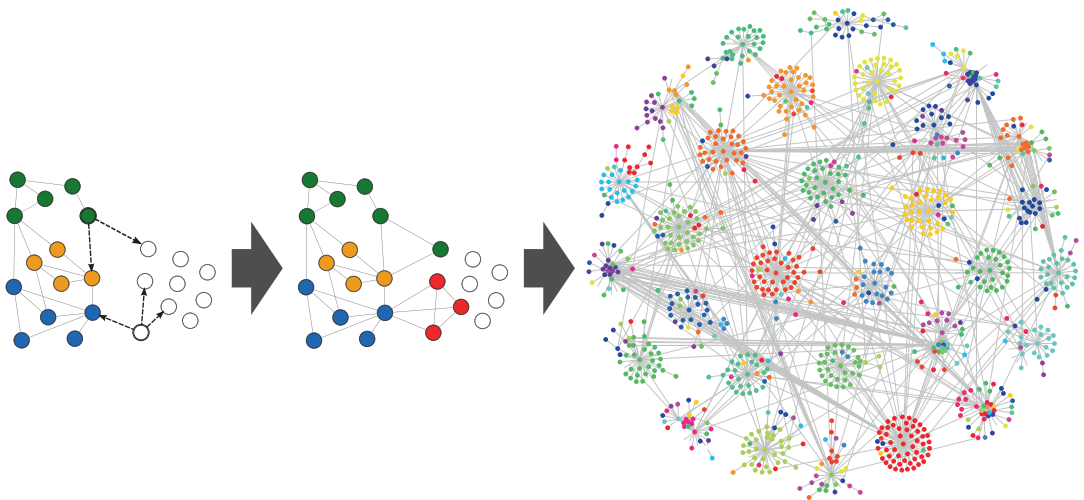
Future Directions

Empirical evidence suggests that innovator networks are not designed, but emerge endogenously.



Innovator Networks, Fig. 2 Snapshots in the years 1989, 1993, 1997, 2001, 2005, and 2009 for five selected sectoral R&D networks: Pharmaceuticals, Computer Software, Communication Equipments, Aircrafts and parts, Medical Supplies. *Blue* nodes represent the firms strictly

belonging to the examined sector, while *orange* nodes represent their alliance partners belonging to different sectors. The peculiar rise-and-fall trend is visible in all sectoral networks shown



Innovator Networks, Fig. 3 The formation of an interfirm innovator network, captured through an agent-based model. The figure depicts a representative example of strategic link formation and community building in a collaboration network. The result is a network whose

synthetic communities (represented by different colors) exhibit a remarkable overlap with the empirical ones (represented by different locations in the plot area) (See Tomasello et al. (2014) for more details)

Agents pursue self-interested goals when forming and dissolving relationships, and thereby create an evolving network that affects all the agents in its turn. Although some models are already able to capture several empirical observations, a comprehensive theory to explain the features of real-world innovator networks is still missing. A complete study should be able to reproduce similarities and differences across the large variety of observed innovation systems, and at the same time unveil the complex interdependencies between the network position of the innovators and their intrinsic knowledge characteristics.

Besides, substantial potential for future work lies in the study of performance, optimization, and resilience of real innovator networks. The exercise of defining and maximizing a performance indicator, so far limited to the field of R&D networks (see Tomasello et al. 2016b), could be extended to other domains. The ultimate goal would be to assess innovator networks in real time, and design policies to make them more resilient and more conducive to knowledge transfer.

Cross-References

- ▶ [Actor-Based Models for Longitudinal Networks](#)
- ▶ [Collaboration Patterns in Software Developer Network](#)
- ▶ [Entrepreneurial Networks](#)
- ▶ [Interorganizational Networks](#)
- ▶ [Network Games](#)
- ▶ [Networks of Practice](#)
- ▶ [R&D Networks](#)

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Instant Message

- [Microtext Processing](#)

Instant Messaging for Detecting Dynamic Ego-Centered Communities

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Synonyms

[Community evolution](#); [Dynamic community detection](#); [Temporal analysis](#); [Temporal networks](#)

Glossary

Dynamic Community Ego-Centered Community Instant Messaging Networks	a community that changes over time
Instant Messaging	an online chat that offers real-time text transmission over the Internet
Spatiotemporal Network	a social network that is built based on individuals, their interaction, and their location over the time

Definition

The development of online social media has created many opportunities to communicate, access, and share information from anywhere and at any-time. The kind of application such as *Viber*, *WhatsApp*, *Imo*, *Line*, as well as *Facebook* affords

plenty of possibilities for getting in touch with friends, colleagues, and relatives at every moment with real-time messages, photos, videos, etc. Data collected from those applications integrate two parameters such as the time and the geographical position from which they were sent and/or received. Therefore, it is worthwhile to build a social network based on instant messaging content to find out “who talk to whom” and/or “who is closed to whom.” Such a network that we call instant messaging network (IMN) can be represented as a graph where individuals are the vertices and their interactions the edges.

Alongside the growth of online social media, there is a rapid expansion of social network analysis (SNA), which is a process of exploring the graph in order to discover knowledge leading to informative decision-making. At the heart of the SNA topics that attract many scientists, we have the community detection. The main reason is related to the fact that individuals tend to form a community in many real-life situations. The first works on this topic had been conducted on static aspects while neglecting that community structures may be dynamic or ad hoc due to specific characters related to the location and/or time. Thus, the major drawbacks of the static communities are the fact they neither depict the real-time situation nor how the communities were shaped over time. To face these limitations, recent works were conducted in order to deal with the dynamic and/or temporal features of the communities.

The purpose of this work falls within this framework while addressing the ego-centered communities in dynamic networks. The key idea is to detect community structures while targeting some nodes we found interesting based on their position.

Introduction

Research in either dynamic networks (DN) or spatiotemporal network (STN) is attracting more and more scientists, thanks to the rapid development in information technology and computing, which allows to retrieve data from everywhere and at anytime and to build scalable solutions. Some examples of complex networks that are

possible to handle today are mobile social networks (Eagle et al. 2009; Gao et al. 2012), disease diffusion (Rocha et al. 2011), electric power systems (Paevere et al. 2014), artificial neural networks (Ermentrout 1998; Zeng and Zhang 2013), etc. Nonetheless, it is worth noticing to remind the little confusion between temporal and dynamic network community. The difference can be portrayed as follows: the dynamic aspect generally focuses on how the topological structure evolves over time, while the temporal aspect focuses more on the historical traces of those changes. Notwithstanding this difference, it is still challenging to deal with STN or DN due to their time and/or space dimensions. Therefore, we rely, for the purpose of this study, on instant messaging network (IMN), which is a good illustration of both STN and DN.

To address the time dimension, many solutions rely on capturing a series of snapshots at different time windows. For each snapshot, one may apply a set of processes and finally compare the outcomes with the ones obtained in the other snapshots. This strategy is widely used in dynamic community studies in order to track changes over time and has the advantage of reusing existing algorithms of static community detection. Even though there are many approaches that try to deal efficiently with the problems raised by the reuse of static algorithms for dynamic community detection, there are less works addressing the problem of how to assess and to set the time windows. Rather, a time window size has a real impact on a community structure evolution since it determines the data that belong to each snapshot. In other words, a bad size definition of the time windows can lead to miss the most interesting structure changes.

Another challenge is the collection, the interpretation of instant messaging platforms, which provide huge and various information with a high velocity. Actually, in many studies, data contents are extracted and analyzed for unveiling knowledge. Important parts of these approaches rely on semantic rules and/or more simply on keywords to identify the network structures (nodes and their links). However, when it comes to dealing with temporal networks, it is worthwhile to integrate the fact that links can either vanish or intensify

over time. That is, everything (nodes as well as links) must be set as dynamic. Moreover, based on the exchange flows, a link direction may switch over time and leads to many changes in the structures of the communities. Therefore, instant messaging networks should be modeled as a directed and weighted graph in order to deal with the frequency and the direction of the interactions. In our best knowledge, there is a glaring lack of studies that target the challenges we pointed out while we count hundreds of works in community detection topic. In this entry, we aim at facing the above challenges by focusing on two aspects: (1) discuss on how to represent data from instant messaging in order to track evolution and (2) propose the building blocks of a dynamic ego-centered community detection in IMN.

Key Points

The objective of this study is to detect dynamic ego-centered network based on instant messaging. To this end, we consider data from an instant messaging platform such as *Facebook* or *WhatsApp* and build a social network based of “who talk to whom” and/or “who is closed to whom.” We aim therefore at finding out communities centered on some special nodes due to their characteristics or social positions. This is very useful since identifying individuals that are closely exposed to a disease is a starting point for controlling an epidemic. With this insight, finding out nodes that share a community with an infected individual may help to point out who is exposed or not and where it is more relevant to make specific actions to break down the disease spread. The goal of this entry is twofold. First, we propose an approach to detect ego-centered network based on instant messaging network. Second, we envision a tracking mechanism to see how communities evolve over time.

The remainder of this entry is as follows. We first present a background related to dynamic community studies before describing instant messaging network. Afterward, we portray how we deal with dynamic community detection. Finally, through some illustrative examples, we explain how our proposal works.

Historical Background

Dynamic communities in social networks have attracted many researchers, and one of the main focuses of their studies is the tracking of community evolution (Hopcroft et al. 2004; Chakrabarti et al. 2006; Wang et al. 2008; Lancichinetti et al. 2009; Chan et al. 2009; Greene et al. 2010; Xu et al. 2011; Li et al. 2012; Xie and Szymanski 2012; Bródka et al. 2013; Shang et al. 2014; Cazabet and Amblard 2014). There are two main trends that emerge from existing algorithms dealing with dynamic communities, namely, snapshot-based approaches and stream-based ones.

The general idea on which snapshot-based approaches rely is capturing a set of successive static networks, each called a snapshot that represents an evolution of a dynamic network at a time slot. The principle of these approaches is based on three steps. First, decompose the network into several snapshots based on a regular time period. Second, apply a static algorithm on each snapshot in a strict or partial sequential manner. Third, compare communities of any couple of snapshots and assess whether the community structures have changed.

Furthermore, the incremental approach represents another alternative, which does not consider the network as a sequence of multiple snapshots. The key idea is to consider changes as a result of a stream of events. Basically, the approach captures the sequence of events and modifies directly current community structures rather than recalculating a new composition from scratch. This approach is particularly useful in the case of a real-time analysis.

The approach presented in this entry is related to a snapshot-based approach since we do not focus on real-time analysis.

Description of Evaluation Network

We consider a mobile-based instant messaging platform such as *WhatsApp* or *Facebook* run on a *mobile* for harvesting data of users and some additional informations such as their geographical position as well as their exchanges flow over time. Based on the collected data, we build an instant messaging network (IMN) that is represented by a social graph

where nodes are the users and link the communication between them (e.g., real-time texts or calls). In this respect, a set of users are identified as the seeds, and once a seed texts or calls a new phone number, we add it as a new node and set the corresponding link. A link is created after a call or a post as well as any reaction related to it. For example, all individuals who comment or who like as well as who share a post of someone are linked to him or her.

Node's Information

For each node, we gather the following informations:

- Identifier: It helps identifying each user in a precise and unique way (e.g., number, phone numbers, user profile, etc.).
- Arrival time: It gives the time that a node comes into the network.

Link's Information

The following informations characterize each link:

- Direction: It shows who initiates the interactions and who reacts.
- Weight: It keeps the intensity of the communication expressed in terms of the number of messages or number of calls during a time window.
- Location: It indicates the geographical position of the nodes at the beginning of their communication.
- Duration: It represents the time that lasts each interaction. Actually, if the nature of the interaction is a voice call, thus, the duration is the period that two nodes talk each other from the beginning till they hang over. However, if the communication is based on real-time texts, we set a short-time threshold (called *user active chat slot* \bar{T}_u) beyond which we consider that the interaction is over. That is, we have the same communication between two nodes if the delay of any two subsequent messages is shorter than \bar{T}_u . If one of the users waits more than the *user active chat slot*, then, we consider he initializes a new communication even if he is answering a message belonging to the previous communication. The intuition behind this copes well with the instant messaging philosophy where

individuals tend to text in a real-time fashion in such a way that messages from a user have their response instantaneously.

Network Changes Information

We distinguish a set of changes that may affect the overall structure of the network and its communities:

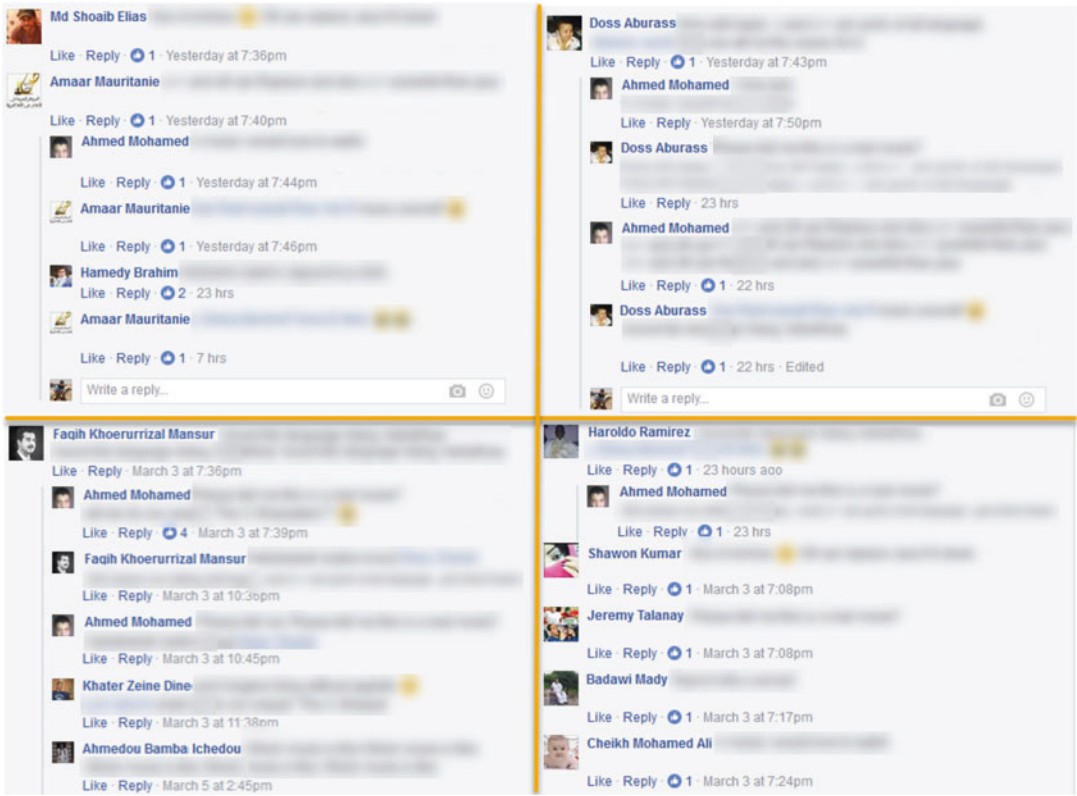
- Incoming node (IN): A node that does not yet belong to the network.
- Vanishing node (VN): A node is considered as vanishing if its communication rate is equal to zero during a given time window.
- Incoming link (IL): It reflects the first time two nodes start exchanging.
- Vanishing link (VL): A link is considered as vanishing if one of the nodes that share it vanishes.
- Growing link (GL): A link with an increasing weight, thanks to the higher communication rate of the corresponding nodes at a given time window.

In short, our social graph is a directed and weighted ego network, and it is made of people who send texts or make call between them. Moreover, the graph is dynamic and we set a variant time windows to capture the network evolution. In other words, for each time window, we extract a snapshot and checks whether a given community structure evolves or not.

Example of Instant Messaging Network

In this section, we show how to build an instant messaging network from a real-time discussion following, for instance, a Facebook post. There are two situations that may happen after a post: (1) someone reacts to the post directly, and (2) someone reacts to one of the comments of the post. Hence, the network is built based on all the individuals that partake to the exchange flow of posts and/or comments. In this respect, we have the following:

- A link is created between a person who adds a post or a comment and the ones who react to it. If the person reacts to a same post/comment several times, we just add a weight on the link that represents the number of comments he/she made.



Instant Messaging for Detecting Dynamic Ego-Centered Communities, Fig. 1 Screenshot of an instant discussion on Ahmed’s post on Facebook

- The link direction goes from the person who makes a reaction to the person that owns the posts or comments.

Figure 1 shows an example of instant discussion on Facebook. The post is created by the user named Ahmed Mohamed. For data privacy reasons, the message contents are hidden. From this discussion, we extract the corresponding social graph as depicted on Fig. 2.

Dynamic Ego-Centered Community Detection

As pointed out in the section, we rely on the snapshot-based approach to find out and track ego-centered communities over time. The algorithm starts by finding out the ego nodes that we choose between the most central nodes. Ego

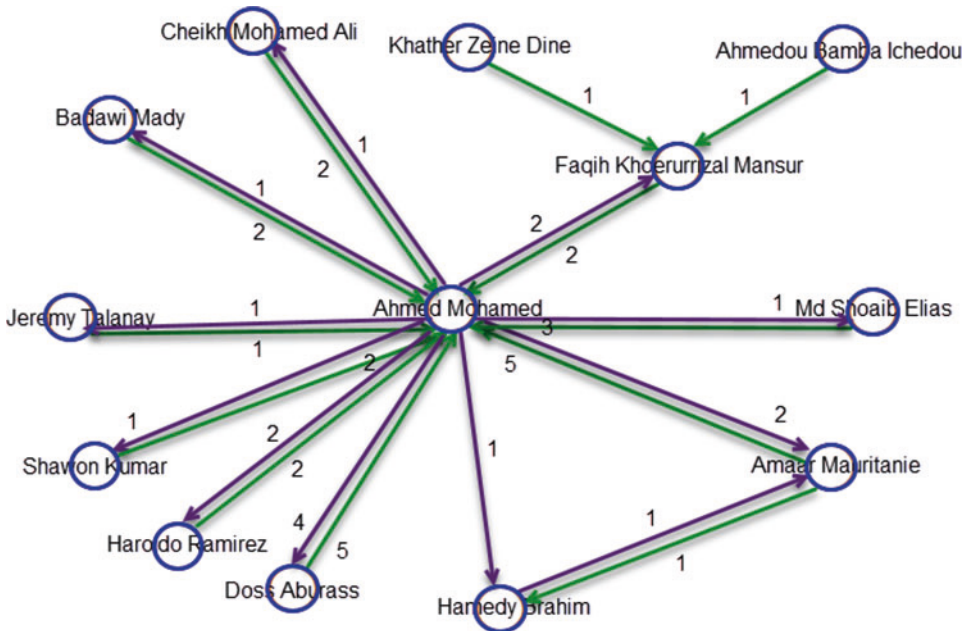
nodes are considered as the ones that may influence, for instance, the rest of the network. Actually, our solution is devised on two aspects: (1) a community detection algorithm that takes into account the features of an instant messaging network (IMN) and (2) a mechanism to find out community changes over time.

IMN Community Detection

For each ego node, the algorithm identifies the nodes that will form its community by creating first a seed before adding any nodes that may increase the cohesion of the group. The group cohesion is estimated by a quality function that we define later in this section.

Finding the Seed

To identify the seed of a given ego node, we rely on its direct neighborhood. In this respect, we have a random and a nonrandom approach. For



Instant Messaging for Detecting Dynamic Ego-Centered Communities, Fig. 2 Illustration of the ego network corresponding to the instant discussion shown in Fig. 1

both cases, the seed is made by considering the ego node with one or more of its neighbors.

Random Seed In this case, we choose randomly one or more nodes among the neighbors of the ego node. It is trivial to observe that the seed is different for successive initializations, which may lead to different community structures.

Nonrandom Seed This approach considers the nodes with a higher degree centrality as well as a higher communication intensity with the ego node. Since the links are oriented, for each neighbor, we sum the weight of the incoming link and the outgoing ones in order to calculate its communication intensity with the ego node. This approach has the advantage to consider almost the same seed for successive initializations.

Shaping the Community of a Given Seed

Since we address a dynamic network, we consider both the topological structure of the network and the intensity of communication over time. Once a seed is identified, we build the ego-centered community by adding in a straightforward manner other nodes

while trying to maximize the cohesiveness of the group. In this respect, we define a quality function that captures how cohesive is a group.

Quality Function to Measure a Group Cohesion

There are several solutions that propose an approach measuring the cohesion of the group, for instance, the functions described in Shi and Malik (2000), Clauset (2005), and Chen et al. (2009). The work of Ngonmang et al. (2012) assesses the community cohesion by using the degree sum of internal nodes versus the outgoing links from the community. However, the main drawback is the noninclusion of communication intensity in the calculation. In a context of a dynamic network, the communication rate may intensify or vanish and should, therefore, be integrated to evaluate the group cohesion over time. The method proposed by Lu et al. (2013) takes into account the intensity of communication by weighting the links. However, it neglects the topological dimension, i.e., two links of the same weight and having different degrees are considered equivalent.

To overcome these drawbacks, we propose a new quality function that combines both the

communicational and the topological aspects to address the IMN features.

Actually, our quality function is based on the two well-known community definition criteria: (i) the separation from the rest of network and (ii) the internal cohesion.

The first criterion is taken into account by dividing the sum of weights of outgoing links by the sum of weights of incoming links $\frac{\sum w_c^{out}}{\sum w_c^{in}}$. Thus,

the lower this ratio, the more the group is isolated or separated from the rest of network. That is, the intensity of the communication within the nodes of C is higher than the communication between nodes of C and the rest of the network.

The second criterion is addressed with the topological aspect of the community. Basically, if the number of links of a community is significantly greater than the number of its nodes, thus, the subgraph representing the community is highly dense. That is, each node of the community is almost linked to all other nodes and indicates how cohesive is the structure. Therefore, the proportion $\frac{|V_C|}{|E_C|}$ assesses at what level the community is cohesive. The quality function is defined as follows:

$$\psi(C) = \frac{\sum w_c^{out}}{\sum w_c^{in}} \times \frac{|V_C|}{|E_C|} \quad (1)$$

where

- $\sum w_{out}$ represents the weights sum of the links shared between a node inside C and another outside of it.
- $\sum w_{in}$ means the weights sum of the links that belong entirely in C .
- $|V_C|$ is the number of nodes in community C .
- $|E_C|$ is the number of links in C .

In conclusion, when $\psi(C) \approx 0$, then, both the internal cohesion and the separation from the rest of the network are satisfied. In other words, a good community, in our definition, is the one with a low-quality function value.

Step-by-Step Ego-Centered Community Detection Let $G = (V, E)$ be a weighted and directed graph, where V represents the set of

nodes and E is the set of links. Assume $u \in V$ being an ego node, its neighbors are represented by the set N_u , while the seed calculated by one of the two methods described before is represented by S_u . The community centered on u is obtained by finding out the nodes that minimize the quality function value $\psi(C_u)$. In this respect, we formalize the definition of C_u as follows:

$$C_u = \begin{cases} \{u \in V\} & \text{if } \psi(C_u) \leq \alpha \text{ and } u \in N_u \\ \{u \in V\} \mid p(u, v) \wedge \psi(C_u) \leq \alpha & \text{if } u \notin N_u \end{cases}$$

The first part of the equation is applied for any node v directly linked to u (i.e., $v \in N_u$).

The second term is introduced to consider nodes that are two or more steps from the ego node. $p(u, v)$ means that there is a weighted path either from u to v or from v to u . The weight of p is calculated by adding the weight of the different links that form the path. The reason for introducing the second term is to avoid having an ego node and its direct neighbors as a community. Moreover, as pointed out in the previous section, the threshold α has to be set approximatively equal to 0 to obtain more cohesive group. To build the community of u , we initialize $C_u = S_u$, and let therefore our algorithm that works in a three-phase fashion doing the rest as follows:

- *Phase 1:* Select a node $v \notin C_u$ from N_u or $V - N_u$;
- *Phase 2:* Check if $\psi(C_u \cup v)$ decreases;
- *Phase 3:* If the outcome of *phase 2* is positive, thus, adds v in C_u .

These three phases are repeated till $\psi(C_u)$ reaches the threshold α . Moreover, in the selection phase, we ensure that the chosen node is the one with a higher connection with the ego node or the members of its community. This strategy ensures to have a partial order with which nodes are selected. Nonetheless, the detail of such algorithm is beyond the scope of this entry, but it works in such a way that both the topological aspect and the communication one are well integrated. Furthermore, it is worth noting that in case we build the communities of several ego nodes, the

algorithm can detect overlapping communities since one node can belong simultaneously to several communities.

Mechanism of Tracking Community Changes

The question we have to address first is how do we track the changes: Should we do it in a continuous manner or a discontinuous one? In our case, we decide to use the discontinuous fashion since we do not aim to have the trace of every single change but the overall change in some particular points. The first problem raised is to define these particular points, which we suppose known in this entry due to a sake of presentation. Basically, we follow the change of a single community over time by checking in each particular point whether its structure has changed or not. In other words, we take a snapshot for each time point and see how the structure evolves. To this end, we apply our ego-centered community construction in a sequential manner on each snapshot. The outcome of the snapshot at T is compared with the one of $T + 1$ in order to identify changes if ever. As in the literature, the evolution that may occur is categorized in seven classes: *growth*, *contraction*, *continuing*, *division*, *merging*, *birth*, and *death*. In opposite with the existing solutions, we do not just limit our analysis on finding the class of the evolution but what kind of changes in terms of nodes (IN or VN), links (IL and VL), as well as communication intensity (GL) are also harvested. The reason of doing so is to be able to explain what factors underline the evolution. In our context, this strategy is very useful since an evolution of a

community is not always conducted by the topological aspects but with also the behavior of a node regarding to an ego node during a time window.

Moreover, we found interesting to introduce vanishing node (VN) in order to identify nodes that stay inactive for a long time. However, an ego node cannot be considered as a VN since it can stop communicating with its neighbors that keep exchanging between them. However, a community disappears when the number of its nodes is lesser than a fixed number. For example, if we consider that a community is composed at least of N nodes, then an ego community will disappear if we have less than $N - 1$ nodes.

Illustrative Examples

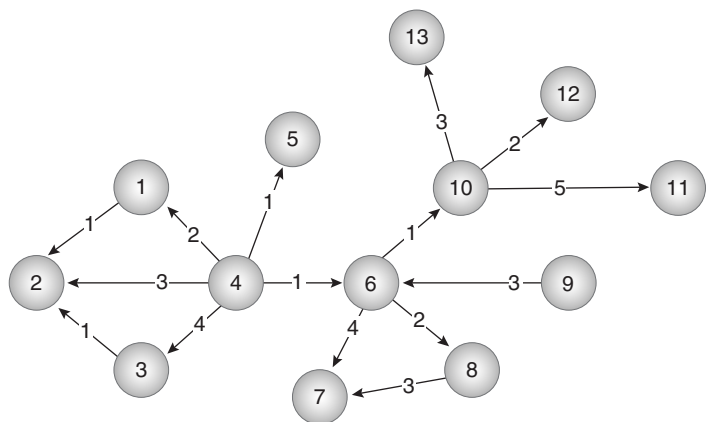
We illustrate our overall approach in this section by first showing how our quality function is used to detect community of a given network and how we track community over time.

Community Detection

Consider the network depicted on Fig. 3 been the initial network made of by 13 nodes and 15 links. We choose three ego nodes based on their central position.

We portray in Table 1 how communities are detected. We start with node 6 and add node 7 to form S_6 and we calculate $\psi(C_6)$. Afterward, we review one by one the neighbors of node 6, and whenever adding of one its neighbors decreases the value of $\psi(C_6)$, we do it and move forward to

Instant Messaging for Detecting Dynamic Ego-Centered Communities,
Fig. 3 Illustration network



the next neighbor. If ever an addition of one neighbor increases $\psi(C_6)$, we reject it and move forward. This is done till there is no more nodes that may minimize $\psi(C_6)$. For instance, if nodes 9 and 8 are added first to C_6 , thus $\psi(C_6)$ value decreases, while the addition of nodes 4 and 10 will do the opposite. That is, nodes 7, 9, and

8 belong to C_6 , while 4 and 10 are rejected. However, if nodes were selected with the following orders 4, 10, 9, and 8, thus, nodes 4 and 10 should be included in C_6 since their addition decreases the $\psi(C_6)$ value. However, this situation will never happen because our algorithm ensures that nodes with a higher connection are first used, which ensures a certain non-strict partial order when adding nodes.

After building C_6 , we find out C_{10} and C_4 by repeating the same process. Finally, we draw the corresponding communities in Fig. 4 with different colors.

Instant Messaging for Detecting Dynamic Ego-Centered Communities, Table 1 Illustration of the community detection procedure

Ego node: node 6		
$V(C_6)$	$\psi(C_6)$	Decision
{6, 7}	1.5	Added
{6, 7, 9}	0.64	Added
{6, 7, 9, 8}	0.08	Added
{6, 7, 9, 8, 4}	0.84	Rejected
{6, 7, 9, 8, 10}	0.76	Rejected
Final C_6 : 6, 7, 8, 9		
Ego node: node 10		
$V(C_{10})$	$\psi(C_{10})$	Decision
{10, 11}	2	Added
{10, 11, 13}	0.375	Added
{10, 11, 13, 12}	0	Added
{10, 11, 13, 12, 6}	0.68	Rejected
Final C_{10} : 10, 11, 12, 13		
Ego node: node 4		
$V(C_4)$	$\psi(C_4)$	Decision
{4, 3}	4	Added
{4, 3, 2}	0.5	Added
{4, 3, 2, 1}	0.14	Added
{4, 3, 2, 1, 6}	0.55	Rejected
{4, 3, 2, 1, 5}	0.06	Added
Final C_4 : 1, 2, 3, 4, 5		

Tracking Community Evolution

In this paragraph, we illustrate our community tracking mechanism. To this end, we consider two snapshots of a network captured at times T and $T + 1$ as depicted on Fig. 5. As one may observe it, the network taken at time T is made of three communities (each community with a given color). Precisely, we have $C_4 = \{1, 2, 3, 4, 5\}$, $C_6 = \{6, 7, 8, 9\}$, and $C_{10} = \{10, 11, 12, 13\}$.

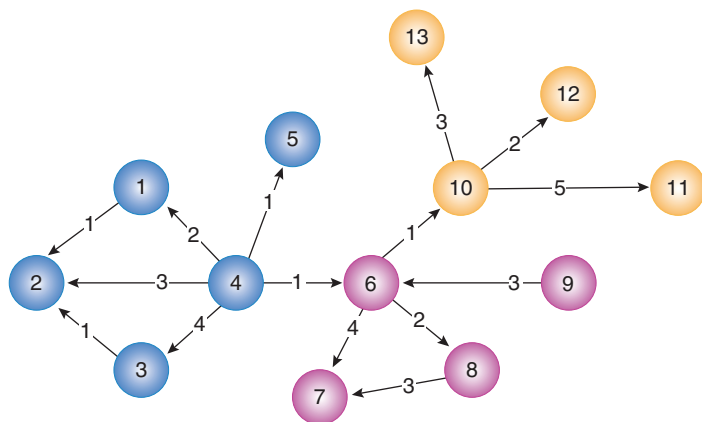
At $T + 1$, the changes that occur on the overall structure are as follows:

- 02 vanishing nodes (VN), namely, nodes 1 and 11
- 02 vanishing links (VL): (4, 5) and (6, 10)
- 04 incoming links (IL): (5, 10), (10, 6), (8, 9), and (8, 10)

Moreover, when we apply our ego-centered algorithm, we found three categories of evolution:

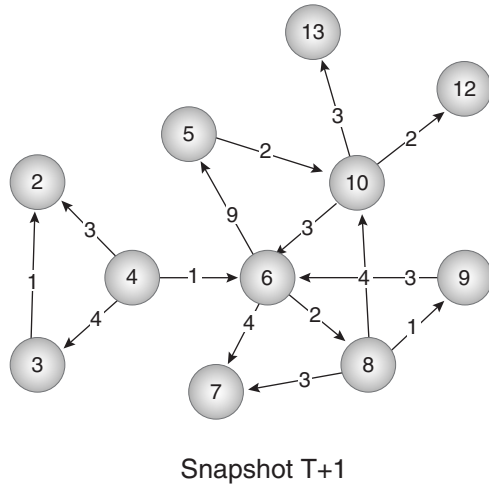
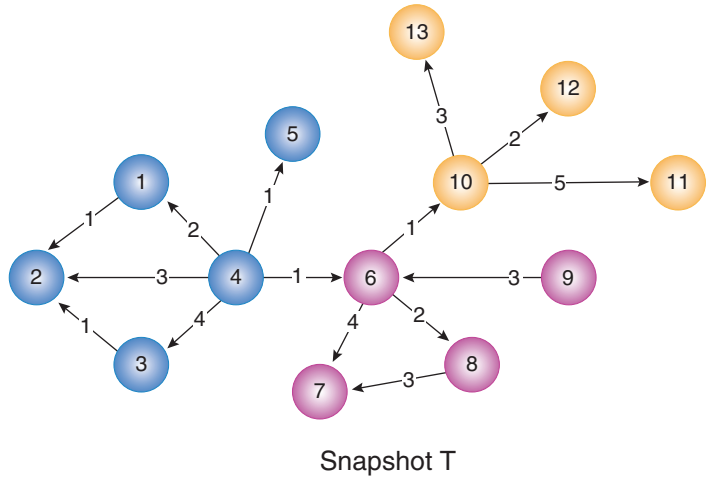
Instant Messaging for Detecting Dynamic Ego-Centered Communities, Fig. 4 Detected communities by our algorithm in the illustration network

Detected communities by our algorithm in the illustration network



Instant Messaging for Detecting Dynamic Ego-Centered Communities,

Fig. 5 Two successive snapshots of a dynamic network



- A growth with C_6 gaining two more nodes and becoming $C_6 = \{6, 5, 7, 8, 9, 10\}$
- A contraction with C_4 losing two nodes $C_4 = \{2, 3, 4\}$
- A death of C_{10}

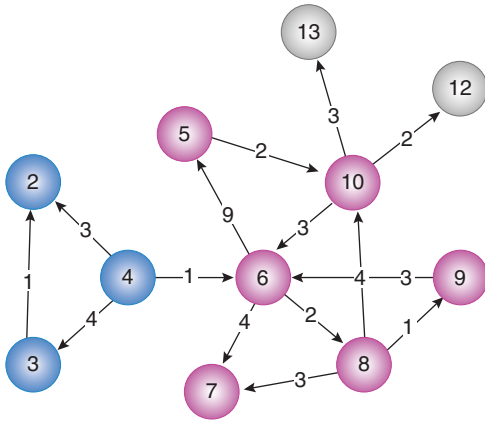
In Fig. 6, we portray the new community structures of the snapshot T+1.

Key Applications

The propositions described here can be applied in several situations such as in epidemiology and information and/or opinion diffusion.

Epidemiology Control

Most infectious diseases spread through close social links or interactions of human populations. These interactions are a good channel within which disease contagion spreads and leads to an outbreak. That is the reason why social network methods have been included in epidemiological studies in the earlier 1980s. The goal of doing so is to elucidate the impact of the human social behavior in the spread of infectious diseases. In this perspective, our ego-centered community detection can be applied to identify nodes with higher risk. In this respect, the ego-node is considered as the infectious individual, and finding out all nodes that belong to its



Instant Messaging for Detecting Dynamic Ego-Centered Communities, Fig. 6 Detected communities by our algorithm in snapshot $T+1$

community gives information about individuals who should be under control for avoiding an outbreak. Moreover, since we keep geographical positions of the nodes, hence, we may assess whether a risk is real or not and when an action has to be taken for preventing new infections.

Information and/or Opinion Diffusion

Our mechanism of tracking community changes figures out how links evolve over time. Therefore, it reveals links that are more likely to spread information, which are not well depicted by the overall structure. Precisely, our mechanism points out intense and strong links as well as infrequent and weak ties. Actually, when a link appears and disappears over time, it is trivial to understand that such a link has a low probability to diffuse information. In other words, based on the outcome of our solution, someone has the information whether an information should be diffused in one time slot or whether it will take a while to reach the targeted node.

Future Directions

The ongoing works related to this entry are twofold:

- *Time window size.* We plan to propose a mechanism for setting the time window size in such a way that captured snapshots contain enough changes. In fact, the time window size has a real impact on the outcomes of our tracking community evolution. A small size is cost consuming, while a large size leads to the miss of paramount and pertinent changes in a real-time basis. In this respect, we aim at calibrating the time window in such a way we capture the effective structure changes while minimizing the computational cost.
- *Strict total order for building a community.* We formalize and generalize our concept of ordering nodes to select when constructing a community. We recall that when two nodes have the same position *w.r.t* the ego node, we choose one of them in a random fashion. That is, our detection algorithm is not deterministic. We envision to face such a drawback by setting a total order selection of nodes in order to make a unique outcome of the algorithm.

Cross-References

- ▶ [Analysis and Visualization of Dynamic Networks](#)
- ▶ [Community Detection: Current and Future Research Trends](#)
- ▶ [Community Evolution](#)
- ▶ [Dynamic Community Detection](#)
- ▶ [Models for Community Dynamics](#)

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Intellectual Property Rights

- ▶ [Legal Implications of Social Networks](#)

Interaction

- ▶ [Computational Trust Models](#)
- ▶ [Human Behavior and Social Networks](#)

Interaction Network

- ▶ [Query Answering in the Semantic Social Web: An Argumentation-Based Approach](#)
- ▶ [Social Communication Network: Case Study](#)
- ▶ [Subgraph Extraction for Trust Inference in Social Networks](#)
- ▶ [Twitter: A System for Collective Social Intelligence](#)

Interaction Patterns

- ▶ [Social Interaction Analysis for Team Collaboration](#)

Interconnected Network

- ▶ [Multilayer Social Networks](#)

Interest Alignment

► [Policy Networks: History](#)

Interfirm Networks

► [Interorganizational Networks](#)

Interlocking Directorate Network

► [Interlocking Directorate Networks](#)

Interlocking Directorate Networks

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Synonyms

[Board interlocks](#); [Corporate elite](#); [Interlocking directorate network](#); [Old boys network](#); [Power elite](#)

Glossary

Interlocking directorate: A link between two organizations that emerges when at least one person is a member of both organizations' board of directors simultaneously

Interlocking directorate network: The cumulative set of interlocking directorates that together form a network of

linkages between a set of boards of directors
 SNA: Social network analysis

Definition

An interlocking directorate is a link between two organizations that emerges when at least one person is a member of both organizations' board of directors simultaneously. The cumulative set of interlocking directorates together forms an interlocking directorate network. These networks are analyzed from a variety of perspectives with a variety of tools. They are often seen as an expression of concentration of corporate power and are studied on a national and a global level.

Introduction

Since the emergence of the modern corporation in the nineteenth century, managerial power has been in the hands of a relatively small group of people, often referred to as the business elite. Within this group, a substantial number of individuals sit on the board of directors of multiple companies. By combining multiple board positions, these individuals create “interlocking directorates” between companies. As a result, this rather small group of individuals can, potentially, coordinate management decisions, share information and practices, and enforce norms in different company contexts.

This publicly visible concentration of economic power in the hands of a few and its manifestation in extensive networks of interlocking directorates led to a rising interest in this phenomenon. A significant set of studies on business elites emerged, beginning in the early twentieth century. Questions include the nature and delineation of the business elite and the causes and consequences of this phenomenon for the company and its stakeholders over time (e.g., Mizruchi 1996). The development of social network analyses further pushed this line of work considering that it is a beautiful and compelling example of a

social network, and data are relatively easy to gather from public sources.

Once the data are collected, the set of companies, directors, and their linkages form an “affiliation,” or “two-mode” network, from which a “one-mode” company-by-company network and a director-by-director network can be induced. These networks can be seen as a reflection of economic power structures and raise a variety of different questions which have been approached with different perspectives and by applying different network analyses.

Historical Background: Network Analyses from Local to Global

Since the early days of interlocking directorate research, most of the empirical research was concerned with national networks of interlocking directorates. The analyses were highly descriptive at first, and the networks were seen as a social structure that fosters communication, coordination, and cohesion among the corporate elite within their specific institutional setting (e.g., Scott 1985a). Occasional comparative approaches initially focused on general statistical network properties such as density and centrality and showed large differences (e.g., Stokman et al. 1985). More recently, cross-country comparative studies benefit from the insight that network properties are typically nonlinear and that both local (such as clustering) and global network (such as average distance) properties need to be taken into account (e.g., Kogut 2012).

National networks of interlocking directorates have lost part of their theoretical and empirical significance in the wake of increasing international board interlocks, which first emerged in the 1970s (Fennema 1982). The global dimension of the interlocking directorate networks draws more and more attention over time. The network spanned the north-Atlantic and remained fairly stable even during times of globalization. Even in 1996, the international network remained a superstructure based on resilient national business communities (Carroll and Fennema 2002). This has changed considerably since the turn of the

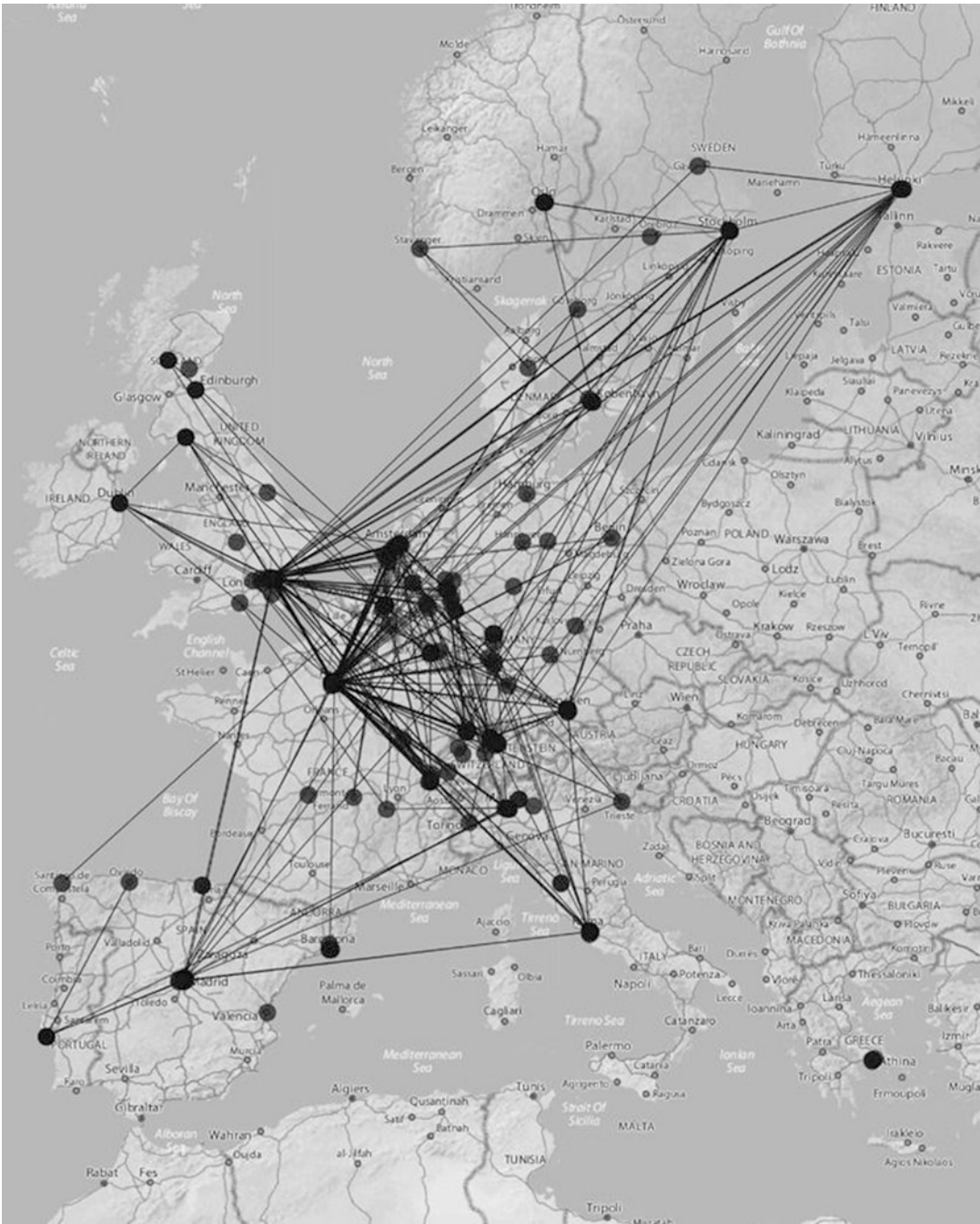
century. Corporate boards are increasingly nationally diverse, and international board membership is becoming common practice among the corporate elite. This builds a transnational elite network of interlocking directorates (Van Veen and Kratzer 2011; Heemskerk 2011; Carroll et al. 2010).

For some, this emerging international network is a tell-tale sign of the long-awaited transnational capitalist class. Others see it as the emergence of a global network of pipes and prisms – to use the wordings of Podolny – that might serve as the cornerstone of a global business community reminiscent of the national business communities throughout the twentieth century. However, in the first decade of the twenty-first century, the transnational network continues to be dominated by western-industrialized world, and the lion’s share of the growth over the past decade is realized within Europe (Carroll 2010) (see Fig. 1 for the European network of border-crossing interlocks in 2010).

Interlocking Directorate Networks; Different Perspectives and Different Questions

Overall, interlocking directorates’ research can be divided along two axes which generates four different fields of study. First, the *level* of analysis can be either the “actor” or the “network.” Second, the *unit* of analysis can be either the individual director or the company. When combining these two axes, four research perspectives can be derived (see Table 1) (Scott 1985b; Heemskerk 2008, pp. 33–38). Scholars, therefore, typically concentrate on either the interorganizational or the interpersonal projection (although, recently, advances have been made to study the full bipartite network) (e.g., Robins and Alexander 2004; Kogut 2012).

When studying **individual directors** on an **actor level**, attention will be drawn to individual attributes of the members in the network. It draws special attention to the “social capital” of individual directors. On the one hand, questions about elite family and educational background and career trajectories are relevant here, as well as



Interlocking Directorate Networks, Fig. 1 The European network of border-crossing interlocks in 2010

studies on how “new” groups, such as women and foreign directors, enter the corporate elite. On the other hand, it raises questions regarding board composition such as how board members use

their social capital for private (such as job search and information gathering) and company-related purposes (such as profit and corporate legitimacy).

Interlocking Directorate Networks, Table 1 Classification of different interlocking directorates approaches

Level of analysis	Units of analysis		
	Actor	Corporation Resource dependence	Individual Social capital
Network	Coordination of markets Financial hegemony	Class hegemony Inner circle Transnational capitalist class	

When studying **individual directors** on a **network level**, a network analysis investigates networks where directors are the nodes connected through their mutual board memberships. Questions include how the “inner circle” of individual directors operates as a group, whether there are traces of power or influence, and how the network of board interlocks reflects class cohesion. More recently, in the wake of globalization, the internationalization of these hitherto national elite networks has become more and more relevant (which we discuss below in more detail).

When we shift to the other mode of the affiliation network, the company perspective becomes dominant. Now, individual directors are instruments to reach company goals. With this idea in mind, scholars study **single companies** on an **actor level** to determine how they are embedded in the wider network. Questions focus on why certain individual directors might be useful for companies in order to manage “resource dependencies.” Board members are seen as a personal linkage to important, but external, resources. Finally, one can study **companies** on a **network level** where firms are the network nodes, connected through mutual board members. This perspective has dominated the literature over the past decades and includes investigations into the dispersion of corporate governance practices through the network and on the influence of network position on company performance. Throughout the twentieth century, a key issue has been whether a restricted set of

companies – such as banks – dominates groups of other companies by being more central in the network.

Research on interlocking directorates flourished during the 1970s and 1980s when it was proven to be a rich empirical foundation to institutionalism and structuralism. The explosion of research on interorganizational relations has increased the importance, and research has become even more prominent in the 1990s. However, despite its virtues, research on interlocks has always attracted critique as well. It has been criticized for being an overstructured approach and not leaving enough room for agency. The theoretical multiplicity in the field made it attractive to many but also hampered a cumulating knowledge base. Furthermore, a call for caution is in order regarding a too simple notion of the utility of interlocks: some matter more than others. Corporate board interlocks can best be understood as an opportunity structure.

Future Directions

After more than a century of interlocking directorates’ research, the field remains vibrant and innovative. As the (global) society and economy rapidly transforms, so do the networks of corporate boards. Some observers stress the decline or fragmentation of national business elites and call into question their relevance in tomorrow’s economies (Mizruchi 2013; Chu and Davis 2016). Others see the stability of the global board interlock network in the wake of the financial crisis (Heemskerk et al. 2016) as an indication of its continuing relevance.

These developing questions about the role, nature, implication, and naissance of interlocking directorates will, therefore, prove a veritable ground for academic attention. Three developments will be especially important in the near future. First, there will be a growing availability of large datasets: big data. This opens up multi-layered analysis of truly global elite networks with hundreds of thousands of nodes. Although data quality continues to be an issue, it assists in

answering key empirical questions on the composition and connection of corporations and its elites across the globe. It also allows for a much needed comparative approach that moves beyond the well-studied cases of a few western industrialized countries (e.g., Naudet and Dubost 2016). This is particularly important since novel work in the realm of business history has revealed how the role and function of board interlock networks differ across time and different institutional settings (David and Westerhuis 2014).

Second, recent methodological advances introduced longitudinal actor based modeling in SNA. Methods such as stochastic actor-based modeling and temporal exponential random graph modeling allow us to study the generative mechanisms that drive interlock formation and the wider network dynamics. Third, the context in which interlocks emerge will receive increasing attention. Although there has been attention paid to rough classifications such as bank-based versus market-based financial systems, more fine-grained attention to governance regimes and elite formation will drive new research (e.g., Van Veen and Elbertsen 2008). This will become even more important when emerging economies such as China, Brazil, and India integrate in the corporate elite networks. For instance, the role of politics in (international) networks of interlocking directorates will rise in relevance, not in the least due to a growing number of corporations and investors in the world being state controlled. As a result, pressing questions will begin to emerge about the precise relationship between political and business elites. This opens important avenues for further research on how economic power is organized in an era characterized by an evolving global economy.

Cross References

- ▶ [Economic Network Analysis Based on Infection Models](#)
- ▶ [Interorganizational Networks](#)
- ▶ [Managerial Networks](#)
- ▶ [Top Management Team Networks](#)

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International Hyperlink Networks

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Synonyms

[Global Internet hyperlink networks](#); [International Telecommunications Networks](#)

Glossary

Hyperlink network analysis (HNA)	network analysis where the links connecting nodes are hyperlinks
International Communication Network (ICN)	a network where the nodes are nation-states linked by information flows
International hyperlink structure (HIS)	the relatively stable pattern of hyperlinks among nations
International Information Flow (IIF)	the exchange of information among nations regardless of specific channel or media
Social network analysis (SNA)	a set of research methods for identifying <i>structures</i> in social systems based on the patterns of relations among system components
Webometrics	the measurement of aspects of the World Wide Web, including websites, web pages, parts of web pages, words in web page hyperlinks, and web search engine results

Definition

An international hyperlink is the technological capability that enables a website to link seamlessly with another globally, generally through the click of a mouse. Hyperlinking between websites functions as a navigational tool allowing for different collections of information throughout the world. International hyperlink network analysis provides a framework not only for examining the structure of global communication networks (the pattern of relations among the nodes based upon the frequency of hyperlink connections) but also for understanding the wider pattern of the ways in which nation-states establish links with one another for the exchange of resources.

Introduction

The World Wide Web may be defined as a distributed hypertext system consisting of a network of content and hyperlinks, with billions of interlinked pages. The Web has no engineered architecture, and it may be considered a self-organized system with a well-defined structure of linkage that implies an underlying social structure (Chakrabarti et al. 1999). This entry examines the evolution of the Web's emergent social structure at the level of nation-states. It reviews the research of international hyperlink networks since 2000, the beginning of international hyperlink studies.

Key Points

Linkages between countries have changed significantly as the global information infrastructure has evolved over the past decade. Appadurai (1990) posited that globalization processes take place via five flows: the movement of people, science and technology, capital, media and information content, and political ideologies. Messages circulated through online networks have been fundamentally altering our perception of globalization. For example, we no longer think of globalization only in terms of economics but also in how it impacts, science, education, and the arts. Castells

et al. (2007) identified the emergence of a “networked society” that is being created by hypertext websites and other new media. He claimed that all messages people communicate through networked media create a new space that is both local and global at the same time. That is, transnational message flows pave the way for increasingly individualized and networked places in cyberspace.

Historical Background

The Internet represents a technological revolution that makes use of computer networks to share distributed information. It has changed the geography-bounded nature of communication toward greater interconnectedness, making globalization possible (Barnett et al. 2001; Berners-Lee 1999; Park and Thelwall 2006). Scholars have argued that its diffusion has the potential to alter the structure of the relations among the nations of the world by limiting the impact of geography on communication (Graham 2001). Some have argued that global communication systems strengthen worldwide social relations in the information age and that cultures are shaped and clustered across national borders (Barnett and Sung 2006). The reconfiguration of communication and the globalization of cultures are indeed important features of an Internet-mediated society.

The structure of international telecommunications may be understood from two different perspectives: the increased centralization and increased diversification of communication flows. According to the first perspective, the global communication network is rooted in a broader perspective of economics emphasizing the asymmetry between information-rich and information-poor countries from the framework of world systems theory (Barnett and Park 2005; Lee et al. 2007). World systems theory calls into question the modernist assumption that nations are independent. This theory claims that nations’ development can only be understood by considering the systematic ways in which societies are linked to one another within the context of a larger

network of material, capital, and information exchanges.

In line with world systems theory, the global structure may be described in terms of three types of structurally equivalent nations – the core, the periphery, and the semi-periphery – which are related to each other economically. A network composed of these three types may show a centralized hub structure where one or several dominant nodes are noticeably more central than the other nations that are not directly connected to each other (Barabási 2002). In the international communication network in which the ties consist of the flows of information, there are interactions between the center hub and the periphery along the spokes but not along the rim from one peripheral nation to another (Galtung 1971).

The lack of direct interactions among peripheral nations can lead to a condition in which communication flows through the core, to asymmetric physical distances where central nodes are geographically proximate to each other and peripheral nodes are relatively closer to the center than to each other, and to a dependent network position that subsequently influences a nation’s ability to control the flow of goods and services and information among nations because the network defines which pathways between nations are available (Lee et al. 2007). Due to the stable differences in national economies, the peripheral societies specialize in the production and export of labor-intensive, low-wage, low-technology goods desired by the core and the semi-periphery. In return, the core produces capital-intensive, high-wage, high-technology goods that are exported to the periphery and the semi-periphery, which engages in both core-like and periphery-like activities. Although some disputes exist regarding the classification of specific nations into the core, semi-peripheral, and peripheral categories (Smith and White 1992), each nation’s membership in one of these three categories tends to be relatively stable.

World systems theory has a number of implications for the examination of international telecommunications and globalization: (1) the structural position of a country determines its potential for social and economic development

and its interaction patterns, (2) the structural position of a country is a result of its interactions with other countries, and (3) the relationships among the network's nations are relatively stable, changing only as the distribution of modes of production changes for semi-peripheral nations (Barnett 2001; Barnett et al. 2001; Barnett et al. 1999).

The second approach for examining the global communication network focuses on the increasing trends of decentralization, regionalism or cultural pluralism, and the emergence of clusters in peripheral areas (Barnett 2001; Lee et al. 2007; Matei 2006; Robertson 1992). Although world systems theory was initially created to describe global interactions during the industrial age, recent research has demonstrated its suitability for examining the flow of international information but concluded that world systems theory is inadequate to describe the complexities of international communication (Barnett and Choi 1995; Barnett et al. 1999). These trends toward both centralization and decentralization may be constitutive features of contemporary global circumstances and be important in determining the future trajectories of the international telecommunications network.

According to this perspective, the structure of the international telecommunications network may be determined by factors other than economic relations, including countries' geographical locations and language spoken (Barnett and Choi 1995), the religion practiced (Barnett et al. 1999), and culture (Barnett and Sung 2006). Prior studies have suggested that subsystems formed by geographical, social, or cultural homophily and interdependence acts against the centralizing forces of globalization (Huntington 1996). Such dynamics can lead to a network different from the traditional core-periphery model and create a multilayered network where central hubs coexist with multiple lower-level core regions (Lee et al. 2007).

Although considerable research has analyzed international telecommunications traffic (Barnett 2001; Lee et al. 2007), few studies have examined the Internet's international structure. The Internet is a packet-switched network unlike the

telephone, which devotes a single circuit to each individual message. Consequently, the origin and destination of individual messages cannot be determined (Barnett and Park 2005). An alternative approach that can allow the examination of the Internet's international traffic is the analysis of inter-domain hyperlinks.

Research Review

Beginning of International Hyperlink Research

The first large-scale study of the international Internet hyperlink examined the bilateral hyperlinks among the Organization for Economic Cooperation and Development (OECD) (Barnett et al. 2001). The number of inter-domain hypertext links embedded in domains associated with all twenty-nine OECD countries (country code top-level domains [ccTLDs] such as .ca for Canada) and six generic top-level domains (gTLDs) (.com, .net, .int, .gov, .edu, and .org) was gathered for July 1998. These countries represented approximately 96% of Internet traffic. Not included in the analysis were non-OECD members. Because no single top-level domain (TLD) represented Internet traffic for the USA, .edu, .us, and .gov were combined to designate the USA. The other gTLDs (e.g., com, .org, .int, and .net) were not included in this group because those were not exclusively from the USA. The results indicated that .com was the most central node, followed by .net.

The USA was the most central country acting as the network's hub or the nucleus of the World Wide Web, followed by the UK, Canada, Germany, and Australia. Most peripheral in the network were Iceland and Turkey. A reasonable explanation for this structure is that the Internet was developed in the USA and that it has low-cost bandwidth. At that time, it accounted for 58% of all Internet hosts and 94 out of the top 100 websites were based in the country (Cukier 1999). The correlation between centrality and GDP was .974 ($p < .000$), indicating that a nation's position in the network was a function of its total wealth. Cluster analysis revealed that

the OECD nations and gTLDs formed a single group centered about the .com–.net dyad. There were no subgroups due to geography, language, or culture.

Based on the assumption that sites are internally fully navigable, and the interlinkage between sites becomes the main factor in determining the accessibility of web-wide content, Bharat et al. (2001) found that there was a much higher number of intranational or site links than ties to other countries. Typically, only 1% of links were to websites in another country. When the links among the most central countries were removed, geographical, linguistic, and political factors impacted the structure of the Web.

The structure of the Web was related to a number of exogenous variables and older networks (Barnett et al. 2001). The older networks include the international telephone, air traffic, trade, science citations, and student flow networks. Other variables were language and asynchrony, defined as the difference in time zones between national capitals. Physical distance, however, was not related to the structure of international hyperlinks. The cost of communicating via the Internet was unrelated to distance. The combined effects of transportation, telecommunications, science, asynchrony, and either trade or student flow accounted for between 62% and 64% of the variance in network structure, with transportation the most significant determinant. These results characterized the Internet as an autopoietic system (Barnett 2005), growing through the self-replication of the existing telecommunications network but evolving to accommodate physical displacement and the ability to rapidly exchange and store vast amounts of information other than voice.

Expansion of the Research

Barnett and Park (2005) expanded on earlier research by gathering data in January 2003 on the number of bilateral inter-domain hyperlinks among nations. They investigated 47 nations including all OECD member countries (except Poland) and six gTLDs. Notable additions were Brazil, India, China, Russia, South Africa, Israel, Singapore, and Indonesia. The TLDs represented approximately 98% of Internet traffic. Again,

because no single TLD totally represents the USA, .edu, .mil, .us, and .gov were combined to represent the USA (.usa). The hyperlink network in 2003 was completely interconnected. As in 1998, the USA, Australia, the UK, China, Japan, Canada, and Germany were central, while Uruguay, Luxemburg, the UAE, Thailand, and Slovakia were peripheral. When link direction was considered, the USA was the most central in terms of in-degree, followed by Indonesia, India, Italy, and France, while Uruguay, the UAE, and the Czech Republic were the most peripheral. While Germany, the UK, US, and Australia were central in out-degrees, the UAE and India were the most peripheral. Again, the results revealed a single group centered about the .usa–.au dyad, the two most central nodes.

Barnett and Park (2005) compared the hyperlink network to one represented by bilateral bandwidth capacity. Bandwidth describes the physical network that transports packets of data from point-to-point as opposed to the TCP/IP for which geography is irrelevant (Townsend 2001).

These connections are nondirectional. The density of the bandwidth network for the countries indicated that 18.5% of the possible direct hyperlinks were present. The USA was by far the most central country, followed by the UK, Germany, Hong Kong, Singapore, Japan, and France; most peripheral were Iceland, Lithuania, Morocco, Croatia, and Guatemala. There were three major groups: (1) English-speaking countries (the USA, the UK, Canada, Australia, and New Zealand) with Northern Europe (Scandinavia, Belgium, and the Netherlands) and East Asia, (2) Latin America, and (3) Franco-German Europe (France, Germany, Austria, Italy, Spain, Switzerland, and Czech Republic). The network resembled a wheel, with the USA at the hub and spokes to individual countries and clusters of nations. The USA dominated Internet flow due to its position in the network. While there were links entirely within Europe and the Asian-Pacific region and limited links within Latin America, intercontinental links primarily went through the USA. Further, even the connections within specific regions may have been routed through the USA because of limited within-region bandwidth. Clearly, the

USA was in position to act as an information broker or gatekeeper.

Previously, Townsend's (2001) examination of Internet bandwidth concluded that every region and nearly every country has a direct Internet connection to the USA and direct connections between other countries are less common. Furthermore, direct connections between different major regions such as Asia and Europe are practically nonexistent. This structure dictates that the US Internet infrastructure functions as a massive switching station for traffic that originates and terminates in foreign countries. The hyperlink and bandwidth networks correlated .412 ($p = .000$) (Barnett and Park 2005). Additionally, there was a strong relationship ($r = .847$, $p = .000$) between both networks' centralities, indicating that the physical infrastructure of the Internet is an important determinant for which countries communicate via this medium.

Reconstruction of the Research

Park et al. (2011) examined the structure of the international hyperlink network in 2009 and how it changed from 2003. Data was collected in May 2009 using Yahoo. Yahoo had indexed about 47 billion websites at that time (<http://www.worldwidewebsite.com/>). Over 9.3 billion hyperlinks among 33.8 billion sites from 273 TLDs were examined. Again, three TLDs reserved for the exclusive use of American institutions, *.edu*, *.gov*, and *.mil*, were combined with *.us* to form a node for the USA. Because *.com*, *.org*, and *.net* are not exclusive to the USA, they were not included. This may have resulted in an underestimate of the centrality of the USA and other countries that rely heavily on gTLDs. The 2009 international hyperlink network was completely interconnected. The USA had the largest in-degree centrality, followed by Germany, the UK, France, Japan, and Spain, while Germany had the highest out-degree centralities. The G7 and several European Union countries were central in the 2009 network. Also, Brazil and Russia emerged as core countries integrating more peripheral nations. Brazil linked South America, and Russia linked the former Soviet Republics. Additionally, it appears that for the first time there were regional,

cultural, and linguistic groupings, a Latin American group, cliques centered about Russia and China, and a Scandinavian group, as well as a core group.

Park et al. (2011) investigated changes in the World Wide Web by comparing the hyperlink relations among 47 countries in 2009 with the same nations from 2003. The results for the two points in time were similar. The USA was still the most central country along with Germany, the UK, France, Japan, and Spain. The semi-peripheral countries included the Netherlands, Austria, Switzerland, Belgium, Australia, Brazil, Mexico, China, India, and Russia. The UAE, Israel, Estonia, Uruguay, and Luxembourg were the most peripheral. Various measures of centrality correlated at an average of .80, suggesting stability in the network. The overall correlation between the 2009 and 2003 networks was only .406 ($p < .01$).

There were some interesting changes. First, the international hyperlink network became more highly centralized. The composite Gini score of 2009 network was 0.466. It was only 0.291 in 2003. The greatest departures from the predicted changes were for the most central countries. Europe as a whole, especially Germany, became much more central. The UK, France, Spain, Italy, and Japan's out-degree centralities grew more than expected. The USA, Germany, the UK, France, Japan, and Spain's in-degrees grew more than expected. Second, Brazil, Russia, India, and China showed various changes. Brazil grew more than predicted, and Russia grew as predicted. China had fewer outward links than expected. This was probably due to internal domestic growth or the use of the Chinese language, which limits its contacts with the West. India had fewer inward links than expected. Third, the centralities are distributed as a power curve (Barabási 2002), suggesting disproportionate growth in the number of hyperlinks by the more central countries, supporting the notion of preferential attachment (Barabási and Albert 1999). Fourth, while there was only one group in 2003, regional, cultural, and linguistic groupings formed in Latin America, Scandinavia, and around China and Russia, suggesting that hybridization, increased centralization toward core-peripheral countries,

and increasing autonomous diversification of semi-peripheral countries took place.

While investigating the structure of global Internet connectedness, Seo and Thorson (2012) attempted to measure key structural changes in bandwidth and the centrality of digital nodes. Using a combination of bandwidth metrics and centrality indicators, they demonstrated how the global information infrastructure evolved between 2002 and 2010 and especially how several countries in the Middle East rose to prominence as good nodes mediating strong intra-regional networks. The results showed that a total amount of international Internet bandwidth has significantly increased in a manner reminiscent of the familiar power law from 931,319 Mbps in 2002 to 37,424,671 Mbps in 2010. The USA was the most important country in the 2002 Internet network based on both eigenvector centrality and degree, followed by the UK, Germany, France, Italy, Singapore, the Netherlands, and China. In 2010, the USA maintained the highest degree, but its eigenvector centrality was second to the UK. The density of the global Internet network was 0.030 in 2002 and it rose to 0.034 in 2010. Total international Internet bandwidth within the Middle East and North Africa (MENA) region has increased considerably between 2002 and 2010. The total MENA bandwidth was 2,096 Mbps in 2002 and increased to 375,798 Mbps in 2010. The growth in the Internet bandwidth within the region showed a power-law pattern similar to that found in worldwide bandwidth capacity.

Chung et al. (2014) investigated international hyperlink networks and their content in terms of the .com domain, the most ubiquitous generic top-level domain. They examined the kinds of global websites linked to .com, what the linked contents were, and who were dealing with the hyperlinks. The results showed the hyperlink network of websites with outgoing hyperlinks to .com websites indicated the dominant centrality of the USA, whereas that of those with incoming hyperlinks from .com websites illustrated a core-periphery structure centered about the USA and other wealthy countries. The most globalized topics covered by websites linked to .com websites

were business, the Internet and computers, recreation and entertainment, and personal interests. Many of the websites with outgoing hyperlinks to .com websites used only one non-English language. The predominant use of English by websites with outgoing hyperlinks to .com websites demonstrated the centrality of countries using English as the mother language in hyperlinked societies on the Web. Inferring international dotcom according to [Alexa.com](#) (Alexa 2012), the most frequently visited gTLD websites were .com websites. For example, of the 120 most frequently visited gTLD websites (including .net and .org) listed on [Alexa.com](#), more than 90% were .com websites, and the percentage of Internet visitors to those websites followed the power-law distribution. However, including other gTLDs should provide a better understanding of the structure of hyperlink.

Barnett et al. (2016) have described how the structure of the international hyperlink network has changed since 1998. In particular, it describes the hyperlink structure in 2010 and how it has changed over the last few years. The international hyperlink network is highly concentrated about a few central domains, the identity of which is dependent on the selected lens. In the overall network, .com is by far the most central node, accounting for 32.8% of all hyperlinks. An additional 13.2% may be attributed to other gTLDs, .org and .net. Among the ccTLDs, Japan, the UK, Germany, and China account for another 22.9%. Together, these seven TLDs account for almost 75% of the world's 14.3 billion hyperlinks. When only the ccTLDs are examined, the six most central nodes (.jp, .uk, .de, .fr, .es, and .usa) account for 38.5% of the hyperlinks, and when focusing only on the 87 nodes with the decomposed website data included, the five most central nodes (.jp, .uk, .de, .cn, and .usa) involve 56.5% of the links. These findings are consistent with world systems theory, which suggests that the global system may be characterized by unequal exchanges between information-rich and information-poor countries (Barnett et al. 1996; Chase-Dunn and Grimes 1995; Choi 2011; Wallerstein 1974). Building on what the McNeills has drawn from the history of the human Web (McNeill and

McNeill 2003), van Dijk (2012) has recently made a similar point that the basic idea of those who think that the Internet is decentralizing on a global scale and necessarily undermines the national state and is just as one sided as the opposite idea of the centralization of control by the national state. The state will not wither away or even dissolve into virtual relationships of horizontal types of organization appearing on the Internet. Both visions are one sided, since networks consist not only of (horizontal) connections but also of (vertical) centers and nodes.

Key Applications

The key application in this area is to collect some inter-hyperlinking data across countries. The interlinking data have been mostly obtained from commercial search engines. AltaVista was frequently employed in early 2000 but closed its hyperlink search options after Yahoo acquired it in 2004. Yahoo renewed the AltaVista's hyperlink commands in the name of "Site Explorer" in September 2005. Furthermore, Yahoo enabled researchers to automatically download a large volume of international interlinkage data using its application programming interface (API) function. Google has also a search option for hyperlink data. Although Google could run incoming hyperlink queries, there is no option in Google for retrieving bidirectional ties between a pair of country code top-level domains (ccTLDs) (e.g., .kr for South Korea). However, Yahoo discontinued its API option for interlinkage data in April 2011 and finally stopped its popular Site Explore service in November 2011. Now Yahoo's web search services are currently being moved to Bing. But, the successful transition of link command to Bing has not been made yet until August 2012. Alternatively, Thelwall and Sud (2012) recently proposed to replace hyperlink searches with URL citation searches with the Bing search API facilities. International hyperlink analysis has been used to predict economic and political development and to describe the changing landscape of international relations, including to predict international conflict and state sponsorship of terrorism (Barnett et al.

2013). Recently, Google's search engine has been employed to determine the hyperlink connections among the world's universities (Barnett et al. 2014).

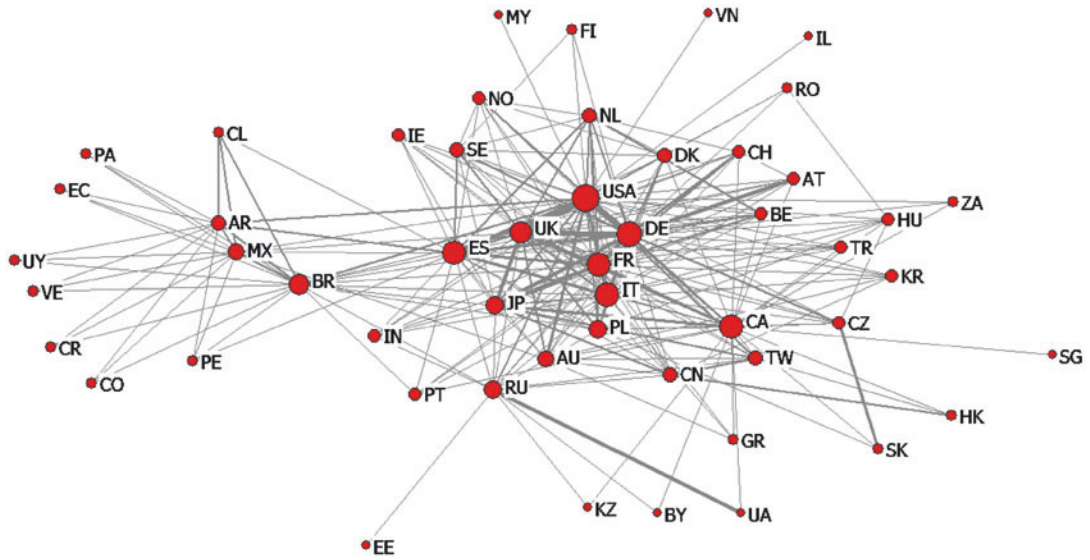
Future Directions

An important issue that remains unresolved in international hyperlink research is how imperfect spatial information (the generic TLDs may be hosted anywhere in the world) alters the structure of the network (Grubestic and Murray 2005). Past research has not included gTLDs, creating an inherent bias in the analysis of the international hyperlink network links in the examination of the links among ccTLDs. That is, it does not account for the geographic locations of .com, .net, or .org. As a result, the connectivity of the USA and other nation-states that rely heavily on domains other than ccTLDs are underreported. The gTLDs were not included due to the difficulty in determining which countries these websites reside in and who links to these sites (Rosen et al. 2011).

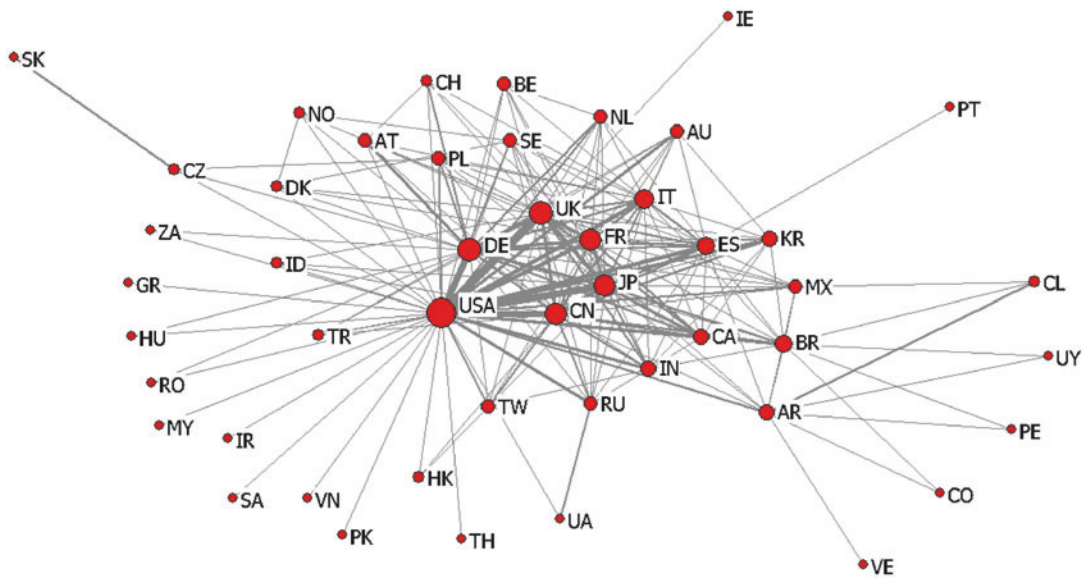
Based on the assumption that decomposing .com leads to a more accurate description of the international hyperlink network, Barnett et al. (2011) investigated adjusting hyperlink networks using information from *Alexa.com* on the percentage of international Internet users for the most frequently visited gTLDs. They decomposed the three gTLDs (.com, .org, and .net) into the countries in which their users reside and distributed the links proportionally to the ccTLDs. Then they compared the results obtained with the traditional methods. The adjusted hyperlink network showed significant changes in the centrality of several countries. The USA's out-degree centrality increased dramatically and its centrality changed in more than any other country (see Figs. 1 and 2).

The size of the concentric circles indicates the hyperlink connection density among countries. The thickness of the line connecting the two nodes is proportional to the connection density between the two nodes. Only those ties exhibiting more than 500,000 hyperlinks are shown. $N = 87$.

Also, the notability of several countries in Asia such as China, Japan, and India increased, probably



International Hyperlink Networks, Fig. 1 International hyperlink structure excluding .com (Chung 2011)



International Hyperlink Networks, Fig. 2 International hyperlink structure including .com (Chung 2011)

due to their economic ties with the USA and due to the Chinese language search engines *baidu.com*, *qq.com*, and *taobao.com*. On the contrary, the centrality of countries that did not heavily rely on gTLDs such as European countries decreased. Correlations between the two sets of centrality scores

showed that the addition of TLDs did not change the network centralities a great deal. The correlations ranged from .90 to .93 depending on the measure. The cell-wise correlation indicated that there were systematic differences between the two networks ($r = .755, p = .00$). The top 20 residuals

involved the USA (13), China (5), Japan (4), the UK (2), France (2), Korea (2), Germany (1), Spain (1), Canada (1), and India (1) (Barnett et al. 2011).

The size of the concentric circles indicates the hyperlink connection density among countries. The thickness of the line connecting the two nodes is proportional to the connection density between the two nodes. Only those ties exhibiting more than 1.5 million hyperlinks (three times the hyperlink network excluding .com, based on three times the degree difference) are shown. $N = 87$.

Although this research more precisely defined countries as nodes on the Internet by decomposing gTLDs based on where their users reside, these adjustments were not based on the volume of hyperlink connections. Rather, they were based on the proportion of Internet users from each country that used certain websites. The hyperlinks to and from gTLDs were distributed to various countries based on their residents' website use, which assumed that this was an accurate proxy for the distribution of hyperlink connections.

Cross-References

- ▶ [Combining Link and Content for Community Detection](#)
- ▶ [Extracting and Inferring Communities via Link Analysis](#)

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International Telecommunications Networks

- ▶ [International Hyperlink Networks](#)

Internet

- ▶ [Mapping Online Social Media Networks](#)
- ▶ [Personal Networks: Ties, Space, and the Internet](#)
- ▶ [Social Media and Social Networking in Political Campaigns/Movements](#)
- ▶ [Social Phishing](#)

Internet Science

- ▶ [Web Science](#)

Internet Support Groups

- ▶ [Research on Online Health Communities: A Systematic Review](#)

Interorganizational Collaborations

- ▶ [Innovator Networks](#)

Inter-organizational Marketing

► [Business-to-Business Marketing](#)

Interorganizational Networks

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Synonyms

[Alliances](#); [Chains](#); [Clusters](#); [Consortia](#); [Emergency response networks](#); [Firm networks](#); [Governance networks](#); [Interfirm networks](#); [Joint ventures](#); [Organization sets](#); [Organizational networks](#); [Policy \(domain\) networks](#); [Problem solving networks](#); [Public sector networks](#); [Regional networks](#); [Service delivery networks](#); [Supplier networks](#); [Temporary organizations](#); [Whole networks](#)

Glossary

Engineered networks	Networks that are consciously created and have common goals, i.e., alliances and consortia. Participants have a mutual awareness or even a common identity as network members (see Doz et al. (2000); Provan et al. (2007))
Formal relation	Organizations connected only through contracts or formal arrangements
Informal relation	Organizations connected by personnel flows or personal communication without a formal relation

Interorganizational relation

Dyadic link between two organizations in the form of exchange of information and knowledge, tangible and intangible resources, board interlocks, alliances, joint venture, consortia, etc. (see Galaskiewicz (1985); Cropper et al. (2008)). Essential building block for the formation of interorganizational networks

Network boundary

Indicates who or what is part of the network and what is defined as being outside the network. Represents first step in every network analysis. For different boundary specification strategies, see Laumann et al. (1983)

Serendipitous or emergent networks

Networks that emerge as an aggregate out of dyadic or triadic interactions and relationships between actors. Network boundaries are often ambiguous and are determined by the researcher (see “Network boundary”)

Definition

Wherever three or more organizations interact, we talk about interorganizational networks (two being a dyadic relationship). Interorganizational networks are in principle social networks. In the broadest definition and based on the general definition of social networks, they can therefore be characterized as a set of organizations connected by a set of linkages. Examples of interorganizational networks are networks in policy making (Laumann and Knoke 1987; Schneider et al. 2003),

innovation (Powell et al. 1996), public service delivery (Provan and Milward 1995), or alliance formation (Doz et al. 2000; Gulati 1998).

Interorganizational networks are the aggregate of the formal and informal relationships between the organizations as independent entities and the formal and informal relations between their members, if they act at least partially in their function as organizational members (interorganizational relations). Interorganizational networks can be emergent or engineered. In the first instance, they exist as social systems on the basis of dyadic or triadic interactions. They do not necessarily have a common goal. Also, the participating organizations do not need to possess a joint identity or even a conscious knowledge of each other beyond their direct contacts. Examples are policy networks or alliance networks where different alliances are connected through joint membership of companies. Engineered networks on the other hand are interorganizational networks that are consciously created often by a lead organization or more bottom-up by professionals in different organizations. Examples are R&D consortia, emergency response networks with detailed plans and exercises, and service delivery networks in health and human services. In empirical reality, however, emergent and engineered processes often coincide or are sequential, i.e., an emergent network might at some point become formalized.

Related to but conceptually distinct, networks of organizations are also often seen as a form of governance, i.e., as a way to coordinate human (inter)action (Powell 1990). In this context, networks are often juxtaposed against markets and hierarchies and characterized as systems, in which actors are autonomous but interdependent and coordination takes place primarily through exchange and negotiations rather than through fiat (hierarchy) or price competition (market). Here, four interaction modes can be distinguished: Buy, make, ally, and join (Raab and Kenis 2009). Make and buy do not constitute interorganizational networks, since no interaction between different organizations is required (make) or the transaction is completed on a (spot) market. In case of ally, an organization

forms bilateral relationships with other organizations to secure access to resources, reduce uncertainty, or gain legitimacy. In case of join, a group of organizations together consciously forms a network and produces an output at the level of the network (Raab and Kenis 2009). It should be noted, however, that these interaction types are mostly analytical and that we often see mixed forms in empirical reality. For example, long-term buyer-supplier relationships that started off as market transactions might gain more characteristics of a network over time, and networks can also include elements of fiat.

The common thread through these different conceptualizations is that interorganizational networks are social systems formed by vertically integrated formal organizations that are involved in complex interactions and the exchange of material and nonmaterial resources (including information and knowledge).

Introduction

Interorganizational networks matter at three levels. For individual organizations, their networks influence their power position within a sector, influence the access to new knowledge, and generally determine opportunities and constraints. For the networks themselves (network level outcomes), how they are structured and governed influences to a great extent what outcomes they produce and in case of engineered networks to what extent they are able to achieve their goals. For the wider community or society, the way they are structured and governed determines the positive or negative effects these networks might have, for example, in terms of innovative spillovers or policy making. For each of these three levels, a large body of literature has emerged. Most of the theories for the explanation of outcomes at the level of the individual organization, however, are applications or adaptations of general social network theory (see Borgatti and Halgin (2011)). On the other hand, networks are often used merely as a metaphor or social network concepts, and analytic techniques are used to test

and further develop other theories like diffusion theories or power theories. It is mainly in public management, where some progress has been made toward a separate theory of network effectiveness (Provan and Milward 1995). Despite conceptual ambiguities, interorganizational network research has been widely conducted in the social sciences and economics. Economists, economic geographers, political scientists, sociologists, organization, and (public) management scholars study organizational, network, and societal effects of interorganizational networks. Economists and management scholars are mostly interested in the questions, which governance mechanism yields transaction cost advantages and which structural positions in networks generate the most benefits for individual organizations. Economic geographers and sociologists are interested in the effects of different network characteristics like centralization, clustering, or density on the performance and innovative capacities of (regional) networks and their wider impact on regional economies. Political scientists study the power structures, decision-making, and policy outcomes that networks of political and societal actors produce in specific policy fields. Public management scholars are interested in the effectiveness of networks in public service delivery or emergency management based on their structural features, resources, context factors, and governance.

Key Points

Interorganizational networks are most of the time analyzed as “pipes,” i.e., organizations are connected by relationships that facilitate the flow of tangible and intangible resources, very often tacit or explicit knowledge and information. These ties are also crucial for the coordinative function of interorganizational networks to produce collective outputs. From the perspective of an individual organization, it is assumed that the formation, change, continuation, and termination of ties are driven by purposeful exchange and calculated investments in social capital and at least a heuristic cost-benefit or a risk-opportunity reasoning. An organizations’ position in the

network is then assumed to determine opportunities and constraints as having access to resources or being able to mobilize political support. The resulting social capital on the level of the individual organization affects a variety of outcomes like innovative capacity, political influence, or financial performance. However, this theoretical reasoning is based on two assumptions. First, it is assumed that material and immaterial resources flow through organizations, despite the fact that two different subunits within the organization which connects two other organizations are involved in the interaction, i.e., it is assumed that differentiated organizations are integrated enough that they function more as valves and not sink-holes and that paths are still completed (see Ghosh and Rosenkopf (2014)). Second, it is assumed that organizational actors have at least partially an accurate assessment of the indirect ties of their direct partners, i.e., have a somewhat accurate representation of the relationships beyond their first-order zone and can at least heuristically predict the centrality and influence position of the key actors in the network. Very recent empirical work shows that the latter assumption has to be at least qualified (Knoben et al. 2012), since the accuracy of perceptions about the network structure and other actors’ positions seems to rapidly decrease beyond the direct contacts (first-order zone).

From the perspective of the whole or entire network, the interactions between organizations form collective social capital that helps govern the network and is usually seen as positive for the achievement of outcomes like innovative capacity of networks, client well-being or satisfaction, the creation of policy outputs, or the general performance of networks. In case of engineered networks, this effect is attempted to be strengthened by making conscious choices about the governance mode of the network (Provan and Kenis 2008); see for a recent summary of the field and related cases Sydow et al. (2016).

Historical Background

Even though the idea of organizations having external relations goes far back into the twentieth

century, it took until the 1960s to really identify interorganizational relations and networks as a specific field of research. In his now seminal piece, Evan (1965) introduced a “theory of interorganizational relations” and the notion of an organization set, i.e., a focal organization and its direct interactions with organizations in its environment (Evan 1965, p. B220). The discussion and research on interorganizational networks intensified in the 1970s both theoretically and empirically, took off in the 1980s, and matured in the 1990s. Interestingly, discussions developed in many different areas in the social sciences, economics, and (economic) geography as a reaction to the increasing connectedness of social, economic, and political processes. Next to the (perceived) changes in empirical reality, advances in organization theory like contingency or resource dependence theory and the development of social network analytic techniques as a toolbox pushed the agenda forward. In that process, sometimes scholars took note of the other fields; very often, however, theoretical insights were developed in ignorance of each other. Conceptually, one can distinguish approaches that were developed by departing from the organization as a single entity and looking beyond its borders from systemic approaches that started with a perspective on a whole industrial sector, a policy field, geographical region, an area of service delivery, or even an entire economy. The first group has mainly been concerned with the opportunities, benefits, and constraints for individual organizations, i.e., characteristics of organizations’ relations, an organization’s structural position in a network, or characteristics of the network it is embedded in have been used as independent variables to explain outcomes at the organizational level. This has been the dominant perspective in economics and management. Theoretically, core discussions revolved around the effects of embeddedness (Granovetter 1985; Uzzi 1997) and structural positioning in networks in terms of brokerage and/or centrality (Burt 1992) on growth, survival, innovative capacity, or other performance indicators of organizations. In terms of methods, we can observe an increasing application of quantitative social network analysis in

studies on interorganizational networks due to the development of concepts and algorithms since the 1970s, user-friendly software like UCINET, and the spread of personal computers in the 1990s. With the improvement of the methodological apparatus for quantitative analysis, more and more quantitative network data have been collected since the 1990s, and it became possible to also build, process, and analyze large databases, for example, in the area of alliance networks (Gulati and Gargiulo 1999) and innovation (Schilling and Phelps 2007).

The second group of approaches investigates what the structures of interorganizational networks look like, what their antecedents are, how they evolve, and what the consequences are for outcomes at the network or system level. In addition, questions of influence and power as well as the governance of economic and political systems have been very important in this area. Within these approaches, it was less network theoretical discussions that initially triggered a lot of research but rather empirical descriptions and the curiosity about the structural characteristics of interorganizational networks. In addition, “network” was often used as an empirical tool to operationalize and test relational concepts from other theories in the social sciences like power or diffusion theories in different fields within the social sciences. First, community power studies in the early 1970s demonstrated the importance of organized action and provided an empirical basis in the controversy with Marxist theories, which dominated the discourse in sociology at the time (Laumann and Pappi 1976). Another prominent application especially in sociology was the study of interlocking directorates as a way to investigate the mechanisms of coordination or power structure in sectors or whole economies (Mintz and Schwartz 1985). Second, the discussion on federalist systems (Hanf and Scharpf 1978) and policy networks (Laumann and Knoke 1987) showed the great importance, organizations, and their evolving networks had gained for policy making and public administration. Third, a broad discussion has evolved since the 1980s in regional economics, economic geography, and economic sociology about which interorganizational network

arrangements between private enterprises, (semi)-state actors, and knowledge institutes or universities are most advantageous for regional development, regional innovative capacity, and competitiveness as production was transformed from mass production to knowledge-intensive flexible specialization (Saxenian 1990). A very broad theoretical discussion, however, that went across these different fields evolved around the question, under what conditions which governance form, i.e., market, hierarchy, or network, would be most effective and efficient. This discussion was very much triggered by the ideas of Williamson (1975) and later Powell (1990). In terms of methodology, the dominant approach in this group has been (comparative) case studies, increasingly with a combination of qualitative narrative analysis and quantitative network analysis.

Interorganizational Networks: Main Theoretical Approaches

Within the field of interorganizational network research, we can distinguish at least nine theoretical perspectives, if one combines the three main units of analysis, the organization, the dyadic relationship, and the whole network, in terms of antecedents and outcomes in a 3×3 table (see Raab et al. (2012)), since many empirical studies and major theoretical contributions combine the perspective of at least two cells. The most prominent combinations in the theoretical discussion in the field so far have been the following: (1) the effect of organizational characteristics including structural network positions like centrality on organizational level outcomes, (2) the effect of dyadic interorganizational relationships on outcomes at the organizational level, (3) the effect of organizational and dyadic characteristics on tie formation, and (4) the effect of network characteristics on network level or system level outcomes. As discussed above in the historical background section, theoretical contributions came from many different fields in the social sciences and economics. However, similar to theory development in the field of intraorganizational

networks, major theoretical discourses originated in sociology on social networks in general which have recently been summarized as core elements of a network theory by Borgatti and Halgin (2011). Core theoretical contributions in this regard, which were strongly picked up in the research on interorganizational networks, have been made by Granovetter (1973) with his idea of the strength of weak ties and the embeddedness of social actors, including organizations in wider webs of social relations (Granovetter 1985). In addition, the discussion on different forms of social capital, i.e., Burt's structural holes approach (1992, 2005) and Coleman's (1988) idea of closure in social systems, has been a major discourse in the field. Actually, a lot of research was triggered by two theoretical controversies, namely, the controversy between Burt and Coleman's ideas on social capital and between Granovetter's (1985) embeddedness approach and Williamson's (1975) transaction cost perspective. The latter controversy revolved around the question, what determinants are most important for the behavior of organizations and especially their tie formation. While Williamson (1975) claimed that ties would be formed on the basis of the nature of an economic exchange like frequency, asset specificity, and uncertainty, Granovetter (1985) argued that such a view was "undersocialized" and neglected the fact that social actors are embedded in social relations with the exchange partners themselves but also with third parties and beyond. As a consequence, predictions about the behavior of social actors and their tie formation had to take these wider social relations into account including their history and potential future. The first controversy (Burt vs. Coleman) revolved around the question, what the most advantageous tie or network structure (form of social capital) is, for an organization to be embedded in. Is it a brokerage position (structural hole, Burt), where the partners of an organization are not linked to each other and an organization can therefore play out its partners against each other or is it a position, in which the partners are all connected to each other (closure, Coleman) and therefore trust and common norms can develop? In 2005, Burt resolved this controversy by combining the two perspectives, i.e., he

now argued that a position is most advantageous for an organization, if it is embedded in a densely connected cluster and that cluster is then in a brokerage position between other clusters. Such a constellation maximizes both local trust building and access to novel information.

A third contrasting discussion was between again Williamson's (1975) transaction cost theory and Powell's (1990) claim that networks formed a third and distinct form of governance that was not simply a combination of characteristics of markets and hierarchies. The questions what distinct forms of governance are under which conditions likely to arise and be effective were broadly discussed in (economic) sociology, in political science, as well as in management and organization science. "Network" was thus mainly discussed with "market" and "hierarchy" as the reference points. As a result, networks were often treated as if they were all the same even though empirical evidence was mounting that very different types of networks exist and that especially engineered networks can be governed quite differently. Consistent with this more recent view, Provan and Kenis (2008) have suggested three ideal types (they call them modes) of network governance, which they argue can lead to effective outcomes depending on a number of contingencies. With regard to these engineered networks, we see a somewhat independent theory development in public management which is concerned with the evolution, functioning, and especially the effectiveness of interorganizational networks (Provan and Milward 1995).

Especially with regard to general conceptualization and the formation of dyadic ties from the perspective of the individual organization, valuable theoretical contributions also came from organization theory. Aldrich (1971) was one of the first who started to conceptualize the environment of organizations as consisting of other organizations with concrete linkages among each other. This perspective further developed in the resource dependence theory (Pfeffer and Salancik 1978) by combining it with general exchange theory. Resource dependence theory states that all organizations are dependent on other organizations for resource inputs, since they cannot

produce everything themselves. Depending how strong this (asymmetric) dependency is, organizations try to control and cope with these dependencies through internal measures but also by forming ties to other organizations, for example, in the form of crossover board memberships to stabilize the relationship.

Illustrative Example(s)

In the following, a few empirical studies are briefly discussed. They have been selected and are grouped according to their primary unit of analysis (individual organization, network, system/community). Two of the most prominent studies that focus on network effects for the individual organization are the study by Uzzi (1997) on "Social Structure and Competition in Interfirm Networks: The Paradox of Embeddedness" and by Powell et al. (1996) on the "Inter-organizational Collaboration and the Locus of Innovation: Networks of Learning in Biotechnology." Uzzi (1997) conducts an ethnographic analysis at 23 women's better-dress firms in the New York City apparel industry and attempts to explore and explain the "links between social structure, micro-behavioral decision-making processes, and economic outcomes within the context of organizational networks" (Uzzi 1997, p. 61). Departing from Granovetter's notion of embeddedness, Uzzi shows that firms perform best, if they manage to position themselves in an "integrated" network environment that is constituted of a mix of "arm's-length" and "embedded ties." Arm's-length ties are understood as relationships on the basis of market exchanges, while embedded ties are long-standing close and special relationships in which tacit knowledge can be transferred.

Powell et al. (1996) demonstrate in their study about the interorganizational network which forms the organizational infrastructure of the biotechnology industry mainly in the USA that in industries like biotech where knowledge is complex and rapidly expanding and the sources are widely dispersed the networks rather than the individual organizations become the carrier of

innovation. This is due to the fact that in such industries new knowledge is and cannot be produced within the confines of a single organization or through simple market transactions but is rather created through access to general knowledge flows and intensive interaction with organizational partners. They show in addition that as the locus of innovation shifts to the network level, that access to the knowledge flows in the network becomes crucial. Therefore, the higher the number of R&D alliances and the better they are managed in t_0 , the higher the number of alliances and central connectedness in t_1 with subsequently stronger organizational growth in t_2 . The network was constructed of firms being active in or having an affinity with the biotechnology industry and various R&D and investment ties.

Compared to the first group of studies that have the organization as the unit of analysis, the number of studies investigating outputs and outcomes at the network level is relatively small. This is due to the fact that organization level outcomes are of greater importance to management scholars and data requirements are very challenging for studies that focus on the network level as the unit of analysis (Provan et al. 2007). In their seminal study “a preliminary theory of network effectiveness,” Provan and Milward (1995) compared four interorganizational networks in the area of mental healthcare in the USA and developed a theoretical framework that could explain the level of network effectiveness in terms of client well-being/quality of life. The networks consisted of around 30 organizations that were involved in taking care of mentally ill people and their linkages such as client referrals, joint programs, case coordination, and service contracts. They found that network structure in the form of centralized integration, direct nonfragmented control, and the context factor resource munificence and system stability had a positive effect on network effectiveness.

Human and Provan (2000) conducted a comparative case study of two networks of 22 and 23 small-and medium-sized companies in the US wood processing industry. Both networks were structurally similar and had a network

administrative organization that coordinated network activities. The types of ties that were included in the analysis were business, friendship, and information linkages with the other firms in the network. Human and Provan were interested in describing the evolution of the two networks especially with regard to the development of legitimacy and its impact on the survival of the networks. The authors collected two rounds of data in order to be able to conduct a longitudinal analysis. They identified three dimensions of network legitimacy that were crucial: “network as an entity”, “network as a form”, and “network as interaction”. They found that the network which first addressed internal and then external legitimacy could sustain its activity, while the other network ceased to exist.

A lot of studies, which investigate the effects or implications of interorganizational networks on the wider system or community, can be found in fields, which have a wider system perspective as regional economics, economic geography, sociology, and especially political science. With regard to the latter, policy (domain) networks were a very prominent object of study during the 1990s and 2000s. In one of the most ambitious empirical research projects, Laumann and Knoke (1987) conceptualized the US energy and labor policy domains as large interorganizational networks that develop out of the participation of political actors (legislators, governmental agencies, interest groups, etc.) in political events and their engagement in political exchange relations. Through these exchanges, power positions of political actors are constituted and become visible. Subsequently, power distributions among these actors in combination with their political interests have significant consequences for policy making and policy outputs.

In a widely cited study, Owen-Smith and Powell (2004) combine insights from economic geo-graphy, economic sociology, and organization theory to analyze the flow of knowledge and information through a regional sectoral interorganizational network, i.e., the Boston Biotechnology community. On the basis of research and development ties between public research organizations such as universities and knowledge

institutes and private companies such as diagnostic biotechnology firms, venture capitalists, and pharmaceutical companies, they conduct a quantitative network analysis to visualize the interorganizational network and calculate the betweenness centrality of organizations. The centrality scores are then used in regression models to determine to what extent membership and centrality of an organization in a network will influence the innovative outputs of firms. Even though the unit of analysis is the single firm, the study is exemplary in systematically collecting detailed relational data on a regional sectoral network and demonstrating the mechanisms that lead to increased innovative capacity also of entire regions or geographical agglomerations. One of their main findings in this regard is that geographic propinquity and centrality matter especially in networks that are dominated by private knowledge regimes as in their biotechnology case. Here, knowledge and information were transferred through rather closed channels and not through open channels as we might find them in networks that are dominated by public research organizations. Thus, Owen-Smith and Powell (2004, p. 17) argue that two characteristics of interorganizational networks that are independent of structure, i.e., propinquity and institutional characteristics with regard to the knowledge regime, transform network effects of information flows.

Key Applications

The field of interorganizational relations and networks has especially since the 1990s produced a large amount of empirical studies on a large variety of topics. During the late 1980s and early 1990s, scholars primarily focused on describing the characteristics of networks. Since the late 1990s, more and more work is also conducted on the antecedents (Gulati and Gargiulo 1999) and especially the outcomes of interorganizational networks. While the 1990s were characterized by a positive normative bias about networks being beneficial for organizations and societies alike, work in recent years has taken a more

neutral and empirical stance. Networks are now regarded as one of the several possible governance mechanisms, and the question then arises, under what circumstances they are effective. In addition, while the discussion during the 1990s focused on the comparison between market, hierarchy, and network as the three ideal typical governance forms, more recent work demonstrated that networks themselves can differ considerably and can be governed quite differently. Provan and Kenis (2008) suggest three governance modes for networks: the self-governed network, in which all participants jointly take on coordination and control; the lead organization mode, where one of the participating organizations mainly determines the course of action; and the network administrative organization, where a separate entity is founded to take on coordination and control. In addition, a strand in the public management literature on “collaborative networks” emphasizes the distinct management processes within networks like “activating, framing, synthesizing, and mobilizing” (Agranoff and McGuire 2001). Also more recently, more and more studies focus on the dynamics and evolution of interorganizational networks (Ahuja et al. 2012) and the temporariness of many interorganizational networks (Jones and Lichtenstein 2008).

Future Directions

Due to a strong structuralist tradition, studies on interorganizational networks have for a long time been rather static, i.e., concentrated on the analysis of relational and organizational data at one point in time. Moreover, data collection about interorganizational networks is very time consuming and risky, since for every network, access to often more than a dozen organizations has to be achieved. In addition, agency was often assumed but not further investigated. Therefore, one of the most promising avenues for further research is the inclusion of network processes in the study of interorganizational networks (see also Ahuja et al. (2012); Kilduff and Brass (2010)). In more detail, these are studies in the following areas:

1. Studies which empirically demonstrate the assumed causal relationships between certain structural characteristics and outcomes of interorganizational relations and networks, for example, between centrality and organizational performance.
2. Studies that look at actions of individuals and organizations in interorganizational relations and networks over time. Especially interesting but also very challenging are studies that apply a multilevel perspective, i.e., combine interorganizational and interpersonal relations (see Lazega et al. (2008)). In addition, once we take agency in networks seriously, we need to know much more what actors know about their network environment, how they use that information, and how it influences their decisions and behavior (see Knoben et al. (2012)).
3. Studies that analyze and explain process patterns of interorganizational networks or use them to explain various outcomes for both engineered and emergent networks.
4. Studies that describe and analyze the dynamic evolution and change of networks and their antecedents for engineered and emergent networks alike.
5. Studies which investigate the effect of time in interorganizational networks. Despite the fact that many interorganizational networks are temporary, i.e., their existence is limited from the outset with regard to a certain time period or until they achieve a certain outcome or state, the effect of time has been rarely questioned in studies on interorganizational networks. The core question in that regard is to what extent the limited or short duration influences tie formation, trust building, or the general coordination process. Since many interorganizational networks are actually constituted by projects, a combination with the project literature is likely to be beneficial (see Jones and Lichtenstein (2008)).

A second interesting and important avenue especially in public management and policy making is the question what makes networks effective and under which conditions do they produce

which outputs or even outcomes. For a long time, networks seemed to be the solution for many problems that were connected to market and state failure. However, the initial optimism in the 1990s has subsided, and after more than two decades of empirical research, scholars are in general much more realistic. However, more research is still necessary in this area and also here we need to combine structural characteristics of networks with (management) processes, agency, network governance, and context factors to make progress in our understanding of interorganizational networks and their outputs or even outcomes. In that endeavor, a configurational approach together with Qualitative Comparative Analysis (QCA) as an analytical tool is very promising (see Raab et al. (2012)). With QCA it is not only possible to more systematically analyze data in cases of small and medium sample sizes, a notorious problem in interorganizational network research (Provan et al. 2007), but it is also possible to systematically explore the different configurations of causal conditions that might lead to a certain output. A configurational approach also focuses on the identification of necessary and sufficient conditions for certain outputs as well as equifinal and conjunctural causation. These characteristics of the configurational approach make it highly valuable not only for academic research and theory building but also for supporting the formulation of relevant management and policy recommendations.

Cross-References

- ▶ [Futures of Social Networks: Where Are Trends Heading?](#)
- ▶ [Interlocking Directorate Networks](#)
- ▶ [Intraorganizational Networks](#)
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- ▶ [Structural Holes](#)

- ▶ [Supply Chain Networks](#)
- ▶ [Social Capital](#)

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Interpersonal Networks

- ▶ [Intraorganizational Networks](#)

Interpersonal Relations

- ▶ [Origins of Social Network Analysis](#)

Intraclass Analysis

- ▶ [Barycentric Discriminant Analysis](#)

Intraorganizational Networks

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Synonyms

[Company chart](#); [Informal network](#); [Informal organization](#); [Interpersonal networks](#); [Organizational blueprint](#); [Organizational grapevine](#); [Organizational social capital](#); [Prescribed versus emergent organizational structure](#)

Glossary

Embedded relation	Organizational members connected through both a formal and an informal relation
Formal relation	Organizational members connected only through authority or workflow interdependence
Informal relation	Organizational members connected only through a personal tie without a formal relation
Membership relation	Organizational members who are part of the same unit but not connected through a formal or an informal relation

Definition

Intraorganizational networks are the aggregate of the formal and informal relationships between the members of an organization. Depending on the presence or absence of formal and informal elements in the tie between two members of the organization, four elementary types of intraorganizational relationships can be distinguished. Together they form the intraorganizational network.

Formal relationships can be based both on vertical authority relations between a hierarchical superior and a collaborator and on horizontal workflow interdependencies between peers. Formal “relations” are often documented in the organizational chart or blueprint. They define legitimate routes for interaction, advice, approval, and the transmission of information. The chart does not necessarily say much about the actual frequency or importance of the specific formal relationships.

Informal relationships in the narrowly defined sense are personal ties between members of the organization who are not connected through a formal relationship. Informal personal ties can be positive or negative and weak or strong, depending on the level of mutual expectations

and obligations, the frequency of interaction, and the degree of multiplexity of their relation (a relation is multiplex if it consists of more than one dimension, e.g., friendship and the exchange of advice). Commonly studied informal relations are often categorized into “affective” (interpersonal trust, friendship) and “instrumental” (e.g., advice, communication) ties. More recent organizational network studies also pay attention to negative relations and “sour social capital,” like distrust, betrayal, mobbing, foes, and gossip.

Embedded relations consist of ties in which both parties are connected through both a formal and an informal relationship (e.g., a boss and his collaborator have a friendship relation). Embedded relations can be consistent or inconsistent, depending on the degree to which the logic and objectives of the formal and the informal relation conflict with or mutually support each other (Soda and Zaheer 2012). For example, an embedded relation is inconsistent if functionally interdependent team members are also friends, and friendship norms lead to positively biased evaluations of the quality of each other’s work.

Finally, even if they are not connected through a formal (authority or workflow interdependence), an informal, or an embedded relationship, employees can still be connected through a membership relation. Simply being part of the same organization subunit or (temporary) project can be highly relevant for how employees behave towards each other. Three types of membership relations can be distinguished. In a one-dimensional membership relation, employees perceive themselves as being part of one unit. In a multi-dimensional membership relation, employees are part of several units (e.g., a project group, a department, a committee). These units can overlap and/or be nested. Ambivalent membership relations emerge where boundaries between organizational (sub)units dilute, and/or formal membership criteria are subject to multiple interpretations (e.g., should interns or workers from temp agencies who join a team – often for considerable periods – be considered as members of the organization?).

Most research on intraorganizational networks focuses on the study of informal relationships and insufficiently specifies the formal or embedded context of the informal ties and has paid only scant attention to the role of membership relations.

Introduction

Intraorganizational networks matter at three levels. For individuals, their personal network at work influences opportunities, perceptions, and behavior during all stages of their contact with the organization: from getting hired to getting promoted and getting fired and from learning the tricks of the trade to getting ones job done. For workgroups, the structure of the informal network matters during all phases of the production process (input, throughput, and output). For example, it can be decisive in a workgroup’s ability to coordinate, to sanction free riders, to prevent and solve conflicts, and to foster creativity and innovation. On the level of the organization, the configuration of formal and informal structures is a key element of its governance structure. Some even see “network forms of organization” as a specific (new) organizational form, characterized by enduring exchange relations that “lack a legitimate organizational authority to arbitrate and resolve disputes that may arise during the exchange.”

For each of these levels, a wide array of literatures and theories has emerged. There seems to be no subfield that did not try to incorporate social networks into their research agendas. Organizational behavior scholars, labor market researchers, decision theorists, to name but a few, study individual level effects of intraorganizational networks. Small-group researchers from all social science disciplines are interested in the antecedents, processes, and consequences of intraorganizational network structures. Economists and sociologists use theories of organizational governance to analyze under which conditions network organizations yield transaction cost advantages compared to other governance structures, in particular hierarchies and markets.

Key Points

Intraorganizational networks are traditionally analyzed as “pipes” connecting organizational positions and their incumbents. The major mechanism is one of instrumental relationalism: the formation, change, and effects of networks are driven by purposeful social exchange, motivated by calculated investments in social capital, and governed by straightforward cost-benefit reasoning. An individual’s network position determines his or her opportunities to have access to scarce material and immaterial resources – like information, resources, and social support. The resulting level of social capital of individuals and workgroups, in turn, affects a large variety of outcomes, ranging from individual performance, career prospects, creativity, well-being, and job satisfaction to the innovativeness and flexibility of workgroups and organizations.

More recent approaches emphasize that intraorganizational ties also function as “prisms” through which individuals frame social expectations and obligations. The major mechanism is one of constructive relationalism: intraorganizational networks are the result of cognitively mediated functional interdependencies. This shifts the focus from social exchange of goods and services to the sustaining or hampering role of social ties for joint production. This approach endorses a more complex behavioral model, in which the cognitive activation of network perceptions, the institutional and cultural context, and relational signaling processes are important elements for modeling the emergence, dynamics, and effects of intraorganizational networks.

Historical Background

The study of intraorganizational networks is strongly intertwined with four developments in the social sciences. First, the “discovery” of the informal organization is usually attributed to the so-called Hawthorne experiments, carried out in Western Electric’s Hawthorne plant between 1924 and 1932. In particular, sociometric data from the “bank wiring room experiments” showed the

impact of informal ties on performance: independently of the formal structure, informal cliques emerged, developing and enforcing productivity restricting group norms to the point that the introduction of individual performance related pay even decreased performance. The Hawthorne studies marked a shift in perspective in organizational research from conceiving organizations as closed and rational systems – self-contained, pre-designed formal structures that functioned according to the principles of a rational bureaucracy – to seeing “the company behind the chart”: a natural and open system, with emergent social relations, highly susceptible to outside influences and “nonrational” impulses affecting the perceptions, emotions, attitudes, and behavioral decisions of its members. The Hawthorne studies and what later should become known as the human relations approach sensitized organization scholars not only to the importance of social relations, group membership, and identities but also to a more complex model of human nature than envisioned by the dominant closed rational system framework. Nevertheless, these theoretical developments had only marginal impact on the emerging field of intraorganizational social network studies.

Second, a major impulse came from organizational ethnographies, carried out in the tradition of the Manchester School. They combine the application of sociometric data collection techniques with an in-depth ethnographic case study approach which allows them to uncover not only relational patterns but also the norms, rules, and mechanisms behind social dynamics. Four early studies exemplify this approach. One of the first longitudinal intraorganizational network studies was carried out in a Canadian furniture retail sales store during the 1950s (French’s 1963). This descriptive study maps the friendship relations of 25 salesmen at three points in time, the negative relations at one point in time, and the pattern of (non)compliance to informal norms (“don’t snitch to management,” “don’t steal a regular customer from a colleague”), regulating the functional interdependencies among them. Kapferer’s (1969) research in a Zambian mine focused on the development and resolution of

conflict among 15 workers. Sociometric information covered were conversation, joking, job assistance, cash assistance, and personal assistance. A “crisis in a cloister” is the focus of Sampson’s (1969) study of a conflict in a monastery, which resulted in the expulsion of four monks and the voluntary departure of many others. Retrospective sociometric information was collected for three time periods and a large variety of relations, including liking, dislike, and influence. Finally, Thurman (1979) studied two major disputes among 15 employees in the overseas office of a large international corporation. The structure of the informal network helped explain the success and failure of “leveling coalitions” against the target of the conflict. A common thread in these and similar studies consists in the finding that organizational networks are often characterized by inconsistent embedded relationships in which informal and formal relations are at odds with each other.

The third early source inspiring intraorganizational network research was social exchange theory and its application to small groups. Homans (1950) analyzed social ties in organizations as an exchange of social approval (“liking”) for compliance with group obligations, building (among others) on a reanalysis of the Hawthorne bank wiring room experiments, and his own sociometric study of the “cash posters” in the accounting division of a firm. In Blau’s (1955) influential sociometric study of two governmental tax agencies, civil servants exchange advice for deference and professional status. But despite the rich empirical insights produced by the early case studies in the tradition of the Manchester School and the strong explanatory potential of the emerging social exchange and human relations paradigms, intraorganizational network research stagnated during the 1960s to the 1980s. Reasons for this decline were the problem of getting access to organizations, the sensitive privacy issues related to sociometric surveys, which usually require that respondents disclose their identity to the researchers, and the limited ability to “generalize” from the findings of a single case study.

As a result, during the 1980s, the focus shifted towards the study of formal characteristics and

structures of organizations (e.g., centralization, span of control, size) and to interorganizational networks (e.g., interlocking directorates), both of which can be more easily accessed through (publicly available) secondary data sources. The rise of transaction cost and institutional theories of organizational governance during the 1980s and 1990s also sparked some renewed interest for detailed ethnographic studies of intraorganizational networks, in particular for their role in processes of organizational control (Gargiulo 1993; Wittek et al. 2003).

Intraorganizational Networks: Theories and Social Mechanisms

Intraorganizational network research always had a strong structuralist legacy, which assumes that the major determinants of human decision-making behavior, cognitions, or emotions are not their individual attributes, attitudes, or other psychological traits but their position in a social structure. Individuals in similar network positions are confronted with similar opportunities and constraints, which in turn trigger the same kind of individual perceptions, action opportunities, and responses. In these constraint-driven structuralist accounts, there already was little room for a more grounded behavioral theory and the problem of “agency.” The surge of new, powerful statistical parameters and algorithms, which allowed to detect positions in and “hidden” structural properties of networks, further reinforced this structuralist legacy during the 1970s.

Theories of action in the form of instrumental relationalism entered intraorganizational network research in the late 1980s (Jansen 2002) in the form of two milestone contributions, which still define the core of the widely applied social capital approach (Flap and Völker 2012). Rational choice-based exchange theory suggests that network closure is beneficial for norm compliance and therefore for group performance, because it fosters social control. Ronald Burt’s (1992) structural hole theory combines ideas from social exchange theory with structuralist theories of power, shifting the attention to the individual

“network entrepreneur” who actively creates and strategically exploits structural opportunities as they emerge due to the absence of ties between his or her contacts. This framework emphasizes the disadvantages of network closure and the corresponding benefits of brokerage positions for individual achievement. Both the closure and the brokerage mechanism were frequently subjected to empirical tests.

During the 1990s, growing dissatisfaction with instrumental relationalism converged into the emergence of an alternative behavioral foundation, relational constructivism (Jansen 2002). It builds on a more complex model of human nature than the thin version of rational choice theory that was at the core of instrumental relationalism. A key role is reserved for individual identities, institutional embeddedness, and the cognitive mediation of functional interdependencies. Rather than being simple “pipes” for the exchange of resources, social relations function as “prisms” (Podolny 2001) framing mutual expectations and obligations. Relational Signaling Theory (Lindenberg 2000) explicates the social mechanisms underlying the creation, maintenance, and decline of cooperative relations in organizations. It suggests that in settings with a high degree of functional interdependence, individuals will constantly screen each other’s actions for positive or negative relational signals, in order to assess whether the other party is still in a cooperative frame.

Illustrative Examples

An empirical study of the impact of network positions on cognitions of 86 employees of a computer software firm (Walker 1985) illustrates a structuralist mechanism. In addition to the formal reporting relationship, respondents indicated, for each colleague, the frequency of “sending” and “receiving” eight different types of ties (feedback, problems, extra time, technical information, marketing information, etc.). “Cognitions” reflected individual employee’s assessment of how strongly 31 different means (e.g., close contact with end user during the development phase) contributed to the accomplishment of four different

types of product goals (performance, generativity, endurance, and coherence). Network positions were assessed through structural equivalence analyses. Two individuals are structurally equivalent if they have the same pattern of relations to similar others. Completely in line with structuralist reasoning, “network position was found to be a stronger and more stable predictor of differences in cognition than the type of function an individual had and the type of product worked on” (Walker 1985:103).

A longitudinal network study modeling the emergence of advice relationships among 57 employees of a Dutch housing corporation (Agneessens and Wittek 2012) illustrates the logic behind instrumental relationalism. The study reconstructs the assumptions between the social capital and the social status approach. Though both are rooted in a social exchange framework, their behavioral micro-foundations differ slightly, with competing hypotheses about the structure of the advice network being the result. For example, where the social capital approach predicts an overrepresentation of reciprocal dyadic relations and cyclical triadic relations, the social status approach predicts the opposite, i.e., an overrepresentation of nonreciprocal dyads and triads. The analysis yields partial support for both perspectives: overrepresentation of reciprocal relations at the dyad level (in line with the social capital approach) and overrepresentation of noncyclical triads (in line with the social status approach).

An ethnographic study on the escalation of informal conflict management in the management team of a German paper factory (Wittek et al. 2003) illustrates the logic behind relational constructivism. It uses data on 67 conflicts involving 22 managers and 4 waves of sociometric information, covering a period of 3 years. Social escalation is defined as the involvement of one or more third parties in a conflict. Building on Lindenberg’s relational signaling theory, strong social ties are expected to foster de-escalation only as long as the organizational context sustains unambiguous exchange of positive relational signals. Multilevel analysis indeed confirms this – but the protective effect of strong ties disappears through time. The result is a decline in frame-stabilizing arrangements in the firm, reflected in a drastic decrease of the

frequency of meetings, and a major organizational change through which acts that were previously considered as strong positive relational signals – like the provision of unsolicited advice – became ambiguous, since they could now also be interpreted as attempts to improve one’s status at the expense of other team members.

Key Applications

The field of intraorganizational network studies meanwhile produced empirical studies on a large variety of topics, covering antecedents, dynamics, and outcomes of networks at the level of individuals, workgroups, and organizations.

Organizational networks can affect individuals during all phases of their contact with an organization. Job seekers with friends occupying a power position in a prospective employing firm are more likely to be hired. Once at work, friendship relations play an important role for the socialization of new colleagues into the culture of the organization and learning the tricks of the trade. A personal network with structural holes increases the chances for and speed of promotions for senior men (Burt 1992). Ties to powerful members in the organization increase the success in salary negotiations, particularly for minorities. Strong ties to colleagues who are satisfied with their job increase the likelihood of being satisfied with one’s own job (Agneessens and Wittek 2008), and consistent embedded relations increase individual performance (Soda and Zaheer 2012). One’s friends are also targets for organizational voice (Pauksztat et al. 2011). Being tied to popular others in the organization protects from becoming the object of negative gossip (Ellwardt et al. 2012). Finally, employees who are only weakly embedded into the informal network of the organization are more likely to leave, and those who see their friends leave also are more likely to leave the organization themselves (Krackhardt and Porter 1985).

At the level of workgroups and organizations, intraorganizational networks play a role during all processes related to the input, throughput, and output. On the input side, the structure of the informal network influences outside information

seeking. With regard to throughput, a study of a multinational electronics company showed that frequent contact between product development units increases shared knowledge, which in turn speeds up projects (Hansen 2002). Finally, a major concern of intraorganizational network research has always been the relationship between network structure and output measures like workgroup performance. A meta-analysis indeed reports a positive relationship between the density of intraorganizational networks and team performance (Balkundi and Harrison 2006).

Future Directions

Of the many developments at the current frontier of the field, three are particularly noteworthy. First, considerable progress can still be made with regard to theory formation. Many assumptions, mechanisms, and implications of key hypotheses are still insufficiently explicated, and formal analysis can yield interesting clarifications. This holds for theorizing in both the instrumental and constructivist traditions.

In the instrumentalist branch, game theoretical analyses of the cohesion-performance mechanism (Flache and Macy 1996) have demonstrated that dense informal networks of strong ties may reduce rather than increase team performance, if one assumes that team members do not only exchange social approval for contributions to group production – as envisioned in the standard account of instrumental relationalism – but might also end up in an unproductive exchange of approval for approval. Similarly, a formal model of what happens “if everyone strives for structural holes” shows that in the long run, stable networks will distribute benefits equally – implying that network entrepreneurs will not be able to sustain their structural advantage (Buskens and van de Rijt 2008). Despite some progress, the study of network games and their potential implications for organizations is still in its infancy.

In the constructivist branch, at least two promising developments can be discerned. The first one relates to multilevel networks. Though intraorganizational networks are multilevel by

nature, it is only recently that they are subject to systematic theorizing and empirical testing as well as to statistical modeling (Lazega et al. 2008). The second one explicates the role of cultural consensus and cognitive social structures for network processes (Krackhardt and Kilduff 2002). These efforts benefit from combining insights from cultural theory and cognitive psychology. For example, Ma et al. (2011) show that national cultural contexts moderate the effect of social network structures on opportunity recognition, with structural holes increasing opportunity recognition in individualistic cultures but decreasing it in collectivistic cultures. Investigating the impact of job threat, another study shows that someone's status affects which parts of the network are cognitively activated in his or her mind (Smith et al. 2012). Low status individuals were found to activate smaller subsections of their network than high status individuals. Cognitive activation of network perceptions is likely to be influenced by unconscious and biologically based "honest signaling" mechanisms, as is demonstrated by an emerging area of research using modern information technology ("sociometric badges") to detect and analyze signaling content of verbal communication (Pentland 2008).

Second, statistical models for the analysis of social network dynamics will continue to have a strong impact on the development of intraorganizational network research. Though questions about the origins of intraorganizational structures are fundamental to the field, the theoretical and methodological tools to answer them are relatively recent. Examples for the former are studies on the origins of structural holes (Zaheer and Soda 2009) and the stability of brokerage positions (Wittek 2001). Stochastic actor-oriented models (Snijders 2001) have already been successfully applied by intraorganizational network researchers to disentangle selection and influence effects (e.g., Agneessens and Wittek 2008) as well as the coevolution of multiple relations (e.g., friendship choices and the allocation of power reputations, Labun 2012). New techniques enabling to model the dynamics of events in social networks will further extend the range of possible applications for the analysis of network dynamics.

A final challenge remains the development and design of network interventions: "purposeful efforts to use social networks or social network data to generate social influence, accelerate behavior change, improve performance and/or achieve desirable outcomes among individuals, communities, organizations, or populations" (Valente 2012:49). Such interventions can take at least four different forms (Valente 2012): identification of individuals (e.g., change agents) based on some network property; "segmentation," i.e., identifying groups of people whose behavior is to be changed at the same time (e.g., detecting core members of a network); "induction," i.e., stimulating peer-to-peer diffusion of information or behavior; and "alteration," i.e., changing the network by adding or removing actors and/or their relationships or changing the content of the ties. An example for an alteration intervention is a study in a call center of a large bank, where company policy required workgroup members to schedule nonoverlapping breaks (Waber et al. 2010). After a change in the structure of the breaks that allowed for more overlap, the social cohesion of the teams increased significantly. Though the power of such network interventions is widely recognized by managers and organizational consultants, controlled experiments that would validate the effectiveness of network interventions are still rare. This is understandable, given the obvious limitations of carrying out such real-life experiments in the field. Given such limitations, it is understandable that researchers search low cost and low effort substitutes for in-depth sociometric field experiments and longitudinal intraorganizational network studies. As a result, there is a big temptation to consider the huge amount of intraorganizational relational data that is currently produced through online communication as a substitute for more traditional forms of intraorganizational network research. In combination with modern data mining techniques, this kind of data certainly has the potential to produce useful new insights. However, the strongest potential for generating new insights almost certainly lies in the application of theory-guided multi-method research designs, which allow to adequately assess the role of

organizational context which will always remain a major driver behind any intraorganizational network process.

Cross-References

- ▶ [Economic Network Analysis Based on Infection Models](#)
- ▶ [Futures of Social Networks: Where Are Trends Heading?](#)
- ▶ [Interorganizational Networks](#)
- ▶ [Managerial Networks](#)
- ▶ [Siena: Statistical Modeling of Longitudinal Network Data](#)
- ▶ [Social Capital](#)
- ▶ [Social Interaction Analysis for Team Collaboration](#)
- ▶ [Structural Holes](#)
- ▶ [Trust in Social Networks](#)

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Intrusion Detection

- ▶ [Network Anomaly Detection Using Co-clustering](#)

Inventor Networks

- ▶ [Innovator Networks](#)

Invertible (Nonsingular) Matrices

- ▶ [Matrix Algebra, Basics of](#)

Invisible Web

- ▶ [Pornography Online](#)

ISO/IEC 10646: The Universal Character Set

- ▶ [Unicode](#)

Iterative Classification

- ▶ [Collective Classification](#)

Iterative Methods for Eigenvalues/Eigenvectors

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Synonyms

[Eigenvalue and eigenvector](#) – characteristic value and characteristic vector; [Eigenvalue with the largest magnitude](#) – dominant eigenvalue; [Non-singular](#) – invertible; [Vertex](#): Node

Glossary

Eigenvalue/ Eigenvector	The fundamental entities that characterize any given matrix and can be obtained by finding the roots of the characteristic polynomial of the matrix or by iterative methods
Social Network Analysis	A research area in social and behavioral sciences that uses networks to represent and hence analyze social phenomena
Iterative Method	A procedure for solving a problem by generating a sequence of improving approximations to the true solution of the given problem

Definition

Eigenvalues and eigenvectors are fundamental concepts in linear algebra (Golub and Van Loan 2012; Golub and Vorst 2000) and are defined as follows:

Definition 1 Let A be an n -by- n real matrix (i.e., in $\mathbb{R}^{n \times n}$). If there exist a scalar $\lambda \in \mathbb{C}$ and a nonzero vector $\mathbf{x} \in \mathbb{C}^n$ such that

$$Ax = \lambda x, \tag{1}$$

then x is called an eigenvector of A with corresponding eigenvalue λ . The pair (λ, x) is called an eigenpair of A .

When A is a triangular matrix, the eigenvalues of A are just its diagonal entries. Thus, an important strategy in obtaining the eigenvalues is to transform a given matrix into a simpler form where the eigenvalues can be computed easily. One transformation that can preserve the eigenvalue structure of a matrix is called the similarity transformation and is defined below:

Definition 2 Matrix B is said to be similar to matrix A if there is a nonsingular matrix X such that $B = X^{-1}AX$. Similar matrices share the same eigenvalues. Moreover, if x is an eigenvector of A , then $X^{-1}x$ is an eigenvector of B .

In social network analysis, some problems can be reduced to finding the eigenvalues and eigenvectors of a matrix (Kamvar et al. 2004; Newman 2009; Page et al. 1998; Yin et al. 2012). The eigenvalues λ are the roots of the characteristic equation $\det(A - \lambda I) = 0$, where I denotes the identity matrix of order n . Unfortunately, there are no direct or easy ways to solve the characteristic equation for $n \geq 5$; therefore, we have to resort to iterative methods (Golub and Vorst 2000). In this entry, we review some classical iterative methods for solving the eigenvalue problem (1).

Eigenvalue Problems from Social Networks

In social network analysis, the centrality of a vertex within a network determines the relative importance of the vertex. It reflects how influential the vertex (which may be a person or a webpage) is within the social network (which may be a social club or the World Wide Web). One main measure of centrality is eigenvector centrality (Newman 2009; Opsahl et al. 2010). It

assigns relative scores to all vertices in the network based on the concept that connections to high-scoring vertices contribute more to the score of the vertex in question than connections to low-scoring vertices. One example is Google’s PageRank (Austin 2012) which is a special case of Katz centrality (Katz 1953). The Google matrix describes the hyperlink structure in the World Wide Web (Kamvar et al. 2004; Page et al. 1998; Yin et al. 2012). Below, we explain the associated eigen-problem in more detail.

A Web graph is a graph where each vertex denotes a page in the Web and an edge from vertex v_i to vertex v_j signifies that there is a link in page v_i pointing to page v_j . Let d_i be the number of links from page v_i . The Google matrix A and the PageRank vector x can be constructed as follows (Kamvar et al. 2004; Yin et al. 2012):

1. Define the matrix $P = (p_{ij})$ by

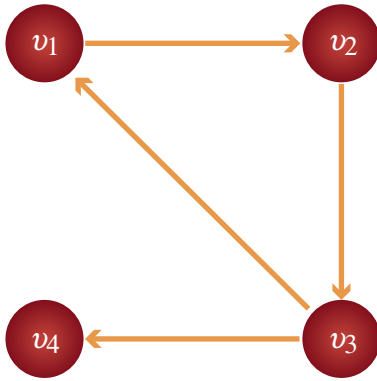
$$p_{ij} = \begin{cases} 1/d_j & \text{if there is a link from } v_j \text{ to } v_i; \\ 0 & \text{if } i = j; \\ 0 & \text{otherwise.} \end{cases}$$

2. The matrix P may contain zero columns. We replace all zero columns with $\frac{1}{n}\mathbf{1}$ and name the resulting matrix as \tilde{P} . Here, $\mathbf{1}$ is the column vector of all ones and n is the order of P .
3. We then construct the Google matrix $A = \alpha\tilde{P} + (1 - \alpha)\mathbf{v}\mathbf{1}^T$, where $\alpha \in [0, 1]$ and \mathbf{v} is a vector satisfying that each component is non-negative and $\mathbf{v}^T\mathbf{1} = 1$. The PageRank vector is the vector x that satisfies

$$Ax = x \tag{2}$$

with all components of x being positive and $\|x\|_1 = 1$.

By the Perron-Frobenius theorem (see Horn and Johnson 1985, p. 508), such a vector x not only exists but also is an eigenvector corresponding to the simple eigenvalue 1 of A . As an example, consider a Web with four pages (vertices) linked as in Fig. 1. Then



Iterative Methods for Eigenvalues/Eigenvectors, Fig. 1 Generally speaking, A is very big in social network analysis. Therefore, it is difficult to solve the eigenvalue problem (1) or (2) directly. One has to resort to iterative methods to find the eigenvalues and eigenvectors approximately. There are two classes of methods: the partial methods which compute only some eigenvalues and the global methods which compute all the eigenvalues

$$P = \begin{pmatrix} 0 & 0 & 1/2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \end{pmatrix}, \tilde{P} = \begin{pmatrix} 0 & 0 & 1/2 & 1/4 \\ 1 & 0 & 0 & 1/4 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 1/2 & 1/4 \end{pmatrix}.$$

Let $\alpha = 0.85$ and $\mathbf{v} = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})^T$, we have

$$A = \begin{pmatrix} 3/80 & 3/80 & 37/80 & 1/4 \\ 71/80 & 3/80 & 3/80 & 1/4 \\ 3/80 & 71/80 & 3/80 & 1/4 \\ 3/80 & 3/80 & 37/80 & 1/4 \end{pmatrix}$$

Solving the PageRank vector \mathbf{x} in (1) gives $\mathbf{x} = (0.2138, 0.2646, 0.3079, 0.2138)^T$. The values of the components of \mathbf{x} represent the relative importance of the corresponding page. Thus, v_3 is the most important page and v_2 is the second important page in Fig. 1.

Partial Methods

Power Method

The simplest eigenvalue problem is to find the eigenvalue with the largest absolute value and a corresponding eigenvector (Householder 1964; Wilkinson 1965). The power method is the simplest iterative method for this task. Suppose that v_1, v_2, \dots, v_n are n linearly independent eigenvectors of A and $\lambda_1, \lambda_2, \dots, \lambda_n$ are the corresponding eigenvalues with $|\lambda_1| \geq |\lambda_2| \geq \dots \geq |\lambda_n|$. Note that v_1, v_2, \dots, v_n form a basis of \mathbb{R}^n .

Given a nonzero initial vector \mathbf{q}_0 , there exist constants c_1, c_2, \dots, c_n such that

$$\mathbf{q}_0 = c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \dots + c_n \mathbf{v}_n.$$

By the equalities $A v_i = \lambda_i v_i, i = 1, 2, \dots, n$, we have

$$A \mathbf{q}_0 = c_1 \lambda_1 \mathbf{v}_1 + c_2 \lambda_2 \mathbf{v}_2 + \dots + c_n \lambda_n \mathbf{v}_n.$$

More generally,

$$\begin{aligned} A^k \mathbf{q}_0 &= c_1 \lambda_1^k \mathbf{v}_1 + c_2 \lambda_2^k \mathbf{v}_2 + \dots + c_n \lambda_n^k \mathbf{v}_n \\ &= \lambda_1^k \left(c_1 \mathbf{v}_1 + c_2 \left(\frac{\lambda_2}{\lambda_1}\right)^k \mathbf{v}_2 \right. \\ &\quad \left. + \dots + c_n \left(\frac{\lambda_n}{\lambda_1}\right)^k \mathbf{v}_n \right). \end{aligned} \tag{3}$$

Due to the fact that $|\lambda_1| > |\lambda_i|, i = 2, \dots, n$, we can conclude that $\left(\frac{\lambda_i}{\lambda_1}\right)^k \rightarrow 0, i = 2, \dots, n$ as $k \rightarrow \infty$. Therefore, $(A^k \mathbf{q}_0) / (c_1 \lambda_1^k)$ is a good approximation to v_1 for sufficiently large k . This is the basic motivation of the power method. Let ϵ denote the error we can tolerate; the power method can be summarized as follows:

Algorithm 1 The Power Method

1. Given an initial vector \mathbf{q}_0
2. for $k = 1, 2, \dots$
3. $x_k = q_{k-1} / \|q_{k-1}\|_2$
4. $q_k = A x_k$

5. $\theta_k = \mathbf{x}_k^T \mathbf{q}_k$
6. if $\|q_k - \theta_k x_k\|_2 \leq \epsilon$, stop
7. end for
8. accept $\lambda_1 = \theta_k$ and $v_1 = x_k$

From (3), we see that the convergence rate of the power method depends on $\left| \frac{\lambda_2}{\lambda_1} \right|$. If this ratio is close to 1, the convergence of the power method is slow.

Inverse Iteration

In order to improve the convergence of the power method and at the same time to find an eigenvalue closest to any given value, say σ , we apply the power method to $(A - \sigma I)^{-1}$ instead of A (Parlett 1980). The resulting method is called inverse iteration, and it is based on the following observation: if λ is an eigenvalue of A , then $1/(\lambda - \sigma)$ is an eigenvalue of $(A - \sigma I)^{-1}$, and the magnitude of $1/(\lambda - \sigma)$ relative to other eigenvalues of $(A - \sigma I)^{-1}$ can be made arbitrarily large by making σ close to λ . This leads us to:

Algorithm 2 Inverse Iteration

1. Given an initial vector q_0 and a shift σ
2. for $k = 1, 3, \dots$
3. $x_k = q_{k-1} / \|q_{k-1}\|_2$
4. $q_k = (A - \sigma I)^{-1} x_k$
5. $\theta_k = \mathbf{x}_k^T \mathbf{q}_k$
6. if $\|q_k - \theta_k x_k\|_2 \leq \epsilon$, stop
7. end for
8. accept $\lambda = \sigma + \frac{1}{\theta_k}$, and the corresponding eigenvector $v = x_k$.

The advantage of the inverse iteration over the power method is its ability to converge to any desired eigenvalue (the one nearest σ). However, inverse iteration in general requires calculating $(A - \sigma I)^{-1} x_k$ (step 4 in Algorithm 2); therefore, it is less attractive when the calculation is

expensive. Finally, we remark that the inverse iteration, as well as the power method, can only compute one eigenvalue along with its eigenvector.

Orthogonal Projection Methods

We now turn to the orthogonal projection methods. Compared to the power method and inverse iteration, the orthogonal projection methods can extract a few eigenvectors from a specified low-dimensional subspace. By choosing the subspace appropriately, the original eigenvalue problem (1) is reduced to a smaller eigenvalue problem. This is the basic idea of the orthogonal projection methods. In this section, we first discuss the general framework of the orthogonal projection methods: the Rayleigh-Ritz procedure. Then we introduce a particular implementation: the Arnoldi method.

Rayleigh-Ritz Procedure

Let \mathcal{S} be an m -dimensional subspace of \mathbb{R}^n , called the search subspace (Saad 2003). Assume that (μ, u) is an approximation to an eigenpair of A . The orthogonal projection method is to seek (μ, u) with u in \mathcal{S} by imposing the so-called Galerkin condition which requires that the residual $r = A u - \mu u$ to be orthogonal to \mathcal{S} , i.e.,

$$(A u - \mu u) \perp \mathcal{S}. \tag{4}$$

Now, we translate (4) into a matrix problem. Let q_1, q_2, \dots, q_m be a basis of \mathcal{S} and denote $Q_m = [q_1, q_2, \dots, q_m]$. Since $u \in \mathcal{S}$, it can be written as

$$u = Q_m y, \quad y \in \mathbb{R}^m. \tag{5}$$

Then, by the Galerkin condition (4), μ and y must satisfy

$$B_m y = \mu y \tag{6}$$

with

$$B_m = Q_m^T A Q_m. \tag{7}$$

Generally, $m \ll n$. As a consequence, we can obtain the approximate eigenvalue μ of A by solving the small eigenvalue problem (6) and then the associated approximate eigenvector u by the relation (5). The pair $(\mu, Q_m v)$ is called a Ritz pair, which is considered as the optimal approximation to the eigenpair of A in the search subspace \mathcal{S} . The process is known as the Rayleigh-Ritz procedure, which can be summarized as follows:

Rayleigh-Ritz Procedure

1. Compute an orthonormal basis $\{q_1, q_2, \dots, q_m\}$ of \mathcal{S} . Let $Q_m = [q_1, q_2, \dots, q_m]$.
2. Compute $B_m = Q_m^T A Q_m$.
3. Compute the eigenvalues of B_m and select the k desired ones $\mu_i, i = 1, 2, \dots, k$, where $k \leq m$.
4. Compute the eigenvectors $y_i, i = 1, 2, \dots, k$, of B_m associated with $\mu_i, i = 1, 2, \dots, k$. Then the corresponding approximate eigenvectors of A are $\mu_i = Q_m y_i, i = 1, 2, \dots, k$.

Arnoldi Method

The construction of the search subspace \mathcal{S} can be done in different ways (Arnoldi 1951). If it is chosen to be the so-called Krylov subspace

$$\mathcal{K}_m(A, \mathbf{q}_0) \equiv \text{span}\{\mathbf{q}_0, A\mathbf{q}_0, A^2\mathbf{q}_0, \dots, A^{m-1}\mathbf{q}_0\},$$

with the initial vector q_0 , then the orthogonal projection method becomes the well-known Krylov subspace method. From the definition of the Krylov subspace, the Krylov subspace method can be viewed as an extension of the power method.

The Arnoldi method is a Krylov subspace method. It first utilizes the Arnoldi process to establish an orthonormal basis, say Q_m , for the Krylov subspace. Then it performs the Rayleigh-Ritz procedure to extract the approximate eigenvalues/eigenvectors of A . We remark that the Arnoldi process, in exact arithmetic, is essentially the Gram-Schmidt procedure applied to $q_0, A q_0, \dots, A^{m-1} q_0$. The process can be described as follows:

Algorithm 3 Arnoldi Process

1. Given the initial vector q_0 and compute $q_1 = q_0 / \|q_0\|_2$
2. for $j = 1, 2, \dots, m$

3. $h_{ij} = \mathbf{q}_i^T A \mathbf{q}_j, i = 1, 2, \dots, j$
4. $\mathbf{w}_j = A \mathbf{q}_j - \sum_{i=1}^j h_{ij} \mathbf{q}_i$
5. $h_{j+1,j} = \|\mathbf{w}_j\|_2$, if $h_{j+1,j} = 0$, quit
6. $q_{j+1} = \mathbf{w}_j / h_{j+1,j}$
7. end for

From lines 4 and 6 of Algorithm 3, we can easily deduce the following fundamental relation:

$$A Q_m = Q_m H_m + h_{m+1,m} \mathbf{q}_{m+1} e_m^T, \quad (8)$$

where e_m denotes the m th column of the $m \times m$ identity matrix, and

$$H_m = \begin{pmatrix} h_{11} & h_{12} & \dots & h_{1n-1} & h_{1n} \\ h_{21} & h_{22} & \dots & h_{2n-1} & h_{2n} \\ 0 & h_{32} & \dots & h_{3n-1} & h_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & h_{nn-1} & h_{nn} \end{pmatrix} \quad (9)$$

is an upper Hessenberg matrix, i.e., $h_{ij} = 0$ if $i > j + 1$.

Multiplying both sides of (8) by Q_m^T and making use of the orthonormality of $\{q_1, \dots, q_m, q_{m+1}\}$, we immediately have

$$Q_m^T A Q_m = H_m \quad (10)$$

It means that H_m is equivalent to B_m in (7) for the case where the search subspace \mathcal{S} is a Krylov subspace. Therefore, we can outline the Arnoldi method as follows:

Algorithm 4 Arnoldi Method

1. Given the initial vector q_0 .
2. Generate H_m and Q_m by performing m steps of Algorithm 3;
3. Compute the Ritz pairs and decide which ones are acceptable;
4. If necessary, increase m and repeat.

Note that in Arnoldi method, the matrix A is only involved in the matrix-vector multiplications.

If A is large sparse or large structured, and only a smaller number of eigenvalues/eigenvectors are needed, the Arnoldi method is an ideal method.

Global Methods

QR Iteration

This method can find all the eigenvalues and eigenvectors of a given matrix (Francis 1961; Parlett 1980). Given $A \in \mathbb{R}^{n \times n}$, we first find an orthogonal $Q_0 \in \mathbb{R}^{n \times n}$, which means that $Q_0^T Q_0 = I$, and compute $T_0 = Q_0^T A Q_0$. Then we construct the QR iteration as:

$$\begin{cases} Q_k R_k = T_{k-1}, \\ T_k = R_k Q_k, \quad k = 1, 2, \dots, \end{cases}$$

where Q_k is orthogonal and R_k is upper triangular. Let $Q_k = Q_0 Q_1 \cdots Q_k$, then Q_k is an orthogonal matrix since all Q_j are. It can be verified that $T_k = Q_k^T A Q_k$, so T_k is orthogonally similar to A . Therefore, T_k and A have the same eigenvalues. For the QR iteration, there are two special cases:

1. If A has real distinct eigenvalues, then T_k converges to an upper triangular matrix T :

$$T = \begin{pmatrix} t_{11} & t_{12} & \dots & t_{1n} \\ 0 & t_{22} & \dots & t_{2n} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & t_{nn} \end{pmatrix}, \quad (11)$$

with t_{ii} , $i = 1, 2, \dots, n$ being the eigenvalues of A .

2. If A has complex eigenvalues, then T_k converges to a block upper triangular matrix T :

$$T = \begin{pmatrix} T_{11} & T_{12} & \dots & T_{1l} \\ 0 & T_{22} & \dots & T_{2l} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & T_{ll} \end{pmatrix}, \quad (12)$$

where each T_{ii} is either order-one or two block. If T_{ii} is an order-one block, then it is a real eigenvalue of A . Otherwise, it contains a pair of complex conjugate eigenvalues of A (see Schur 1909, p. 341).

The computational cost of each QR iteration will reach $O(n^3)$, and the convergence is linear. However, the computational cost can be reduced to $O(n^2)$ if we choose a suitable Q_0 such that T_0 is in the upper Hessenberg form (see (9)). The reduction can be realized with Householder transformations (Householder 1958).

The QR Iteration with Shifting Techniques

In order to make the QR iteration more efficient and robust, the shifted QR iteration strategy was developed (Stewart 1973). Given $\mu \in \mathbb{R}$ and an upper Hessenberg matrix T_0 , construct Hessenberg QR iteration with shift μ :

$$\begin{cases} Q_k R_k = T_{k-1} - \mu I, \\ T_k = R_k Q_k + \mu I. \end{cases}$$

Denote $T_{k-1} = (t_{ij}^{(k-1)})$. There exist two ways to select the shift μ at each iteration (see Golub and Van Loan 2012, pp. 385–388).

1. If $\mu = t_{nn}^{(k-1)}$, the iteration is named as the QR method with single shift.

2. If the two eigenvalues of $\begin{bmatrix} t_{n-1n-1}^{(k-1)} & t_{n-1n}^{(k-1)} \\ t_{nn-1}^{(k-1)} & t_{nn}^{(k-1)} \end{bmatrix}$ are complex, say $\sigma, \bar{\sigma}$, we perform two consecutive QR iteration steps with complex conjugate shifts σ and $\bar{\sigma}$:

$$\begin{cases} Q_k R_k = T_{k-1} - \sigma I, \\ T_k = R_k Q_k + \sigma I, \\ Q_k R_k = T_k - \bar{\sigma} I, \\ T_{k+1} = R_k Q_k + \bar{\sigma} I. \end{cases}$$

The QR iteration is an effective method for dense and moderate eigenvalue problems, especially when all eigenvalues/eigenvectors are required. The first step of a practical QR iteration is to reduce the matrix A to upper Hessenberg form via Householder transformations. However, for large eigenvalue problems, Householder transformations cannot be used as they destroy the sparsity or the structure of A . In these cases, the

orthogonal projection method, such as the Arnoldi method, is an alternative.

Cross-References

- ▶ [Centrality Measures](#)
- ▶ [Ranking Methods for Networks](#)

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