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ARE OUTPUT GROWTH-RATE DISTRIBUTIONS FAT-TAILED? SOME EVIDENCE FROM OECD COUNTRIES

GIORGIO FAGIOLO,[a]* MAURO NAPOLETANO[a,b] AND ANDREA ROVENTINI[a,c]

[a] *Sant'Anna School of Advanced Studies, Pisa, Italy*
[b] *Chair of Systems Design, ETH Zürich, Zürich, Switzerland*
[c] *University of Modena and Reggio Emilia, Italy*

SUMMARY

This work explores some distributional properties of aggregate output growth-rate time series. We show that, in the majority of OECD countries, output growth-rate distributions are well approximated by symmetric exponential power densities with tails much fatter than those of a Gaussian (but with finite moments of any order). Fat tails robustly emerge in output growth rates independently of: (i) the way we measure aggregate output; (ii) the family of densities employed in the estimation; (iii) the length of time lags used to compute growth rates. We also show that fat tails still characterize output growth-rate distributions even after one washes away outliers, autocorrelation and heteroscedasticity.

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1. INTRODUCTION

This work investigates the statistical properties of output growth-rate time-series distributions. In each given country, we consider the time series of aggregate output growth rates and we study the shape of the resulting distribution. More precisely, we follow a parametric approach and we fit via maximum likelihood growth-rate distributions with the exponential power (EP) family of densities (Subbotin, 1923), which includes as special cases the Gaussian and the Laplace.

Our main finding is that in the USA, and in many other OECD countries, growth-rate distributions can be well approximated by EP densities with tails much fatter than those of a Normal distribution. This implies that output growth patterns tend to be quite lumpy: large growth events, either positive or negative, seem to be more frequent than what a Gaussian model would predict.[1]

We show that this result is robust to a series of alternative specifications of the analysis. First, our findings are not affected by the way we measure aggregate output (e.g., GDP or Industrial Production index). Second, fat tails in growth rates still emerge if one removes from the original time series outliers, autocorrelation and heteroscedasticity (if any), and then studies the shape of the ensuing distribution of residuals. Third, the existence of tails fatter than Gaussian ones is confirmed even if one employs non-parametric estimates (e.g., the Hill estimator) and alternative thick-tailed density families, such as the Lévy-stable (Zolotarev, 1986; Uchaikin and Zolotarev,

* Correspondence to: Giorgio Fagiolo, Sant'Anna School of Advanced Studies, Piazza Martiri della Libertà 33, I-56127 Pisa, Italy. E-mail: giorgio.fagiolo@sssup.it

[1] Fat-tailed distributions arise in many empirical contexts. Application areas, apart from economics and finance, include engineering, computer science, social networks, physics, astronomy, etc.: see Adler *et al.* (1998) and Embrechts *et al.* (1997) for an introduction.

1999; Nolan, 2006) and the Student-*t*. However, the EP density turns out to be the family that best fits the data for the majority of countries. Interestingly, this finding supports the view that growth-rate distributions have finite moments of any order. Fourth, we show that growth-rate distributions do not display any significant evidence for skewness. Hence, positive and negative large growth events have almost the same likelihood, thus confirming recent results on symmetry of the magnitude of GDP fluctuations (McKay and Reis, 2006). Finally, fat tails persist even if one computes growth rates using longer time-lags.

Our work is motivated by two, seemingly unrelated, streams of literature. On the one hand, we refer to the rich body of contributions that in the last twenty years have been attempting to single out robust statistical properties of within-country output dynamics (see, among others, Nelson and Plosser, 1982; Cochrane, 1988; Brock and Sayers, 1988; Rudebusch, 1993; Cochrane, 1994; Potter, 1999; Murray and Nelson, 2000). For example, as far as the USA is concerned, output growth-rate time series was found to be positively autocorrelated over short horizons and to have a weak negative autocorrelation over longer time spans. Moreover, still unsettled debates have focused on the questions of whether US GNP is characterized by a deterministic or a stochastic trend and whether output dynamics is better captured by linear or nonlinear models. Following this line of research, our study suggests that fat tails in the distributions of growth rates and their residuals may be considered as a candidate to become an additional stylized fact of within-country output dynamics.

On the other hand, the quest for stylized facts of aggregate output dynamics has been more recently revived by a new body of contributions investigating the properties of *cross-country* output growth-rate distributions. The main finding of these studies was indeed that GDP growth rates tend to *cross-sectionally* distribute according to densities that display tails fatter than Gaussian ones (Canning *et al*., 1998; Lee *et al*., 1998; Castaldi and Dosi, 2004).[2] The basic exercise performed in these works, however, has been focusing only on *cross-section* distributions, i.e. across all countries at a given year, possibly pooling all cross-section distributions together under the assumption of stationarity of moments. In this paper, on the contrary, we show that fat-tailed distributions also emerge *across time within a single country*.

Therefore, by studying the shape of within-country growth distributions, we attempt to bridge earlier studies focusing on the statistical properties of within-country output dynamics to the new stream of research on the distributional properties of cross-sectional growth rates.

Our analysis differs from previous, similar ones (Canning *et al*., 1998; Lee *et al*., 1998; Castaldi and Dosi, 2004) in several additional respects. First, we depart from the common practice of using annual data to build output growth-rate distributions. We instead employ monthly and quarterly data. This allows us to get longer series and better appreciate their business cycle features. Second, as mentioned above, we double-check the results obtained with the EP by both using non-parametric tests (Hill tail-index estimator) and parametrically fitting output growth-rate distributions with a number of alternative fat-tailed densities (Student-*t*, Lévy-stable). In the case of the EP, the Student-*t* and the Lévy-stable, one can actually measure how far empirical growth-rate distributions are from the Normal benchmark. Third, we perform a detailed goodness-of-fit analysis in order to test whether the data are well proxied by theoretical densities. Finally, we ask whether our findings are robust to controlling for the presence of outliers, autocorrelation

[2] Interestingly, similar results were also found for cross-section firm and industry growth rates (see Stanley *et al*., 1996; Lee *et al*., 1998; Amaral *et al*., 1997; Bottazzi and Secchi, 2003a, 2003b; Castaldi and Dosi, 2004; Fu *et al*., 2005; Sapio and Thoma, 2006). Hence, fat tails *cross-sectionally* emerge no matter the level of aggregation.

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and heteroscedasticity in output growth-rate dynamics, and we test for possible asymmetries in growth-rate distributions.

Our results have several theoretical and empirical implications. From an empirical perspective, the emergence of fat-tailed distributions for both growth rates and residuals can be interpreted as a candidate new stylized fact for within-country output dynamics. Furthermore, the widespread evidence in favor of fat-tailed output growth rates suggests that econometric estimation and testing procedures that are heavily sensitive to normality of residuals should be replaced, when necessary, with estimators and testing procedures that are either robust to non-Gaussian errors or are based on thick-tailed errors.

From a theoretical perspective, our findings call for models that are able to reproduce and explain this candidate new stylized fact. At the same time, theoretical models might employ this new evidence in their set of assumptions, so as to possibly improve their performance. In any case, the implications stemming from these models should first be checked against the assumption of fat-tailed growth-rate distributions. Indeed, it has been shown that economic models failing to account for fat tails in their data-generating process may deliver invalid implications, especially if tails are heavy (so that moments might not exist; see Ibragimov, 2005). Our results imply that output growth rates, albeit fat-tailed, are characterized by finite moments of any order. This might be good news for all models requiring the knowledge of the shape of growth-rate distributions, as their implications might end up to be robust to the assumption of EP growth-rate densities, which are in line with empirical evidence.

Furthermore, gaining empirical knowledge on the shape of output growth rates may shed some light on the underlying generating processes (with all the caveats discussed in Brock, 1999). For example, the fact that fat tails characterize not only growth-rate distributions of countries (both time-series and cross-sectionally), but also of industries and firms, hints to the existence of some common forces operating at very different aggregation levels (Lee *et al.*, 1998). In addition, if one thinks of the growth of country output as the outcome of aggregation of firm- and industry-level growth profiles, the emergence of fat tails in country-level growth rates seems to strongly reject the hypothesis that some form of *central limit theorem* is at work (Castaldi and Dosi, 2004).

The paper is organized as follows. In Section 2 we describe the data and the methodology we employ in our analysis. Empirical results on growth-rate distributions for the USA and other OECD countries are presented in Section 3. Robustness checks are discussed in Section 4. Section 5 discusses the implications of our results in light of the existing theoretical and empirical literature on output dynamics in macroeconomics. Finally, Section 6 concludes.

2. DATA AND METHODOLOGY

The main objects of our analysis are output growth rates $g(t)$, defined as

$$g(t) = \frac{Y(t) - Y(t-1)}{Y(t-1)} \cong y(t) - y(t-1) = (1-L)y(t) \tag{1}$$

where $Y(t)$ is the output level (GDP or IP) at time t in a given country, $y(t) = \log[Y(t)]$ and L is the lag operator.

We exploit two sources of (seasonally adjusted) data. As far as the USA is concerned, we employ the FRED database. We consider two output growth-rate series: (i) quarterly real GDP, ranging

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from 1947Q1 to 2005Q3 (GDP, 234 observations); (ii) monthly industrial production (IP), ranging from 1921M1 to 2005M10 (IP1921, 1017 observations). Moreover, in order to better compare the IP growth-rate distribution with the GDP one, we carry out an investigation on the post WWII period only, using IP observations from 1947 to 2005 (IP1947, 702 observations).[3] The analyses for other OECD countries are performed by relying on monthly IP data drawn from the 'OECD Historical Indicators for Industry and Services' database (1975M1–1998M12, 286 observations).[4] Note that, by focusing on IP as a measure of aggregate activity, we can study a longer time span and thus improve our estimates. IP is typically a good proxy of output levels, as it tracks GDP series very closely. In fact, as Figure 1 shows, the GDP–IP cross-correlation profile mimics from time $t - 6$ to time $t + 6$ the GDP auto-correlation profile.

Let $T_n = \{t_1, \ldots, t_n\}$ be the time interval over which we observe growth rates. We define within-country, time-series distribution of output growth rates as

$$G_{T_n} = \{g(t), t \in T_n\} \tag{2}$$

We start by studying the shape of G_{T_n} with a parametric approach. More precisely, we fit growth rates with the exponential power (EP) family of densities, also known as the Subbotin

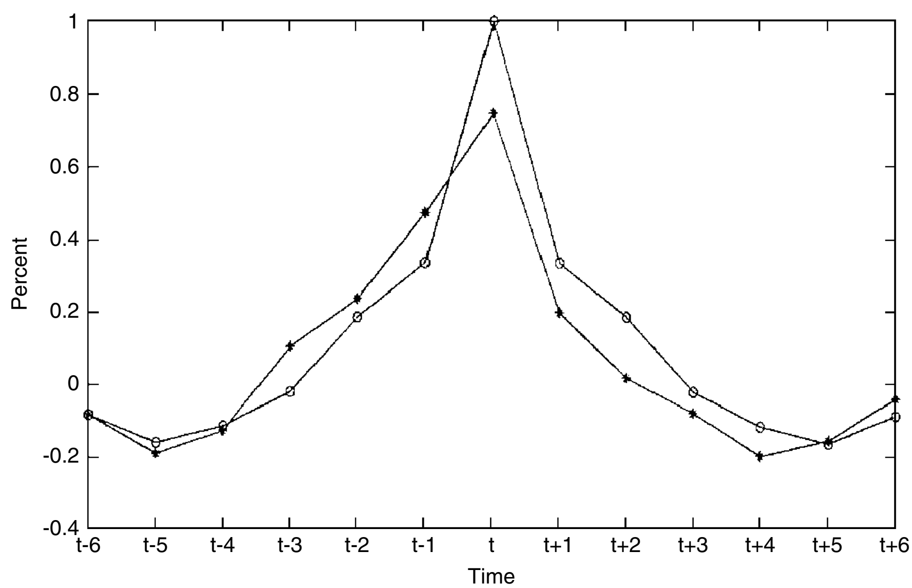

Figure 1. GDP auto-correlation vs. GDP–IP cross-correlations for US FRED data. Circles, GDP auto-correlation; asterisks, GDP–IP cross-correlations

[3] To double-check our results, we also consider the complementary time span 1921M1–1946M12. More on that below.
[4] We study growth-rate distributions of the following countries: Canada, Japan, Austria, Belgium, Denmark, France, Germany, Italy, the Netherlands, Spain, Sweden and the UK.

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distribution,[5] whose functional form reads

$$f(x; b, a, m) = \frac{1}{2ab^{\frac{1}{b}}\Gamma(1 + \frac{1}{b})} e^{-\frac{1}{b}|\frac{x-m}{a}|^{b}} \tag{3}$$

where $a > 0$, $b > 0$ and $\Gamma(\cdot)$ is the Gamma function. The EP distribution is thus characterized by three parameters: a *location* parameter m, a *scale* parameter a and a *shape* parameter b. The location parameter controls for the mean of the distribution, whereas the scale parameter is proportional to the absolute deviation.[6]

The shape parameter is the crucial one for our analysis: the larger is b, the thinner are the tails. In fact, the EP density encompasses both the Laplace and the Gaussian distributions. If $b = 2$, the distribution reduces to a Gaussian. If $b < 2$, the distribution displays tails fatter than those of a Gaussian (henceforth 'super-Normal' tails). If $b = 1$, one recovers a Laplace. Finally, values of b smaller than one indicate tails fatter than those of a Laplace ('super-Laplace' tails in what follows). Figure 2 illustrates the three cases above. The above property is the value-added of the EP density, as it allows one to precisely measure how far the empirical distribution is from the normal benchmark and how close it is instead to the Laplace one. Another important property of the EP density is that it is characterized by exponentially shaped tails, which are less thick than those of power-law distributions. In Section 4.2 we shall come back to a comparison of how alternative density families, characterized by tails even fatter than those of the EP, perform in fitting our data.

In the exercises that follow, we fit empirical distributions G_{T_n} with the EP density in equation (3) by jointly estimating the three parameters via maximum likelihood (ML). Note that ML estimation of EP parameters is not an easy task. For theoretical and computational issues, we refer to Agrò (1995) and Bottazzi and Secchi (2006b). In what follows, we perform estimation by employing the package SUBBOTOOLS.[7] Note that, despite ML estimators being asymptotically unbiased and always unique for $n > 100$, some upward bias may emerge in the estimation of the shape coefficient for small samples. However, Monte Carlo studies (available from the authors upon request) show that, for sample sizes similar to those considered in this work, ML estimators of EP parameters are nearly unbiased and are characterized by a reasonably small variance. This confirms results obtained by Agrò (1995), who also shows that estimation of b is not affected by the other two parameters (a, m).

3. FITTING THE EP DENSITY: PARAMETER ESTIMATION AND GOODNESS OF FIT

In this section we report the results of EP fits. We begin with a detailed analysis of US growth-rate distributions and we then check whether the main findings of the analysis hold also for the other OECD countries under study.

[5] More on fitting EP distributions to economic data is in Bottazzi and Secchi (2003a, 2003b).
[6] On the links between moments and parameters of the EP distribution, see Bottazzi (2004).
[7] Available online at http://cafim.sssup.it/~giulio/software/subbotools/. See Bottazzi (2004) for details.

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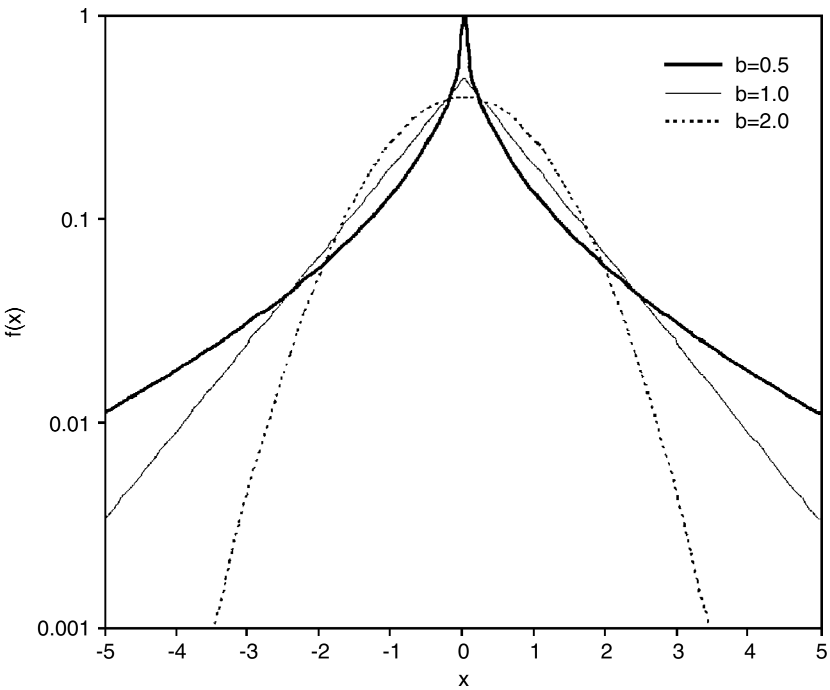

Figure 2. The exponential-power (EP) density for $m = 0$, $a = 1$ and different shape parameter values: (i) $b = 2$: Gaussian density; (ii) $b = 1$: Laplace density; (iii) $b = 0.5$: EP with super-Laplace tails. *Note*: Log scale on the y-axis

3.1. US Growth-Rate Distributions

Let us start by some descriptive statistics on US output growth rates. Table I reports the first four moments of US time series, together with a battery of normality tests for the null hypothesis that the series come from a Gaussian distribution with unknown parameters.

Note first that skewness levels are quite small. This justifies using a symmetric theoretical density like (3) to fit the data.[8] The relatively large figures for kurtosis suggest, however, that output growth-rate distributions display fat tails. Indeed, all normality tests reject the hypothesis that US series are normally distributed. This is confirmed by Anscombe–Glynn's test (Anscombe and Glynn, 1983), which clearly detects that non-normality is due to excess kurtosis.

In order to better explore this evidence, we fit US output growth-rate distributions with the EP density (see equation (3)). Maximum-likelihood estimates, together with standard errors (in parentheses), Cramér–Rao confidence intervals, and hypothesis testing results are reported in Table II.

Estimates indicate that, as expected, all three growth-rate time series are markedly non-Normal. Growth rates seem instead to distribute according to a Laplace for GDP ($\widehat{b}$ very close to one). Furthermore, they display tails even fatter than Laplace ones for IP1921 ($\widehat{b}$ smaller than one),

[8] In Section 4.3 we will explore in more detail departures from symmetry and we will check whether asymmetric EP densities might perform better than a symmetric one.

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Table I. US output growth-rate time series: summary statistics and p-values of normality and kurtosis tests

	Series		
	GDP	IP1921	IP1947
Statistic		Value	
Obs.	234	1017	702
Mean	0.0084	0.0031	0.0028
SD	0.0099	0.0193	0.0098
Skewness	−0.0891	0.3495	0.3295
Kurtosis	4.2816	14.3074	8.1588
Normality test		p-Value	
Anderson–Darling	0.0019	0.0000	0.0000
Adj Jarque Bera LM	0.0000	0.0000	0.0000
Adj Jarque Bera ALM	0.0000	0.0000	0.0000
Cramér–Von Mises	0.0020	0.0002	0.0000
Lilliefors	0.0279	0.0000	0.0000
D'Agostino	0.0120	0.0000	0.0000
Shapiro–Wilk	0.0038	0.0000	0.0000
Shapiro–Francia	0.0023	0.0000	0.0000
Kurtosis test		p-Value	
Anscombe–Glynn	0.0036	0.0000	0.0000

Table II. US output growth-rate distributions: estimated EP parameters

	GDP	IP1921	IP1947
$\widehat{m}$	0.0082 (0.0006)	0.0031 (0.0002)	0.0030 (0.0003)
$\widehat{a}$	0.0078 (0.0006)	0.0091 (0.0004)	0.0068 (0.0003)
$\widehat{b}$	1.1771 (0.1484)	0.6215 (0.0331)	0.9940 (0.0700)
$[\widehat{b}-2\sigma(\widehat{b}), \widehat{b}+2\sigma(\widehat{b})]$	[0.8803, 1.4739]	[0.5553, 0.6877]	[0.8540, 1.1340]
p-Value for $H_0: b=1$ $H_1: b \neq 1$	0.1071	0.0000	0.5268
p-Value for $H_0: b=2$ $H_1: b<2$	0.0001	0.0000	0.0000

Note: Standard errors of estimates $\sigma(\cdot)$ are in parentheses. $\widehat{b} \pm 2\sigma(\widehat{b})$ are Cramér–Rao confidence intervals. p-Values in the last two rows are computed by bootstrapping the distribution of $\widehat{b}$ under H_0 and econometric sample sizes equal to those of the empirical time series. Bootstrap sample size: $M = 10,000$.

whereas the estimated coefficient for IP1947 goes back to a value close to one. The last result seems to suggest that super-Laplace tails could be due to the turmoils of the Great Depression. However, this is not the case: additional estimation exercises (not shown) indicate that the $\widehat{b}$ parameter for the $1921M1$–$1946M12$ period is not significantly different from one.

These results are statistically substantiated by Cramér–Rao confidence intervals (CI), which show that $b=1$ lies in both GDP and IP1947 CIs. Conversely, the CI for IP1921 spans entirely on the left of $b=1$. Of course, $b=2$ does not belong to any CIs. Since these CIs are only valid asymptotically, we also estimate exact p-values—via a standard bootstrap procedure—for two hypothesis tests: (i) H_0: $b=1$ vs. H_1: $b \neq 1$; (ii) H_0: $b=2$ vs. H_1: $b<2$. Estimated p-values

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indicate that normality is strongly rejected for all three time series. For both GDP and IP1947 it is not possible to reject the Laplace hypothesis, whereas for IP1921 the coefficient is statistically smaller than one.

We turn now to a battery of goodness-of-fit tests to explore the performance of the above EP estimates. Indeed, point estimates and parameter testing suggest that US growth-rate distributions are fat-tailed. But how good is the EP fit for the USA? A first visual assessment is contained in Figure 3–5, where we plot the binned empirical density against the ML fitted one (in semi-log scale): the EP seems to describe growth-rate distributions nicely, especially when tails turn out to be super-Laplacian.

As Table III suggests, the above graphical evidence is corroborated by standard goodness-of-fit (GoF) tests (see D'Agostino and Stephens, 1986, for details). In fact, no GoF test rejects the null hypothesis that data come from the fitted distributions. Moreover, both GDP and IP1947 seem to come from a Laplace distribution, whereas IP1921 appears to be well approximated by an EP with super-Laplace tails.

Similar findings are obtained if one performs generalized likelihood-ratio tests (LRTs). Table IV reports LRTs for the null hypotheses that data come from a Laplace or a Normal distribution. Again, normality is rejected in favor of Laplace for GDP and IP1947, and in favor of a super-Laplace distribution for IP1921.

3.2. Do Fat Tails Emerge Also in Other OECD Countries?

The foregoing results are replicated to a large extent also in other OECD countries. Descriptive statistics indicate that, in half of the countries under scrutiny, the distributions of IP growth rates seem to be slightly right-skewed, whereas in the other half they appear to be slightly left-skewed (see Table V). Belgium seems to be characterized by a quite relevant skewness (more on

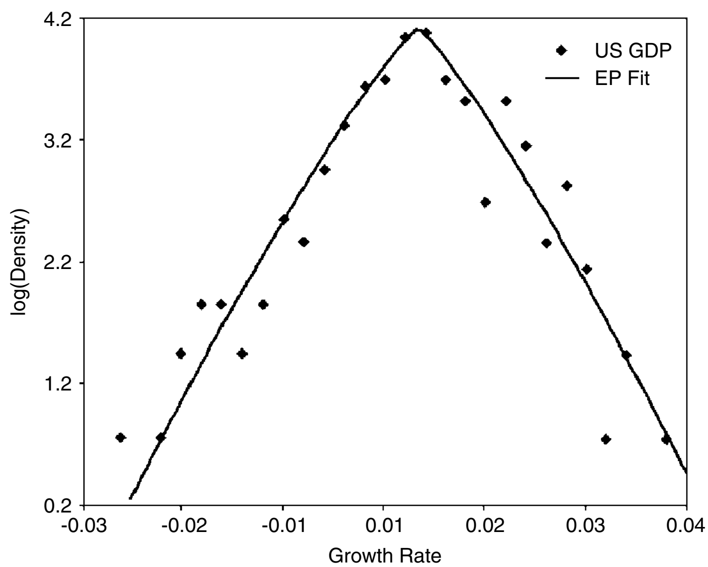

Figure 3. Binned empirical densities of US GDP growth rates vs. EP fit

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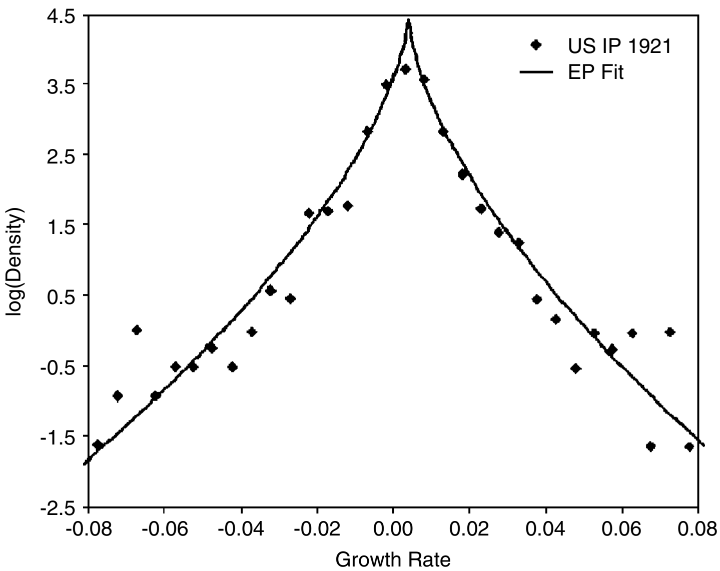

Figure 4. Binned empirical densities of US IP1921 growth rates vs. EP fit

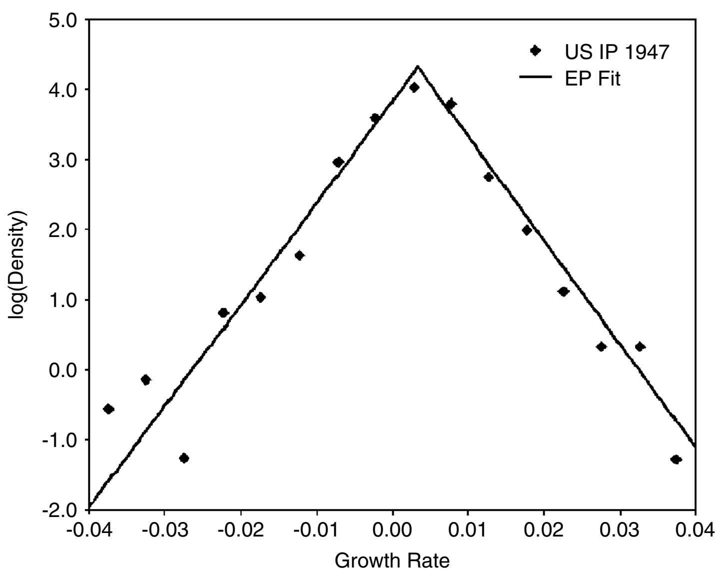

Figure 5. Binned empirical densities of US IP1947 growth rates vs. EP fit

that in Section 4.3). Normality tests show that almost all growth-rate distributions are markedly non-Normal due to excess kurtosis (see Anscombe–Glynn's test). The only exception is Canada, where the evidence from normality and kurtosis tests is mixed. Notice also that Anscombe–Glynn's p-value for France is higher than in other countries, but still lower than 5%. EP fits confirm this

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Table III. Goodness-of-fit tests: test statistics and estimated exact p-values

GoF test	GDP		IP1921		IP1947	
	Statistic	p-Value	Statistic	p-Value	Statistic	p-Value
$H_0: b = \widehat{b}, H_1: b \neq \widehat{b}$						
KSM	0.6772	0.7354	1.1744	0.1303	0.7907	0.5540
KUI	0.8971	0.9155	0.2711	0.1928	1.2742	0.4209
CVM	0.0410	0.9166	0.2090	0.2519	0.1100	0.5301
AD2	0.2934	0.9427	0.9905	0.3637	0.7471	0.5186
$H_0: b = 1, H_1: b \neq 1$						
KSM	0.6069	0.8450	1.6195	0.0042	0.7847	0.5659
KUI	0.9998	0.8083	2.1735	0.0028	1.2643	0.4391
CVM	0.0409	0.9134	0.5025	0.0397	0.1083	0.5421
AD2	0.3697	0.8724	5.6816	0.0015	0.7351	0.5261

Note: KSM, Kolmogorow–Smirnov (D) test; KUI, Kuiper (V) test; CVM, Cramér–Von Mises (W^2) test; AD2, Anderson–Darling quadratic (A^2) test. Test statistics adjusted for small-sample bias according to D'Agostino and Stephens (1986, Table IV.2, p. 105). Exact p-values estimated by bootstrapping the distribution of the test statistics under the null hypothesis H_0, with econometric sample sizes equal to those of the empirical time series and $(a, m) = (\widehat{a}, \widehat{m})$. Bootstrap sample size: $M = 10,000$.

Table IV. Likelihood ratio tests

	GDP	IP1921	IP1947
LL($\widehat{b}$)	755.844	2822.742	2314.933
LL($b = 1$)	755.099	2801.059	2314.928
LL($b = 2$)	747.978	2570.602	2249.369
$H_0: b = 1, (a, m) = (a_1^*, m_1^*);\ H_1: b \neq 1$			
Statistics	1.490	43.365	0.010
p-Value	0.685	0.000	1.000
$H_0: b = 2, (a, m) = (a_2^*, m_2^*);\ H_1: b < 2$			
Statistics	15.731	504.279	131.128
p-Value	0.001	0.000	0.000

Note: LL($\widehat{b}$), log-likelihood associated to ML estimates $(\widehat{b}, \widehat{a}, \widehat{m})$. LL($b = 1$), log-likelihood associated to (a_1^*, m_1^*), i.e., ML estimates of (a, m) subject to $b = 1$. LL($b = 2$), log-likelihood associated to (a_2^*, m_2^*), i.e., ML estimates of (a, m) subject to $b = 2$. Test statistics: $-2 \cdot \Delta LLT(b = b_0) = -2[LL(b = b_0) - LL(\widehat{b})]$, for $b_0 = 1, 2$. p-Values are computed using the fact that $-2 \cdot \Delta LLT(b = b_0) \to \chi^2(3)$.

evidence (see Table VI). All estimated shape coefficients are significantly smaller than two (at 5%, see last column). The only exception is again Canada, where the null hypothesis of normality cannot be rejected (although the p-value is very close to 5%). A quick inspection of p-values for the null hypothesis $b = 1$ (see column before the last one) shows that Spain is the only clear-cut case of a growth-rate distribution with tails fatter than a Gaussian but thinner than a Laplace. Austria, France and the Netherlands seem to have tails slightly thinner than a Laplace (p-values smaller than—or close to—5% but larger than 1%). Conversely, Japan, Belgium, Denmark, Germany, Italy, Sweden and the UK display Laplace tails.[9]

[9] In Denmark there is weak evidence for super-Laplace tails, since the $b = 1$ null hypothesis is rejected at 10% significance level, but not at 5%.

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Table V. OECD countries' output growth-rate time series: summary statistics and p-values of normality and kurtosis tests

	Canada	Japan	Austria	Belgium	Denmark	France	Germany	Italy	Netherlands	Spain	Sweden	UK
Statistic						Value						
Obs.	286	286	286	286	286	286	286	286	286	286	286	286
Mean	0.0021	0.0027	0.0024	0.0013	0.0025	0.0013	0.0015	0.0017	0.0015	0.0017	0.0016	0.0012
SD	0.0113	0.0404	0.0253	0.0401	0.0340	0.0130	0.0212	0.0321	0.0285	0.0401	0.0302	0.0140
Skewness	−0.2317	−0.2250	0.1707	−0.5689	0.1214	0.1525	0.0098	0.0453	−0.0350	0.2559	−0.2955	−0.1631
Kurtosis	3.5631	4.6895	5.7806	5.9446	7.2748	3.7251	9.2312	5.8380	6.5731	4.0067	37.0700	8.4090
Normality test						p-Value						
Anderson−Darling	0.2398	0.0000	0.0012	0.0000	0.0000	0.0025	0.0000	0.0000	0.0000	0.0754	0.0000	0.0000
Adj. Jarque−Bera LM	0.0400	0.0000	0.0000	0.0000	0.0000	0.0300	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Adj. Jarque−Bera ALM	0.0400	0.0000	0.0000	0.0000	0.0000	0.0300	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Cramér−Von Mises	0.2382	0.0000	0.0026	0.0000	0.0000	0.0012	0.0000	0.0000	0.0000	0.0611	0.0000	0.0000
Lilliefors	0.3556	0.0000	0.0279	0.0000	0.0000	0.0007	0.0000	0.0021	0.0007	0.1310	0.0000	0.0013
D'Agostino	0.0523	0.0003	0.0000	0.0000	0.0000	0.0541	0.0000	0.0000	0.0000	0.0053	0.0000	0.0000
Shapiro−Wilk	0.1706	0.0000	0.0000	0.0000	0.0000	0.0108	0.0000	0.0000	0.0000	0.0096	0.0000	0.0000
Shapiro−Francia	0.1036	0.0000	0.0000	0.0000	0.0000	0.0091	0.0000	0.0000	0.0000	0.0059	0.0000	0.0000
Kurtosis test						p-Value						
Anscombe−Glynn	0.0707	0.0002	0.0000	0.0000	0.0000	0.0309	0.0000	0.0000	0.0000	0.0071	0.0000	0.0000

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Table VI. OECD countries' output growth-rate distributions: estimated EP parameters

Country	$\widehat{m}$	$\widehat{a}$	$\widehat{b}$	$[\widehat{b}-2\sigma(\widehat{b}), \widehat{b}+2\sigma(\widehat{b})]$	p-Value for $H_0: b=1$ $H_1: b \neq 1$	p-Value for $H_0: b=2$ $H_1: b<2$
Canada	0.0020 (0.0010)	0.0104 (0.0007)	1.6452 (0.2047)	[1.2358, 2.0546]	0.0000	0.0516
Japan	0.0021 (0.0014)	0.0259 (0.0020)	0.8491 (0.0901)	[0.6689, 1.0293]	0.9301	0.0000
Austria	0.0010 (0.0014)	0.0204 (0.0014)	1.2499 (0.1446)	[0.9607, 1.5391]	0.0328	0.0000
Belgium	0.0011 (0.0017)	0.0284 (0.0021)	1.0202 (0.1125)	[0.7952, 1.2452]	0.4277	0.0000
Denmark	0.0000 (0.0012)	0.0215 (0.0017)	0.8063 (0.0847)	[0.6369, 0.9757]	0.9756	0.0000
France	0.0010 (0.0007)	0.0106 (0.0008)	1.2623 (0.1464)	[0.9695, 1.5551]	0.0256	0.0000
Germany	0.0024 (0.0008)	0.0144 (0.0011)	0.9768 (0.1067)	[0.7634, 1.1902]	0.5812	0.0000
Italy	0.0010 (0.0015)	0.0237 (0.0017)	1.0778 (0.1204)	[0.8370, 1.3186]	0.2627	0.0000
Netherlands	0.0019 (0.0015)	0.0223 (0.0016)	1.2133 (0.1393)	[0.9347, 1.4919]	0.0521	0.0000
Spain	0.0021 (0.0029)	0.0352 (0.0024)	1.4583 (0.1755)	[1.1073, 1.8093]	0.0014	0.0056
Sweden	0.0010 (0.0009)	0.0168 (0.0013)	0.8826 (0.0944)	[0.6938, 1.0714]	0.8666	0.0000
UK	0.0019 (0.0006)	0.0103 (0.0008)	1.0972 (0.1230)	[0.8512, 1.3432]	0.2148	0.0000

Note: Standard errors of estimates $\sigma(\cdot)$ in parentheses. $\widehat{b} \pm 2\sigma(\widehat{b})$ are Cramér–Rao confidence intervals. p-Values in the last two rows are computed by bootstrapping the distribution of $\widehat{b}$ under H_0 and econometric sample sizes equal to those of the empirical time series. Bootstrap sample size: $M = 10,000$.

Overall, the ML fit performs well in describing the data. Apart from the case of Denmark, GoF tests do not reject the hypothesis that data come from the ML fitted EP distribution (see Table VII, top panel). Moreover, as the bottom panel of Table VII shows, GoF tests do not reject the Laplace null hypothesis in almost all countries.

To further check the robustness of the results in Table VI, we turn to likelihood ratio tests. Table VIII confirms that apart from Canada (which have almost-Normal tails) and Spain (with tails fatter than a Normal but thinner than a Laplace), growth-rate distributions of all remaining countries are well approximated by a Laplace density.

4. ROBUSTNESS CHECKS

The foregoing discussion has pointed out that within-country output growth-rate distributions are markedly non-Gaussian. The evidence in favor of Laplace (or super-Laplace) densities robustly arises in the majority of OECD countries, it does not depend on the way we measure output (GDP or IP), and it emerges also at frequencies more amenable for the study of business cycles dynamics (i.e., quarterly and monthly). As a consequence, along the time dimension, big growth events, either positive or negative, are more likely than what a Gaussian model would predict.

Nevertheless, this clear-cut evidence in favor of fat tails can be biased by at least four problems. First, growth-rate series may contain outliers, some autocorrelation structure, and be possibly characterized by heteroscedasticity. This may generate spurious results due to an inappropriate pooling of time-series observations. Second, the emergence of fat tails may depend on the particular type of density employed in our fitting exercises (i.e., the EP one) and, more generally, on the fact that we only employed a parametric approach. Third, while data exhibit some (very mild) evidence for skewness in growth-rate distributions, we have fitted a symmetric EP. What happens when one allows for asymmetric EP densities? Finally, super-Normal tails in the IP growth-rate distributions, both for the USA and for the other OECD countries we have analyzed, may depend

Table VII. Goodness-of-fit tests for OECD countries: test statistics and estimated exact p-values

Country	GoF test							
	KSM		KUI		CVM		AD2	
	Statistic	p-Value	Statistic	p-Value	Statistic	p-Value	Statistic	p-Value
$H_0 : b = \widehat{b}, H_1 : b \neq \widehat{b}$								
Canada	0.6527	0.7766	1.0328	0.7679	0.0545	0.8378	0.3166	0.9251
Japan	0.4406	0.9864	0.8068	0.9694	0.0295	0.9717	0.3681	0.8773
Austria	0.6426	0.7987	0.9574	0.8578	0.0589	0.8099	0.5408	0.7071
Belgium	1.0041	0.2631	1.4815	0.1899	0.1231	0.4773	0.7552	0.5185
Denmark	1.7276	0.0041	2.2713	0.0009	0.3687	0.0844	1.9317	0.0997
France	1.2595	0.0804	2.2097	0.0022	0.1013	0.5552	0.5205	0.7183
Germany	0.5857	0.8764	1.0035	0.8106	0.0481	0.8815	0.4603	0.7879
Italy	0.6566	0.7784	1.3115	0.3716	0.0503	0.8642	0.3096	0.9270
Netherlands	1.3145	0.0647	2.3895	0.0004	0.1737	0.3141	1.0135	0.3453
Spain	0.5622	0.9040	1.0095	0.7963	0.0452	0.8933	0.3572	0.8853
Sweden	1.0848	0.1826	1.4733	0.1911	0.1287	0.4461	0.8782	0.4141
UK	0.8382	0.4802	1.2528	0.4470	0.0721	0.7307	0.7476	0.5155
$H_0 : b = 1, H_1 : b \neq 1$								
Canada	0.7307	0.6551	1.4641	0.2034	0.1547	0.3760	1.5626	0.1648
Japan	0.5475	0.9165	1.0153	0.7919	0.0402	0.9238	0.7433	0.5262
Austria	0.8943	0.3974	1.2702	0.4203	0.0855	0.6540	0.8876	0.4239
Belgium	1.0107	0.2528	1.5065	0.1701	0.1261	0.4638	0.7689	0.5055
Denmark	1.7276	0.0054	2.2140	0.0021	0.3882	0.0805	2.1346	0.0811
France	1.1990	0.1098	2.2097	0.0026	0.0967	0.5923	0.6522	0.6036
Germany	0.5755	0.8915	0.9835	0.8299	0.0454	0.8912	0.4435	0.8013
Italy	0.6678	0.7556	1.3231	0.3522	0.0574	0.8231	0.3754	0.8745
Netherlands	1.3861	0.0419	2.5051	0.0000	0.2186	0.2310	1.4519	0.1885
Spain	0.7280	0.6649	1.4188	0.2477	0.0765	0.7115	1.0192	0.3521
Sweden	1.0312	0.2351	1.2939	0.3912	0.1115	0.5232	0.6802	0.5728
UK	0.9394	0.3314	1.3881	0.2745	0.0822	0.6726	0.8632	0.4390

Note: KSM, Kolmogorow–Smirnov (D) test; KUI, Kuiper (V) test; CVM, Cramér-Von Mises (W^2) test; AD2, Anderson–Darling quadratic (A^2) test. Test statistics are adjusted for small-sample bias according to D'Agostino and Stephens (1986, Table IV.2, p. 105). Exact p-values are estimated by bootstrapping the distribution of the test statistics under the null hypothesis H_0, with econometric sample sizes equal to those of the empirical time series and $(a, m) = (\widehat{a}, \widehat{m})$. Bootstrap sample size: $M = 10,000$.

on the relatively high (monthly) frequency of IP output observations. How do estimated shape coefficients behave when growth rates are computed over longer time lags? In the remainder of this section, we will discuss these issues in more detail.

4.1. Outliers, Autocorrelation, and Heteroscedasticity

A first explanation for the presence of lumpiness in growth-rate time series might refer to the presence of outliers in the raw series (Chen and Liu, 1993). Moreover, our time-series analysis relies on pooling together growth-rate observations over time. Therefore, the observations contained in G_{T_n} should come from i.i.d. random variables. If growth-rate time series exhibit (as they typically do) autocorrelation and/or heteroscedasticity, the process is no longer i.i.d. and fat tails may emerge as a spurious result due to an inappropriate pooling procedure.

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Table VIII. Likelihood ratio tests for OECD countries

Country	$\text{LL}(\widehat{b})$	$\text{LL}(b=1)$	$\text{LL}(b=2)$	$H_0: b=1$, $(a,m)=(a_1^*,m_1^*)$ $H_1: b \neq 1$		$H_0: b=2$, $(a,m)=(a_2^*,m_2^*)$ $H_1: b < 2$	
				Statistic	p-Value	Statistic	p-Value
Canada	878.7356	870.3870	877.4225	16.6971	0.0008	2.6262	0.4529
Japan	540.4682	539.3700	512.0700	2.1963	0.5327	56.7963	0.0000
Austria	655.4767	653.5069	645.3409	3.9396	0.2681	20.2717	0.0001
Belgium	537.1539	537.1403	514.0219	0.0274	0.9988	46.2641	0.0000
Denmark	587.7118	586.7298	561.1264	1.9640	0.5799	53.1708	0.0000
France	842.6275	841.8152	836.8383	1.6247	0.6538	11.5784	0.0090
Germany	725.8727	725.8421	695.7202	0.0612	0.9960	60.3050	0.0000
Italy	594.9010	594.6809	577.7194	0.4400	0.9319	34.3631	0.0000
Netherlands	626.4250	624.5917	612.1028	3.6666	0.2998	28.6443	0.0000
Spain	517.4279	512.9920	514.1326	8.8718	0.0310	6.5907	0.0862
Sweden	668.3535	667.2673	595.3823	2.1726	0.5374	145.9425	0.0000
UK	834.9834	834.5914	814.9782	0.7839	0.8533	40.0103	0.0000

Note: $\text{LL}(\widehat{b})$, log-likelihood associated to ML estimates $(\widehat{b}, \widehat{a}, \widehat{m})$. $\text{LL}(b=1)$, log-likelihood associated to (a_1^*, m_1^*), i.e., ML estimates of (a, m) subject to $b=1$. $\text{LL}(b=2)$, log-likelihood associated to (a_2^*, m_2^*), i.e., ML estimates of (a, m) subject to $b=2$. Test statistics: $-2 \cdot \Delta LLT(b=b_0) = -2[LL(b=b_0) - LL(\widehat{b})]$, for $b_0 = 1, 2$. p-Values are computed using the fact that $-2 \cdot \Delta LLT(b=b_0) \rightarrow \chi^2(3)$.

To control for these alleged problems, we subsequently removed from our original growth-rate series outliers, autocorrelation and heteroscedasticity (whenever detected). Incidentally, none of the growth-rate series considered reveals the presence of unit-root in the data according to standard testing procedures (Phillips–Perron and Dickey–Fuller tests). This is an important result, since the analysis of fat tails could lead to invalid results in the case of time series exhibiting strong persistence. We performed identification and correction of outliers by employing standard procedures available in TRAMO (Gómez and Maravall, 2001). We then fitted a battery of ARMA specifications (see, for example, Blanchard and Simon, 2001) to outlier-free growth series and selected the best model via the Box–Jenkins procedure. Finally, we checked for the presence of heteroscedasticity on ARMA residuals by running Ljung–Box and Engle's ARCH tests. If heteroscedasticity was detected, we fitted the best GARCH specification to obtain a 'fully depurated' series, i.e., an outlier-free series without autocorrelation and (possibly) heteroscedasticity.

Table IX reports summary statistics and normality tests for US series depurated from outliers only, and for 'fully depurated' series.[10] Normality is still rejected in all series (due to high kurtosis). Note also that the standard deviation of 'fully depurated' IP series substantially increases.

Next, we fitted an EP density to outlier-free and 'fully depurated' US series. Notwithstanding outliers and any 'structure' due to autocorrelation and/or heteroscedasticity have been washed away, fat tails still emerge in the distributions of residuals (see Table X). Although all estimated shape coefficients are now larger than one, the null hypothesis of normality is strongly rejected (see last row). According to our tests, the distributions of GDP growth-rate residuals are still close to a Laplace (the p-values for H_0: $b=1$ are slightly larger than 5% for the outlier-free series, but

[10] The best model for GDP growth-rate series is an AR(1) without drift, while for IP1921 and IP1947 we employed an ARMA(1,1) and a GARCH(1,1) with an additional seasonal component in order to account for residual seasonality. More detailed results are available from the authors on request.

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Table IX. Controlling for outliers, autocorrelation and heteroscedasticity. US output growth-rate time series: summary statistics and p-values of normality and kurtosis tests

	Series					
	Outliers only			'Fully depurated' series		
	GDP	IP1921	IP1947	GDP[a]	IP1921[b]	IP1947[b]
Statistic	Value					
Mean	0.0000	0.0019	0.0023	−0.0001	−0.0444	−0.0104
SD	0.0091	0.0129	0.0084	0.0087	0.9957	1.0021
Skewness	−0.1636	−0.3911	−0.1015	−0.0692	−0.1858	0.1156
Kurtosis	3.7148	4.6464	4.4879	3.9990	4.6179	4.0278
Normality test	p-Value					
Anderson–Darling	0.0035	0.0000	0.0000	0.0042	0.0000	0.0001
Adj. Jarque–Bera LM	0.0400	0.0000	0.0000	0.0100	0.0000	0.0000
Adj. Jarque–Bera ALM	0.0400	0.0000	0.0000	0.0100	0.0000	0.0000
Cramér–Von Mises	0.0027	0.0000	0.0000	0.0081	0.0000	0.0002
Lilliefors	0.0147	0.0000	0.0000	0.0493	0.0000	0.0081
D'Agostino	0.0697	0.0000	0.0000	0.0366	0.0000	0.0001
Shapiro–Wilk	0.0243	0.0000	0.0000	0.0066	0.0000	0.0001
Shapiro–Francia	0.0148	0.0000	0.0000	0.0040	0.0000	0.0001
Kurtosis test	p-Value					
Anscombe-Glynn	0.0402	0.0000	0.0000	0.0114	0.0000	0.0001

Note: Columns 1–3 refer to original series depurated of outliers only. Columns 4–6 ('fully depurated' series) refer to original series depurated from outliers, autocorrelation and (possibly) heteroscedasticity. Outlier removal is performed using TRAMO (Gómez and Maravall, 2001). ARMA fitting is performed by selecting the best ARMA model using a Box–Jenkins selection procedure on outlier-free residuals. Best GARCH filtering is applied if both Ljung–Box and Engle's ARCH heteroscedasticity tests (on ARMA residuals) were rejected.
[a] Residuals from ARMA only.
[b] Residuals from ARMA + GARCH.

smaller than 5% for the 'fully depurated' one). The outlier-free IP1921 growth-rate series displays now a coefficient which is not significantly different from one. Although Cramér–Rao confidence intervals indicate that the Laplace distribution should be rejected, estimated exact p-values suggest instead not to reject H_0: $b = 1$. In all other cases, coefficients are significantly larger than one (but smaller than two). GoF and likelihood-ratio tests confirm these results. See also Figure 6 for an illustration of EP fits to outlier-free (left) and 'fully depurated' (right) GDP growth series.

Analogous findings are obtained also for the other OECD countries. Table XI reports results for 'fully depurated' series only.[11] Although estimated shape coefficients typically increase as compared to the non-depurated ones (see Table VI), in all cases (but Japan) the distributions of residuals display tails statistically thinner than those of a Laplace, but much fatter than a Normal density. Note that in Canada the growth-rate distribution is now back to a Laplace, while Japan strikingly exhibits super-Laplace tails.

[11] Best models are as follows. Austria*, Spain*, Sweden: AR(2); Japan: AR(4) with drift; France, Germany, Italy: MA(1); UK: ARMA(3,1); Canada: ARMA(3,1)+GARCH(1,1); Belgium*: MA(1)+GARCH(1,1). (*): Additional seasonal component. Again, detailed ARMA results are available from the authors on request.

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Table X. Controlling for outliers, autocorrelation and heteroscedasticity. US output growth-rate distributions: estimated EP parameters

	Series					
	Outlier-free series			'Fully depurated' series		
	GDP	IP1921	IP1947	GDP[a]	IP1921[b]	IP1947[b]
$\widehat{m}$	0.0000 (0.0006)	0.0027 (0.0000)	0.0026 (0.0000)	0.0000 (0.0006)	−0.0353 (0.0009)	−0.0202 (0.0013)
$\widehat{a}$	0.0073 (0.0006)	0.0094 (0.0000)	0.0067 (0.0000)	0.0071 (0.0005)	0.8280 (0.0010)	0.8480 (0.0014)
$\widehat{b}$	1.2308 (0.1568)	1.0367 (0.0019)	1.2227 (0.0034)	1.2696 (0.1628)	1.3205 (0.0026)	1.3578 (0.0039)
$[\widehat{b} - 2\sigma(\widehat{b}), \widehat{b} + 2\sigma(\widehat{b})]$	[0.9172, 1.5444]	[1.0329, 1.0405]	[1.2159, 1.2295]	[0.9440, 1.5952]	[1.3153, 1.3257]	[1.3501, 1.3655]
p-Value for H_0: b = 1 H_1: b $\neq$ 1	0.0575	0.2719	0.0028	0.0378	0.0000	0.0000
p-Value for H_0: b = 2 H_1: b $<$ 2	0.0002	0.0000	0.0000	0.0005	0.0000	0.0000

Note: Columns 1–3 refer to original series depurated of outliers only. Columns 4–6 ('fully depurated' series) refer to original series depurated from outliers, autocorrelation and (possibly) heteroscedasticity. Outlier removal is performed using TRAMO (Gómez and Maravall, 2001). ARMA fitting is performed by selecting the best ARMA model using a Box–Jenkins selection procedure on outlier-free residuals. Best GARCH filtering is applied if both Ljung–Box and Engle's ARCH heteroscedasticity tests (on ARMA residuals) were rejected. Standard errors of estimates $\sigma(\cdot)$ are in parentheses. $\widehat{b} \pm 2\sigma(\widehat{b})$ are Cramér–Rao confidence intervals. p-Values in the last two rows are computed by bootstrapping the distribution of $\widehat{b}$ under H_0 and with econometric sample sizes equal to those of the empirical time series and $(a, m) = (\widehat{a}, \widehat{m})$. Bootstrap sample size: $M = 10,000$.

[a] Residuals from ARMA only.

[b] Residuals from ARMA + GARCH.

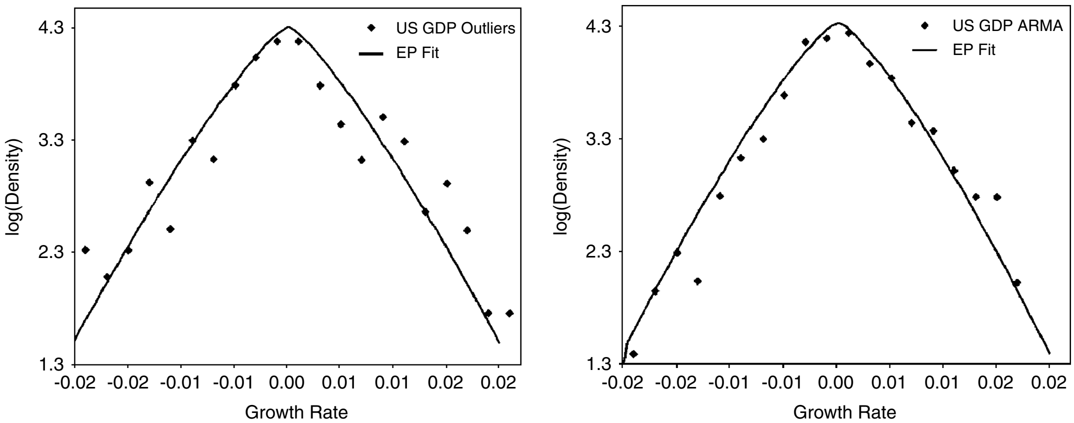

Figure 6. Controlling for outliers and autocorrelation in US GDP growth rates. Binned empirical densities vs. EP fit. Left: residuals after removing outliers only. Right: residuals after removing outliers and autocorrelation ('fully depurated series'). Outlier removal was performed using TRAMO (Gómez and Maravall, 2001). Autocorrelation removal was performed by fitting an ARMA model to outlier-free residuals. No evidence for heteroscedasticity was detected

Table XI. Controlling for outliers, autocorrelation and heteroscedasticity. Output growth-rate distributions of OECD countries: estimated EP parameters

	$\widehat{m}$	$\widehat{a}$	$\widehat{b}$	$[\widehat{b}-2\sigma(\widehat{b}), \widehat{b}+2\sigma(\widehat{b})]$	p-Value for $H_0: \text{b}=1$ $H_1: \text{b}\neq 1$	p-Value for $H_0: \text{b}=2$ $H_1: \text{b}<2$
Canada[b]	0.0122 (0.0032)	0.8508 (0.0035)	1.4007 (0.0099)	[1.3809,1.4205]	0.0042	0.0022
Japan[a]	0.0000 (0.0000)	0.0019 (0.0000)	0.7617 (0.0047)	[0.7522,0.7712]	0.9917	0.0000
Austria[a]	−0.0016 (0.0001)	0.0176 (0.0001)	1.4767 (0.0106)	[1.4555,1.4979]	0.0009	0.0081
Belgium[b]	0.0104 (0.0032)	0.8537 (0.0035)	1.3728 (0.0096)	[1.3536,1.3920]	0.0053	0.0010
Denmark[a]	0.0004 (0.0001)	0.0217 (0.0001)	1.2967 (0.0090)	[1.2787,1.3147]	0.0177	0.0004
France[a]	0.0019 (0.0000)	0.0106 (0.0000)	1.6051 (0.0119)	[1.5813,1.6289]	0.0000	0.0359
Germany[a]	0.0023 (0.0001)	0.0142 (0.0001)	1.4293 (0.0103)	[1.4088,1.4498]	0.0026	0.0035
Italy[a]	0.0038 (0.0001)	0.0207 (0.0001)	1.2437 (0.0086)	[1.2265,1.2609]	0.0367	0.0000
Netherlands[a]	0.0024 (0.0001)	0.0201 (0.0001)	1.7805 (0.0136)	[1.7533,1.8077]	0.0000	0.0162
Spain[a]	0.0038 (0.0001)	0.0253 (0.0001)	1.5058 (0.0110)	[1.4839,1.5277]	0.0003	0.0109
Sweden[a]	0.0000 (0.0001)	0.0176 (0.0001)	1.5842 (0.0117)	[1.5608,1.6076]	0.0000	0.0286
UK[a]	0.0021 (0.0000)	0.0105 (0.0000)	1.4821 (0.0108)	[1.4605,1.5037]	0.0006	0.0086

Note: Estimates are performed on 'fully depurated' series, i.e., original series subsequently depurated from outliers, autocorrelation and (possibly) heteroscedasticity. Outlier removal is performed using TRAMO (Gómez and Maravall, 2001). ARMA fitting is performed by selecting the best ARMA model using a Box–Jenkins selection procedure on outlier-free residuals. Best GARCH filtering is applied if both Ljung–Box and Engle's ARCH heteroscedasticity tests (on ARMA residuals) were rejected. Standard errors of estimates $\sigma(\widehat{\cdot})$ are in parentheses. $\widehat{b}\pm 2\sigma(\widehat{b})$ are Cramér–Rao confidence intervals for b. p-Values in the last two columns are computed by bootstrapping the distribution of $\widehat{b}$ under H_0 and with econometric sample sizes equal to those of the empirical time series and $(a, m) = (\widehat{a}, \widehat{m})$. Bootstrap sample size: $M = 10,000$.
[a] Residuals from outliers and ARMA only.
[b] Residuals from outliers, ARMA and GARCH.

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The foregoing analysis suggests that fat tails still characterize our series even when growth residuals are considered as the object of analysis, i.e., after one washes away from the growth process outliers and any structure possibly due to autocorrelation and heteroscedasticity. This has an important implication. If the 'true' data-generating process were characterized by a fat-tailed distribution, outliers should be observed in the original growth-rate data more frequently than one would have expected under a Gaussian model. The fact that fat tails persist even after washing away outliers (and additional structure) from original growth series indicates either a strong degree of fat-tailedness in the original growth-rate series or a poor performance of standard outlier-removal procedures against fat-tailed data-generating processes (or both).

4.2. Are Growth-Rate Tails Really Fat? Non-parametric Tests vs. Alternative Fat-Tailed Density Fits

Another possible weakness of the above analysis resides in the fact that it relies on fitting via maximum likelihood a particular type of (fat-tailed) density. As mentioned, the use of an EP family is justified by its extreme flexibility: if the goal is to understand not only if fat tails do emerge, but also how far they are from those of a Normal (or Laplace) distribution, the EP density turns out to be very useful. Furthermore, our parametric approach allows one to characterize the shape of the whole growth-rate distribution, and not only the behavior of its tails.

This strategy can be criticized from two perspectives. First, instead of looking for a complete parametric characterization of growth-rate distributions (i.e., fitting via maximum likelihood a family of densities to the whole G_{T_n}), one might simply estimate the heaviness of their tails by using non-parametric testing procedures (see Pictet *et al.*, 1998). Although this approach does not generally convey any information on the shape of the whole growth-rate distribution, it could be employed to give us an alternative measure of growth-rate tail fatness. Second, there exist other well-known examples of densities which are well suited to fit fat- and medium-tailed distributions (for a review of theoretical underpinnings and economic applications, see Embrechts *et al.*, 1997). Thus, one might ask whether the emergence of fat tails is confirmed when alternative densities are fitted via maximum likelihood to the whole growth-rate distribution.

In order to assess the heaviness of growth-rate tails from a non-parametric perspective, we apply the Hill estimator (Hill, 1975), which has become a standard tool as far as the study of tail behavior of economic data is concerned. Since growth-rate distributions studied here appear to be sufficiently symmetric (see also Section 4.3), we compute the Hill estimator (HE) on $R_{T_n} = \{r(t), t \in T_n\}$, where $r(t) = |g(t)|$. This allows us to increase the number of available tail observations. The inverse of the HE is thus defined as

$$\widehat{h}_k^{-1} = k^{-1} \sum_{i=1}^{k} [\log(r_{n-i+1}) - \log(r_{n-k})] \tag{4}$$

where elements in R_{T_n} are sorted in descending order, i.e., $r_n \geq r_{n-1} \geq \cdots \geq r_1$, and k is the number of tail observations employed in the estimation of tail behavior. Of course, the HE crucially depends on the choice for k. To select the most appropriate value for this parameter (k^*), many statistical procedures have been proposed (see Lux, 2001, for an excellent review with applications). Here we report results obtained by using the method developed by Drees and Kaufmann (1998). Similar findings were obtained by employing the bootstrap procedure discussed

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in Danielsson *et al.* (2001).[12] As Table XII shows, the HE returns values always greater than 2 and smaller than 3. Denmark, Spain and USIP1921 series display the lowest tail-index values. Overall, Hill estimates confirm the emergence of fatness in growth-rate tails. Furthermore, in line with our previous results, point estimates suggest finiteness of the first two moments of growth distributions.[13] Note, however, that according to our parametric EP fits, the entire growth-rate distributions can be satisfactorily characterized by a fat-tailed density that preserves the existence of *all the moments*.[14]

Given the above findings, we now turn to additional parametric exercises involving fat-tailed densities different from the EP. We want to answer two related questions: (i) Is the main result on fat-tails emergence affected by the particular choice of the density used in maximum-likelihood estimations? (ii) How do alternative fat-tailed densities, possibly characterized by tails even fatter than those of a super-Normal EP, perform as compared to the EP in characterizing the shape of growth-rate distributions? In what follows, we shall first employ the Student-t distribution, whose density reads

$$t(x;\lambda,\theta,\nu) = \frac{\Gamma\left(\dfrac{\nu+1}{2}\right)}{\theta\Gamma\left(\dfrac{\nu}{2}\right)\sqrt{\nu\pi}}\left[1+\nu^{-1}\left(\frac{x-\lambda}{\theta}\right)^2\right]^{-\frac{\nu+1}{2}} \tag{5}$$

Table XII. Hill tail-index estimator

Series	k*	$\widehat{h}_{k^*}$	Conf. int.
US GDP	77	2.4313	(1.8883, 2.9744)
US IP1921	70	2.1375	(1.6367, 2.6382)
US IP1947	108	2.3415	(1.8999, 2.7831)
Canada	91	2.2791	(1.8108, 2.7474)
Japan	46	2.2653	(1.6106, 2.9199)
Austria	69	2.7135	(2.0732, 3.3538)
Belgium	47	2.2180	(1.5839, 2.8521)
Denmark	109	2.0982	(1.7043, 2.4921)
France	73	2.5998	(2.0034, 3.1962)
Germany	54	2.3098	(1.6938, 2.9259)
Italy	49	2.4933	(1.7952, 3.1915)
Netherlands	72	2.7130	(2.0863, 3.3396)
Spain	96	2.0684	(1.6546, 2.4821)
Sweden	66	2.3388	(1.7746, 2.9031)
UK	85	2.3563	(1.8554, 2.8572)

Note: The Hill estimator ($\widehat{h}$) is computed on absolute values of growth-rate distributions. The optimal cut-off value k^* has been calculated using Drees and Kaufmann's algorithm (Dress and Kaufmann, 1998). Confidence intervals for $\widehat{h}_{k^*}$ have been obtained using asymptotic normality of $\widehat{h}^{-1}$.

[12] We are grateful to Thomas Lux for providing Gauss scripts that implement several existing algorithms for the automatic selection of k^*.
[13] Confidence intervals reported in Table XII are valid only asymptotically and may not be very informative for finite samples compared to the ones employed here.
[14] Despite the Hill estimator being routinely employed as a non-parametric estimate for the fatness of the tails of a distribution, it can be shown to be a maximum-likelihood estimator of the parameter ξ of the Pareto distribution, whose cumulated density is $F(x;\xi) = 1 - x^{-\xi}, x > 0$. Thus, if the (absolute value of the) underlying growth-rate distribution is not Pareto, as happens in our case, the interpretation of the properties of the Hill estimator becomes ambiguous from a parametric point of view.

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where λ is a location parameter, θ is a scale parameter and ν controls for the heaviness of tails (i.e., the degrees of freedom). Note that the larger ν, the thinner are the tails. In fact, as $\nu \to \infty$, tails (slowly) converge to those of a Gaussian. For finite values of ν, the Student-t exhibits instead a power-law tail behavior (i.e., fatter than those of a super-Normal EP).

We shall also fit our data with the Cauchy distribution:

$$C(x; \rho, \varphi) = \frac{1}{\varphi\pi}\left[1 + \left(\frac{x-\rho}{\varphi}\right)^2\right]^{-1} \tag{6}$$

where again ρ is a location parameter and φ is a scale parameter. Since we have that

$$t(x; \lambda, \theta, 1) = C(x; \lambda, \theta) \tag{7}$$

it turns out that the Cauchy can be considered as a special case of a Student-t (with extremely heavy tails).

Finally, we employ the family of Lévy-Stable (or simply 'Stable') distributions, popularized by Nolan (2006).[15] The Lévy-Stable is a four-parameter family of distributions with density

$$S(x; \alpha, \beta, \gamma, \delta) \tag{8}$$

The only condition that a random variable must obey to be a Stable one is that its shape must be preserved under addition, up to scale and shift. Two important remarks are in order. First, the Stable distribution is the only possible non-trivial limit of a normalized sum of independent, identically distributed terms. This result is known as the 'generalized' central limit theorem, because it extends the standard central limit theorem by dropping the finite-variance assumption. Second, the density of a Stable random variable cannot be generally given in closed form. Exceptions are the Gaussian and the Cauchy distributions, which belong to the Stable family. In fact, it can be shown that the parameter α works in the same way b does for the EP distribution: if $\alpha = 2$ we recover the Gaussian distribution, while if $\alpha = 1$ the Stable family boils down to a Cauchy (the densities of Lévy-Stable distributions also have closed-form expressions when $\alpha = 1/2$ and $\beta = \pm 1$). Unfortunately, the Laplace is not a Stable distribution. This prevents us from comparing Stable fits thoroughly with EP ones. Moreover, the parameter β controls for the skewness: the distribution is symmetric if $\beta = 0$. Finally, δ controls for location and γ for the scale.

Tables XIII, XIV and XV report the results of our fitting exercises. Given the robustness results obtained above, we go back to our original growth-rate series and we estimate density parameters via ML.[16] Furthermore, we perform GoF tests and we estimate p-values by bootstrapping the GoF test statistics under the null hypothesis that data come from the fitted distribution.

It is easy to see (Table XIII) that, according to GoF tests, the Student-t distribution does not satisfactorily fit the data, especially as far as OECD IP growth rates are concerned. However, the estimates of the 'degrees of freedom' parameter $\widehat{\nu}$ are quite small, suggesting the presence of fat tails. Note also that the estimates of $\widehat{\nu}$ imply tail-index parameters always belonging to the (2, 4) interval. This is in line with the empirical literature on power laws in economic and financial time

[15] See also Zolotarev (1986) and Uchaikin and Zolotarev (1999) for previous works on stable distributions.

[16] To fit stable densities to the data we employed the package provided by John Nolan (see http://academic2.american.edu/~jpnolan/stable/stable.html).

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Table XIII. Fitting a Student-t distribution: estimated parameters and GoF tests

Series	$\widehat{\lambda}$	$\widehat{\theta}$	$\widehat{\nu}$	KSM	KUI	CVM	AD2
US GDP	0.0085 (0.0594)	0.0084 (0.0693)	2.2513 (1.8039)	0.8903 (0.3961)	1.7803 (0.0408)	0.2843 (0.1454)	2.2639 (0.0652)
US IP1921	0.0031 (0.0171)	0.0083 (0.0206)	1.5685 (0.1425)	0.8643 (0.4480)	1.6801 (0.0715)	0.1228 (0.4906)	0.8533 (0.4492)
US IP1947	0.0028 (0.0296)	0.0076 (0.0324)	2.4727 (0.4114)	1.7118 (0.0050)	3.0975 (0.0000)	0.8816 (0.0029)	6.2509 (0.0004)
Canada	0.0022 (0.0582)	0.0102 (0.0641)	1.5223 (0.6816)	1.4331 (0.0306)	2.6902 (0.0000)	0.6427 (0.0159)	5.9064 (0.0009)
Japan	0.0023 (0.0425)	0.0233 (0.0545)	1.6188 (0.4339)	3.8494 (0.0000)	7.4333 (0.0000)	6.3625 (0.0000)	36.7449 (0.0000)
Austria	0.0018 (0.0538)	0.0200 (0.0560)	2.8749 (1.6001)	3.0482 (0.0000)	6.0317 (0.0000)	4.1182 (0.0000)	25.3073 (0.0000)
Belgium	0.0027 (0.0462)	0.0257 (0.0499)	2.0767 (0.5859)	4.0784 (0.0000)	7.7385 (0.0000)	6.8927 (0.0000)	39.1799 (0.0000)
Denmark	0.0022 (0.0480)	0.0230 (0.0523)	2.2737 (0.7732)	3.6595 (0.0000)	7.0501 (0.0000)	5.7041 (0.0000)	33.4318 (0.0000)
France	0.0013 (0.0565)	0.0115 (0.0698)	1.8036 (0.7438)	1.6605 (0.0081)	3.0668 (0.0000)	1.0403 (0.0009)	7.9628 (0.0001)
Germany	0.0023 (0.0457)	0.0134 (0.0493)	2.1329 (0.6327)	2.0276 (0.0003)	3.7465 (0.0000)	1.4933 (0.0002)	10.9291 (0.0000)
Italy	0.0011 (0.0499)	0.0228 (0.0555)	2.3389 (0.9017)	3.5696 (0.0000)	6.9689 (0.0000)	5.6170 (0.0000)	32.9150 (0.0000)
Netherlands	0.0021 (0.0513)	0.0210 (0.0510)	2.7550 (1.0626)	3.2766 (0.0000)	6.3648 (0.0000)	4.5994 (0.0000)	27.8259 (0.0000)
Spain	0.0014 (0.0573)	0.0358 (0.0615)	1.9141 (0.2811)	4.8194 (0.0000)	9.5354 (0.0000)	10.2404 (0.0000)	54.5191 (0.0000)
Sweden	0.0019 (0.0388)	0.0163 (0.0388)	2.3133 (0.6513)	2.5150 (0.0000)	4.9236 (0.0000)	2.7009 (0.0000)	17.8114 (0.0000)
UK	0.0017 (0.0505)	0.0101 (0.0508)	2.6640 (1.0157)	0.8612 (0.4500)	1.6314 (0.0898)	0.2451 (0.1885)	2.2372 (0.0695)

Note: Standard errors of estimates and p-values of GoF tests are in parentheses. GoF tests refer to the null hypothesis that data come from a Student-t distribution with parameters $(\widehat{\lambda}, \widehat{\theta}, \widehat{\nu})$. KSM, Kolmogorow–Smirnov (D) test; KUI, Kuiper (V) test; CVM, Cramér–Von Mises (W^2) test; AD2, Anderson–Darling quadratic (A^2) test. Test statistics are adjusted for small-sample bias according to D'Agostino and Stephens (1986, Table IV.2, p. 105). Exact p-values are estimated by bootstrapping the distribution of the test statistics under the null hypothesis that data come from a Student-t distribution with parameters $(\widehat{\lambda}, \widehat{\theta}, \widehat{\nu})$, with econometric sample sizes equal to those of the empirical time series. Bootstrap sample size: $M = 10,000$.

Table XIV. Fitting a Cauchy distribution: estimated parameters and GoF tests

Series	$\widehat{\rho}$	$\widehat{\phi}$	KSM	KUI	CVM	AD2
US GDP	0.0083 (0.0005)	0.0050 (0.0004)	0.8846 (0.4159)	1.7276 (0.0545)	0.1256 (0.4713)	1.8003 (0.1215)
US IP1921	0.0031 (0.0003)	0.0065 (0.0003)	0.9394 (0.3251)	1.8606 (0.0232)	0.1494 (0.3800)	2.5632 (0.0435)
US IP1947	0.0032 (0.0003)	0.0045 (0.0002)	1.3302 (0.0600)	2.5820 (0.0000)	0.4523 (0.0507)	5.2932 (0.0016)
Canada	0.0023 (0.0006)	0.0066 (0.0005)	1.2348 (0.0938)	2.3886 (0.0004)	0.3211 (0.1115)	3.6149 (0.0134)
Japan	0.0018 (0.0014)	0.0167 (0.0013)	3.2014 (0.0000)	6.3667 (0.0000)	4.5965 (0.0000)	28.3805 (0.0000)
Austria	0.0010 (0.0012)	0.0132 (0.0010)	2.6382 (0.0000)	5.0762 (0.0000)	2.9971 (0.0000)	19.9725 (0.0000)
Belgium	0.0022 (0.0018)	0.0186 (0.0014)	3.5606 (0.0000)	6.9658 (0.0000)	5.4883 (0.0000)	32.7753 (0.0000)
Denmark	0.0016 (0.0014)	0.0157 (0.0012)	3.0038 (0.0000)	6.0214 (0.0000)	4.1113 (0.0000)	25.9262 (0.0000)
France	0.0011 (0.0006)	0.0067 (0.0005)	1.4089 (0.0355)	2.6171 (0.0000)	0.6079 (0.0211)	5.0267 (0.0025)
Germany	0.0027 (0.0009)	0.0096 (0.0007)	1.8818 (0.0009)	3.6053 (0.0000)	1.1116 (0.0009)	9.4588 (0.0000)
Italy	0.0001 (0.0014)	0.0155 (0.0012)	3.2657 (0.0000)	5.9850 (0.0000)	4.3901 (0.0000)	26.9120 (0.0000)
Netherlands	0.0033 (0.0014)	0.0147 (0.0011)	3.1575 (0.0000)	5.7933 (0.0000)	3.6824 (0.0000)	23.6593 (0.0000)
Spain	0.0031 (0.0020)	0.0220 (0.0017)	4.1418 (0.0000)	7.8756 (0.0000)	7.0009 (0.0000)	39.9212 (0.0000)
Sweden	0.0020 (0.0011)	0.0117 (0.0009)	2.2684 (0.0001)	4.4879 (0.0000)	2.0555 (0.0000)	15.0456 (0.0000)
UK	0.0022 (0.0006)	0.0070 (0.0005)	1.2959 (0.0690)	2.5191 (0.0000)	0.3446 (0.1015)	3.9675 (0.0097)

Note: Standard errors of estimates and p-values of GoF tests are in parentheses. GoF tests refer to the null hypothesis that data come from a Cauchy distribution with parameters $(\widehat{\rho}, \widehat{\varphi})$. KSM, Kolmogorow–Smirnov (D) test; KUI, Kuiper (V) test; CVM, Cramér–Von Mises (W^2) test; AD2, Anderson–Darling quadratic (A^2) test. Test statistics are adjusted for small-sample bias according to D'Agostino and Stephens (1986, Table IV.2, p. 105). Exact p-values are estimated by bootstrapping the distribution of the test statistics under the null hypothesis that data come from a Cauchy distribution with parameters $(\widehat{\rho}, \widehat{\varphi})$, with econometric sample sizes equal to those of the empirical time series. Bootstrap sample size: $M = 10,000$.

series (see, for example, Loretan and Phillips, 1994; Gabaix *et al.*, 2003). The Student-t is, on the contrary, a good choice for US GDP and, to some extent, for the IP1921 series. Similar evidence is obtained by fitting a Cauchy distribution (see Table XIV). Again, GoF p-values are overall poor (apart from US GDP and IP1921), suggesting that the Student-t parameterization $\widehat{\nu} = 1$ is not a good one to fit our data.

Finally, Table XV reports the results from Stable fits. Notice first that GoF tests slightly improve, indicating that the Lévy-Stable distribution does a better job compared to the Student-t and the Cauchy. The latter result is not surprising given that the Cauchy density is a particular case of a Lévy-Stable. Nevertheless, the Lévy-Stable seems to be outperformed by the EP (compare Table XV with Tables III and VII).[17] More importantly, values of $\widehat{\alpha}$ are always between 1 and 2, strongly indicating the presence of medium tails in all distributions. This implication is further supported by standard errors of estimates (in parentheses), which are quite small for all four parameters.

We can then confidently conclude that fat tails robustly arise independently of the particular density employed. Yet the EP seems to outperform all the other three density families in describing our growth-rate data. This in turn implies that growth-rate distributions have finite moments of any order. In other words, despite their fat-tailedness, the distributions of growth-rate series remain 'well behaved' from a statistical point of view.

[17] This casts serious doubts on whether some sort of generalized central limit theorem is in place. We shall come back to this point in Section 5.

Table XV. Fitting a Lévy-stable distribution: estimated parameters and GoF tests

Series	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\gamma}$	$\widehat{\delta}$	KSM	KUI	CVM	AD2
US GDP	1.5220 (0.0918)	−0.1790 (0.3254)	0.0055 (0.0004)	0.0084 (0.0007)	0.9343 (0.3432)	1.1846 (0.5346)	0.1075 (0.5496)	0.8347 (0.4580)
US IP1921	1.1990 (0.0467)	0.0060 (0.0724)	0.0067 (0.0003)	0.0031 (0.0004)	0.5195 (0.9494)	1.0204 (0.7628)	0.0472 (0.8856)	0.6460 (0.6849)
US IP1947	1.4840 (0.0584)	−0.0550 (0.1287)	0.0047 (0.0002)	0.0030 (0.0003)	0.7817 (0.6178)	1.1429 (0.6569)	0.0761 (0.7387)	0.6186 (0.6899)
Canada	1.7760 (0.0448)	−0.0460 (0.0469)	0.0074 (0.0003)	0.0020 (0.0007)	0.6699 (0.7374)	1.0089 (0.7962)	0.0647 (0.7552)	0.3945 (0.8458)
Japan	1.2290 (0.0899)	0.0800 (0.1463)	0.0175 (0.0013)	0.0016 (0.0019)	3.4767 (0.0010)	6.6273 (0.0004)	5.0419 (0.0008)	30.2501 (0.0582)
Austria	1.7990 (0.0763)	0.6080 (0.3572)	0.0149 (0.0008)	0.0000 (0.0016)	2.9260 (0.0010)	5.6914 (0.0007)	3.6128 (0.0004)	22.1102 (0.0123)
Belgium	1.4300 (0.0917)	−0.0750 (0.1896)	0.0204 (0.0012)	0.0004 (0.0021)	4.0941 (0.0003)	7.5255 (0.0008)	6.7418 (0.0001)	38.1219 (0.0323)
Denmark	1.4070 (0.0895)	0.1260 (0.2318)	0.0161 (0.0011)	−0.0006 (0.0019)	3.4170 (0.0007)	6.2171 (0.0002)	4.6578 (0.0004)	27.6751 (0.0359)
France	1.5520 (0.0595)	0.0520 (0.5995)	0.0079 (0.0004)	−0.0001 (0.0008)	1.9064 (0.0001)	2.4177 (0.0005)	1.1587 (0.0005)	6.0807 (0.0147)
Germany	1.5500 (0.0902)	−0.3500 (0.1971)	0.0104 (0.0006)	0.0032 (0.0011)	1.6486 (0.0115)	3.2940 (0.0010)	1.0430 (0.0003)	7.6443 (0.0153)
Italy	1.4740 (0.0890)	0.1320 (0.2277)	0.0165 (0.0010)	0.0003 (0.0019)	3.2672 (0.0001)	6.3374 (0.0010)	4.6340 (0.0005)	27.7981 (0.0315)
Netherlands	1.7830 (0.0850)	−0.3320 (0.2659)	0.0167 (0.0009)	0.0017 (0.0017)	3.3676 (0.0009)	6.3382 (0.0007)	4.6194 (0.0009)	27.4775 (0.0079)
Spain	1.7820 (0.0632)	−0.0410 (0.5152)	0.0255 (0.0013)	0.0025 (0.0026)	4.5076 (0.0008)	8.7702 (0.0005)	8.7814 (0.0009)	47.8478 (0.0060)
Sweden	1.5860 (0.0839)	−0.0680 (0.2915)	0.0127 (0.0007)	0.0014 (0.0014)	2.4971 (0.0005)	4.6093 (0.0005)	2.3793 (0.0005)	15.4577 (0.0203)
UK	1.7510 (0.0832)	−0.9500 (0.2288)	0.0079 (0.0004)	0.0030 (0.0008)	0.9512 (0.3153)	1.2665 (0.4096)	0.2025 (0.2518)	1.6222 (0.1574)

Note: Standard errors of estimates and p-values of GoF tests are in parentheses. GoF tests refer to the null hypothesis that data come from a Lévy-Stable distribution with parameters $(\widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}, \widehat{\delta})$, under the S_0 parametrization. KSM, Kolmogorow–Smirnov (D) test; KUI, Kuiper (V) test; CVM, Cramér–Von Mises (W^2) test; AD2, Anderson–Darling quadratic (A^2) test. Test statistics are adjusted for small-sample bias according to D'Agostino and Stephens (1986, Table IV.2, p. 105). Exact p-values are estimated by bootstrapping the distribution of the test statistics under the null hypothesis that data come from a Lévy-Stable distribution with parameters $(\widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}, \widehat{\delta})$, under the S_0 parametrization, with econometric sample sizes equal to those of the empirical time series. Bootstrap sample size: $M = 10,000$.

Table XVI. Checking for asymmetry in growth-rate distributions: D'Agostino skewness test, asymmetric EP fits and likelihood ratio tests

Series	D'Agostino skewness test		$\widehat{b}_l$	$\widehat{b}_r$	Likelihood ratio test	
	Statistic	p-Value			Statistic	p-Value
US GDP	−0.3770	0.7062	—	—	—	—
US IP 1921	2.9369	0.0033	0.7441 (0.0490)	0.6639 (0.0423)	7.6122	0.1789
US IP 1947	2.3147	0.0206	1.1372 (0.1101)	0.8549 (0.0810)	6.5632	0.2552
Canada	−1.0673	0.2858	—	—	—	—
Japan	−1.0370	0.2997	—	—	—	—
Austria	0.7905	0.4292	—	—	—	—
Belgium	−2.4857	0.0129	0.7427 (0.0993)	1.3424 (0.2042)	7.4858	0.1869
Denmark	0.5642	0.5726	—	—	—	—
France	0.7073	0.4794	—	—	—	—
Germany	0.0458	0.9635	—	—	—	—
Italy	0.2110	0.8329	—	—	—	—
Netherlands	−0.1630	0.8705	—	—	—	—
Spain	1.1755	0.2398	—	—	—	—
Sweden	−1.3512	0.1766	—	—	—	—
UK	−0.7560	0.4497	—	—	—	—

Note: D'Agostino skewness test (D'Agostino, 1970) is rejected only for US IP1921, US IP1947, and Belgium. Asymmetric EP fits: standard errors of estimates $\sigma(\widehat{b}_l)$ and $\sigma(\widehat{b}_r)$ are in parentheses. Likelihood ratio tests (LRT) refer to the null hypothesis that data come from an asymmetric EP density (see equation (9)) wherein parameters are restricted to be homogeneous and equal to maximum-likelihood estimates computed for symmetric EP fits (see Tables II and VI), i.e., $\widehat{b}_l = \widehat{b}_r = \widehat{b}, \widehat{a}_l = \widehat{a}_r = \widehat{a}$ and $\widehat{u} = \widehat{m}$. p-Values for the LRT are computed using the fact that LRT $\rightarrow \chi^2(5)$.

4.3. Skewness and Asymmetric EP Fits

Both descriptive statistics and estimates of the symmetry parameter (β) for the Stable density have suggested the presence of some skewness in growth-rate distributions. In our previous analyses, conversely, we have always employed a symmetric EP (equation (3)). In what follows, then, we test whether our results are robust to fitting *asymmetric* EP densities, whenever significant skewness in the data is detected.

We begin by performing the D'Agostino skewness test (D'Agostino, 1970) on both US and OECD data.[18] As Table XVI shows, only three series display statistically significant skewness levels: US IP1921, US IP1947, and Belgium. It should be noted that this evidence in favor of an overall absence of skewness is in line with recent results showing some symmetry in the magnitude of expansions and recessions of business cycles (see McKay and Reis, 2006).

We fit the data of the three, apparently skewed, series with an asymmetric EP distribution (Bottazzi and Secchi, 2006b), whose density is given by

$$g(x; a_l, a_r, b_l, b_r, u) = \begin{cases} K^{-1} e^{-\frac{1}{b_l}\left|\frac{x-u}{a_l}\right|^{b_l}}, & x < u \\ K^{-1} e^{-\frac{1}{b_r}\left|\frac{x-u}{a_r}\right|^{b_r}}, & x \geq u \end{cases} \quad (9)$$

[18] The D'Agostino skewness test performs quite well in detecting departures from symmetry for given values of kurtosis, even if the distribution is not Gaussian.

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where $K = a_l b_l^{1/b_l}\Gamma(1+1/b_l) + a_r b_r^{1/b_r}\Gamma(1+1/b_r)$. Note that in the asymmetric EP density the parameters b_l and b_r allow for different tail fatness levels on the right and on the left of the mean u, respectively. Right and left scaling is instead controlled by the parameters a_l and a_r.

As Table XVI reports, estimates of b_l and b_r seem actually to differ. In the case of US IP1921, both coefficients indicate super-Laplacian tails, but positive growth events seem to be more likely than negative ones. After WWII, on the contrary, IP growth becomes almost Laplacian as far as negative jumps are concerned, whereas positive growth rates seem to be super-Laplacian. A different story holds for Belgium, where negative-growth large events are far more likely than positive ones.

In order to further explore the robustness of the evidence conveyed by the analysis of parameter estimates, we performed likelihood-ratio tests to check for the null hypothesis that data come from an *asymmetric* EP that has been forced to have *symmetric* parameters, the latter being equal to those obtained in our symmetric EP exercises.[19] Table XVI (last two columns) shows that, despite estimates of the shape coefficients differ, there is no gain whatsoever in fitting an asymmetric density to our data. This happens in all three cases where the D'Agostino skewness test detected some asymmetry in growth-rate distributions. Indeed, the improvement in the goodness of fit does not counterbalance either the larger degrees of freedom or the ensuing increase in the standard deviation of estimates, both implied by an asymmetric fit.

4.4. Increasing Growth-Rate Time Lags

As mentioned above, our work departs from existing studies also because we employ monthly and quarterly data. While this choice might allow us to better appreciate the business-cycle features of growth-rate distributions, it might also generate a potential problem. Indeed, lumpiness in growth events might simply depend on the fact that we have considered output data at a too high frequency, taking on board, for instance, temporary measurement errors. The question then becomes: What happens when one computes growth rates at different (increasing) time lags?

To explore this issue, we inspect the distribution of output growth rates where the latter are now defined as

$$g_\tau(t) = \frac{Y(t) - Y(t-\tau)}{Y(t-\tau)} \cong y(t) - y(t-\tau) = (1 - L^\tau)y(t) \tag{10}$$

where $\tau = 1, 2, \ldots, 6$ when GDP (quarterly) series are employed, and $\tau = 1, 2, \ldots, 12$ when IP (monthly) series are under study.

In line with the results that Bottazzi and Secchi (2006a) report for firm growth rates, we find that the shape parameter becomes higher as τ increases (see, for the US GDP case, the left panel of Figure 7). When we consider the US IP1947 series, $\widehat{b}$ first falls and then starts rising (see the right panel of Figure 7). Therefore, as the 'growth lag' increases, tails become slightly thinner; see Silva *et al.* (2004) for similar evidence in the context of stock returns. Nevertheless, estimated shape coefficients remain significantly smaller than 2, especially for the US IP1947 series. Interestingly,

[19] More precisely, we compute the likelihood of an asymmetric density (9) with a null hypothesis given by: $\widehat{b}_l = \widehat{b}_r = \widehat{b}, \widehat{a}_l = \widehat{a}_r = \widehat{a}$ and $\widehat{u} = \widehat{m}$; i.e., we force the shape and scale parameters of the asymmetric EP to be equal to the corresponding ML estimate of the symmetric distribution.

the lag-4 US IP growth-rate distribution exhibits super-Laplacian tails. Analogous findings were obtained also for the other OECD countries. This means that, even if one considers longer time spans, big growth events remain more likely than what a Gaussian model would predict.

5. IMPLICATIONS

The foregoing evidence brings strong support to the claim that fat tails are an extremely robust stylized fact characterizing the time series of aggregate-output growth in most industrialized economies. In addition, it shows that EP densities provide better fit to the observed data than other classes of distributions that exhibit even heavier tails (e.g., the Lévy-Stable family). This has several implications, from both empirical and theoretical perspectives.

From an empirical perspective, the very emergence of fat-tailed distributions for within-country time series of both growth rates and residuals (see Section 4.1) can be interpreted as a candidate additional stylized fact of within-country output dynamics. Furthermore, this finding confirms from a time-series point of view what seems to be a general property of cross-section growth-rate distributions. As mentioned, fat tails have indeed been discovered to be the case not only for cross-sections of countries, but also for plants, firms and industries (see Stanley *et al.*, 1996; Lee *et al.*, 1998; Amaral *et al.*, 1997; Bottazzi and Secchi, 2003a, 2003b; Castaldi and Dosi, 2004; Fu *et al.*, 2005; Sapio and Thoma, 2006; among others). The general hint coming from this stream of literature is in favor of an increasingly 'non-Gaussian' economics and econometrics. A consequence of this suggestion is that one should be very careful in using econometric estimation and testing procedures that are heavily sensitive to normality of residuals. On the contrary, estimation and testing procedures that are robust to non-Gaussian errors and/or estimators and tests based on fat-tailed errors should be employed when necessary.[20] For instance, if errors follow a Laplace and are independently and identically distributed, the maximum likelihood estimator is the so-called 'least

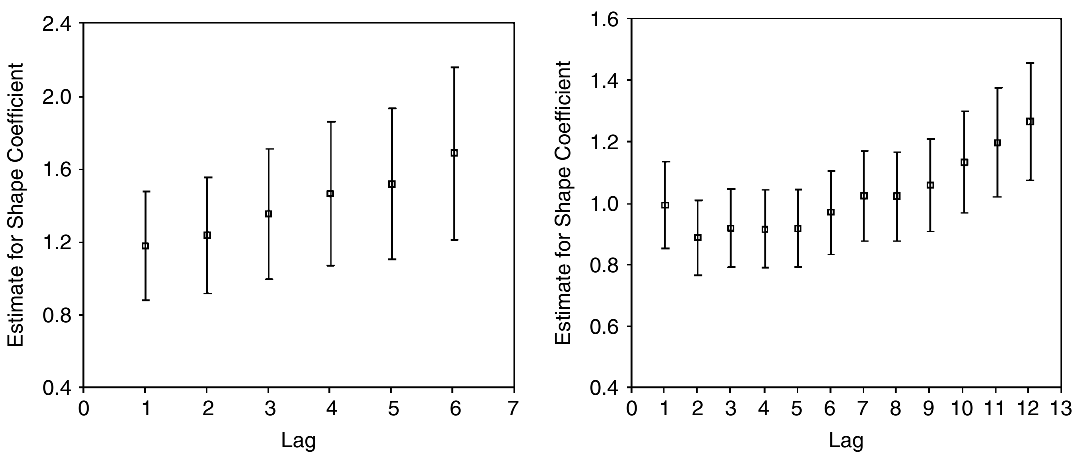

Figure 7. Increasing the time lag in the computation of growth rates. Estimates of the shape coefficient (b). Left: US GDP. Right: US IP1947. Bars represent Cramér–Rao intervals ($\widehat{b} \pm 2\sigma(\widehat{b})$)

[20] See Temple (1998), Zaman *et al.* (2001), Bottazzi *et al.* (2005) and Loretan and Phillips (1994) for specific applications of econometric techniques robust to non-Gaussian errors.

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absolute deviations' (LAD) estimator, instead of the commonly employed 'ordinary least squares' (OLS) estimator. Examples of applications here range from unit-root modeling of aggregate output (e.g., Nelson and Plosser, 1982), to Gibrat-like regressions for the dependence of firm growth on size (Sutton, 1997) and cross-section country growth-rate analyses (see, for example, Barro and Sala-i-Martin, 1992; see also Durlauf *et al.*, 2005, for a critical review).

From a theoretical perspective, our findings may pose both an opportunity and a challenge to modelers. On the one hand, the emergence of fat-tailed distributed growth rates suggests one should go for models that are able to replicate not only the first two moments of output growth distributions, but also higher ones. On the other hand, the hypothesis of fat-tailed distributed growth shocks might directly be embodied in economic models. This might be an important step forward, because, as Ibragimov (2005) shows, the implications of many models in economics and finance are very sensitive to the thickness of the tails of the distributions involved in their assumptions. For instance, the property that diversification is always preferable in portfolio 'value-at-risk' analysis (Dempster, 2002) no longer holds under very thick-tailed distributed risks. Likewise, optimal bundling strategies in multiproduct monopolistic models (Schmalensee, 1984) or the properties of firms' size dynamics in demand-driven innovation models (Jovanovic and Rob, 1987) are not robust to the degree of heavy-tailedness in the distributions of, respectively, consumer preferences and product design shocks. As mentioned, the fact that fat-tailed EP densities provide a good approximation of growth-rate data indicates that all the moments of growth-rate distribution exist and are finite. This might be good news for all the models requiring the specification of the shape of growth-rate distributions among their assumptions, as typically their implications may change in the presence of heavy-tailed distributions that do not allow for finite mean and variance.

Furthermore, gaining some knowledge on the shape of the country-level output growth-rate distributions may provide some hints on its generating process. For instance, suppose one interprets the country-level output growth rate in a certain time period as the result of the propagation of i.i.d. shocks. The emergence of non-Gaussian distributions at the country level strongly militates against the idea that the mechanisms of shock propagation, which drive country-growth dynamics, fulfil the conditions of the central limit theorem. In addition, our evidence shows that a Lévy-Stable density is not in general a good proxy of country growth-rate distributions (see Section 4.2). This implies that not even a generalized version of the central limit theorem (where the finiteness of the variance of shocks is dropped) seems to govern aggregation of shocks in our data. It is worth remarking that this property is independent of the particular shock dynamics one assumes in the model. That is, it refers to time-series propagation of macroeconomic shocks (as in dynamic stochastic general equilibrium models, see, for example, King and Rebelo, 1999; Clarida *et al.*, 1999; Forni and Lippi, 1999) as well as to the propagation of microeconomic (firm- or industry-level) shocks (as in models with locally interacting agents, see Durlauf, 1993; Scheinkman and Woodford, 1994; Horvath, 1998). In addition, the fact that fat tails characterize the shape of growth-rate distributions both cross-sectionally and time-series—and at very different aggregation levels—corroborates the 'intriguing possibility that similar mechanisms are responsible for the observed growth dynamics of, at least, two complex organizations: firms and countries' (Lee *et al.*, 1998, p. 3275).[21]

It must be stressed, however, that such distributional findings relate to 'unconditional' distributions (Brock, 1999). This has two implications. First, making inference on the generating mechanism responsible for fat tails at different aggregation levels becomes an extremely difficult task: many data-generating processes can induce such distributions in the limit. Nonetheless, since

[21] And, in fact, industries as well (see Sapio and Thoma, 2006).

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not *every* data-generating process is compatible with Laplace-distributed growth shocks, our findings might place a first restriction on the set of possible models. This may hopefully help one to discriminate among different theories (e.g., business cycle explanations). Second, and relatedly, our findings could simply be due to the fact that some of the factors driving country growth (or a combination of them) are characterized by a fat-tailed distribution. In other words, the emergence of fat tails in output growth-rate series might be the sheer outcome of not taking into account a fully specified model of country growth. It follows that a fascinating challenge involves the attempt to shed more light on the distributional features of the growth determinants that are responsible for the emergence of fat-tailed distributions at the country level.[22]

6. CONCLUDING REMARKS

In this paper, we have investigated the statistical properties of GDP and industrial production (IP) growth-rate distributions in OECD countries by employing monthly and quarterly time-series data.

We have found that such distributions appear to be well approximated by a symmetric exponential power (EP) density, with tails much fatter than Gaussian ones (but with finite moments of any order). Hence, in the last century, large 'growth events' have been more likely than what one would have expected under a Gaussian model. We have shown that lumpiness of growth patterns robustly emerges independently of: (i) the way we measure output (GDP or IP); (ii) the family of density employed in the ML estimation; and (iii) the length of time lags used to compute growth rates. Furthermore, we have shown that fat tails characterize growth rates even after one washes away outliers, autocorrelation and heteroscedasticity (if any). Finally, we did not find any strong evidence in favor of asymmetric growth-rate distributions.

Our work can be extended in at least two ways. First, we have intrinsically assumed time invariance of the underlying generating mechanism governing output dynamics. Conversely, many studies indicate some evidence towards rejecting the assumption of temporal homogeneity of per capita GDP time series over long time spans (Balke and Fomby, 1991; Gaffeo *et al*., 2005). More specifically, several contributions seem to suggest the presence of structural breaks in the mean growth rate of the USA and of other OECD countries (Stock and Watson, 1999; Ben-David *et al*., 2003). Moreover, the recent debate on the 'Great Moderation' (i.e., the decline in output volatility observed since the end of the 1980s) suggests that also the second moment of GDP growth rates may not be stable over time.[23] There are two lines of future research that could contribute to a better understanding of the possible non-stationary nature of the GDP generating processes. One could study more carefully the robustness of fat tails in growth-rate distributions over distinct time spans. Alternatively, regime-switching models (Hamilton, 1989; Potter, 1995) could be employed in order to deal with potential nonlinearities.

Second, one might check whether the application of different detrending filters on output growth-rate distributions might have a significant impact on our results. As pointed out by Canova (1998), different detrending methods do indeed affect business cycle stylized facts. One could then apply to the output time series the most common filters employed in the business cycle literature (e.g., the Hodrick–Prescott and bandpass filters) and study the ensuing output growth-rate distributions.

[22] Such an issue could, for example, be addressed through robust growth-regression techniques accounting for the problems discussed in Temple (2000).

[23] See, among a growing body of literature, McConnell and Perez-Quiros (2000); Blanchard and Simon (2001); Stock and Watson (2002); Kim *et al*. (2004).

Similarly, one might use a bandpass filter to straightforwardly isolate different frequency bands on growth-rate time series, and then investigate which frequency intervals are more conducive to fat tails.

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