

# Multi-Agent Approach to the Self-Organization of Networks<sup>1</sup>

Frank Schweitzer

Chair of Systems Design, ETH Zurich, Kreuzplatz 5, 8032 Zurich, Switzerland

[fschweitzer@ethz.ch](mailto:fschweitzer@ethz.ch)

## 1 Self-Organization of Networks

*Self-organization* denotes “the process by which individual subunits achieve, through their cooperative interactions, states characterized by new, emergent properties transcending the properties of their constitutive parts.” (Biebricher *et al.*, 1995) Whether these emergent properties occur or not depends of course not only on the properties of the subsystems and their interactions, but also on suitable external conditions, such as global boundary conditions, the in/outflux of resources (free energy, matter, or information). A description that tries to include these conditions is given by the following heuristic definition: “Self-organization is defined as spontaneous formation, evolution and differentiation of complex order structures forming in non-linear dynamic systems by way of feedback mechanisms involving the elements of the systems, when these systems have passed a critical distance from the statical equilibrium as a result of the influx of unspecific energy, matter or information.” (Teichmann and Wilke, 1996)

In this sense, the self-organized structure formation can be considered as the opposite of a hierarchical design of structures which basically proceeds *from top down to bottom*: here, structures are *originated* bottom up, leading to an emerging *hierarchy*, where the structures of the “higher” level appear as a new quality of the system (Darley, 1994; Haken, 1978). For the prediction of these global qualities from local interactions fundamental limitations exist which are discussed e.g. in chaos theory. Moreover, stochastic fluctuations also give unlikely events a certain chance to occur, which in turn affects the real history of the system. This means, the properties of complex systems cannot be determined by a hierarchy of conditions, the system creates its complexity in the course of evolution with respect to its global constraints. Considering that also the boundary conditions may evolve and new degrees of freedom appear, *coevolutionary processes* become important, and the evolution may occur on a qualitatively new level.

The self-organization of *network structures*, i.e. the *emergence* of links between a set of nodes is of crucial importance in many different fields. In electronic engineering, for instance, one is interested in the *self-assembling* and *self-repairing* of electronic circuits (Cui and Lieber, 2001; Mange and Stauffer, 1994; Wang, 2001), while in biology models for the self-wiring of *neuronal networks* are investigated (Segev and Ben-Jacob, 1998, 2000). On the social level, the self-organization of human trail networks between different destinations is a similar problem

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(Helbing *et al.*, 1997). Also the establishment of connections on demand in telecommunication or logistics is related to the problem discussed here.

In general, the self-organized formation of a network is rather different from drawing lines between a set of nodes. Often no *a priori* information about the network structure is provided, i.e. the network has to self-organize itself not only regarding the links but also regarding the nodes. This is the case for instance if the nodes to be linked to the network are “unknown” in the sense, that they *first* have to be *discovered* and only *then* can be *connected*.

A common biological example is the formation of a trail system in ants (cf also Secs. 6, 7) to connect a nest to a set of food sources that first have to be found (Crist and Haefner, 1994; Schweitzer *et al.*, 1997). Such networks are known to be rather flexible and adaptive. After food sources are exhausted, they are “disconnected” from the existing network, because they are no longer visited and the respective trail is no longer maintained, but newly found food sources can be linked to the existing network as well. Hence, *adaptivity* is a desirable feature of self-organized networks. This means that new nodes can be linked to the existing network or linked nodes can be disconnected from the network if this is required, e.g. by the change of some external conditions.

Noteworthy, such a behavior should be not governed by a “supervisor” or “dispatcher”, it should rather result from the adaptive capabilities of the network itself. In the self-wiring of neural structures, for example, a neuron grows from the retina of the eye towards the optic tectum (or superior colliculus) of the brain, without “knowing” from the outset about its destination node in the brain. Hence it has to navigate through an unknown environment in order to detect and to reach the appropriate area. It is known that gradients of different chemical cues play a considerable role in this navigation process. They provide a kind of *positional information* for the navigation of the growth cones (Gierer, 1983). But in the very beginning, this positional information has to be generated interactively, and only in later stages may lead to established pathways.

This points to the questions that shall be discussed in this chapter: Is it possible to link a set of nodes without using preexisting positional information or any kind of long-range attraction of the nodes? Can the process of generating positional information, i.e. the detection of “unknown” nodes and the establishment of chemical gradients, *and* the process of network formation, i.e. the establishment of links between nodes, occur in parallel, on a comparable time scale, as a process of co-evolution?

In the following, we will discuss a model where the generation of relevant information for establishing the links between nodes results from the interaction of many *agents*, i.e. subunits of the system that are capable of performing some activities. Their collective interaction is based on (indirect) communication, which also includes memory effects and the dissemination of information in the system. The relevant (“pragmatic”) information that leads to the establishment of the links then emerges from an evolutionary interplay of selection and reamplification. In the

next section, we will introduce the general framework for modeling such processes, while in Secs. 3-7 two different applications of the model will demonstrate its suitability.

## 2 Brownian Agents

Recently, different computer architectures in distributed artificial intelligence have been developed to simulate the collective behavior of interacting agents (cf. for instance the SWARM project at <http://www.swarm.org/>). However, due to their rather complex simulation facilities many of the currently available simulation tools lack the possibility to investigate systematically and in depth the influence of specific interactions and parameters. Instead of incorporating only as much detail as is *necessary* to produce a certain emergent behavior, they put in as much detail *as possible*, and thus reduce the chance to understand *how* emergent behavior occurs and *what* it depends on.

Therefore, it would be feasible to have multi-agent systems (MAS) that can be also investigated by means of *analytical methods* (from statistical physics or mathematics) – in addition to their computational suitability. The concept of *Brownian agents* (Schweitzer, 2003) is one of the possible approaches to serve for this purpose. It denotes a particular class of agents that combines features of reactive and reflexive agent concepts. A reflexive agent has an (internal) model or at least some knowledge about its environment that allows its to draw conclusions about some certain actions in a given situation. The reactive agent, on the other hand, simply “reacts” to signals from the environment without referring to internal knowledge. This signal-response type of action implies the same reaction to the same signal. From this perspective, the reactive agent is a rather *simplex* or *minimalistic* agent, while the reflexive agent is a rather *complex* agent.

The *Brownian agent* (Schweitzer, 2003) is more advanced than the reactive agent but not yet of the internal complexity of the reflexive agent. It can be described by a set of state variables  $u_i^{(k)}$ . The index  $i = 1, \dots, N$  refers to the individual agent  $i$ , while  $k$  indicates the different variables. These could be either *external* variables that can be observed from the outside, or *internal degrees of freedom* that can only be indirectly concluded from observable actions. Important external variables are  $u_i^{(1)} = \mathbf{r}_i$ , which denotes the *space coordinate* (mostly a vector in the two-dimensional physical space), or  $u_i^{(2)} = \mathbf{v}_i$ , which is the individual *velocity* in the case of a moving agent. Both are assumed as *continuous* variables.

The internal degrees of freedom, on the other hand, cannot be directly observed. They could be continuous or discrete variables. For instance, the state variable  $u_i^{(3)} = \theta_i \in \{-1, 0, +1\}$  may distinguish between three different responses to certain environmental conditions or to incoming information. For example, agents with  $\theta = -1$  may not be affected by a particular signal, while agents with  $\theta = +1$  may respond to it. An important continuous state variable in the context of Brownian agents is the *internal energy depot*  $u_i^{(4)} = e_i$ , which determines whether agent  $i$  may perform a certain action or not. This includes the assumption that all actions – be it active

motion or communication or environmental changes – need to use “energy”. In general, this term describes not just the physical free energy that is dissipated e.g. during active motion, it intends to cover also other resources needed to perform a certain action.

Noteworthy, the different (external or internal) state variables can change in the course of time, either due to impacts from the surrounding, or due to an internal dynamics. Thus, in a most general way, we may express the dynamics of the different state variables as follows:

$$\frac{d u_i^{(k)}}{dt} = f_i^{(k)} + \mathcal{F}_i^{\text{stoch}} \quad (1)$$

For the Brownian agents, it is assumed that the causes for the temporal change of  $u_i$  may be described as a *superposition* of *deterministic* and *stochastic* influences, imposed on agent  $i$ . This picks up the ingenious idea first used by Langevin in early 1900 to describe the motion of *Brownian particles* – and is basically the reason why this agent concept is denoted as *Brownian agent*. A Brownian particle moves due to the impacts of the surrounding molecules whose motion however can be observed only on a much smaller time and length scale compared to the motion of the Brownian particle. Thus, Langevin invented the idea to sum up all these impacts in a stochastic force with certain statistical properties.

For the Brownian agent, we will exploit Langevins idea in a similar manner, i.e. we will sum up influences which may exist on a microscopic level, but are not observable on the time and length scale of the Brownian agent, in a stochastic term  $\mathcal{F}_i^{\text{stoch}}$ , while all those influences that can be directly specified on these time and length scales are summed up in a *deterministic* term  $f_i^{(k)}$ . Such a distinction basically defines the level of coarse-grained description for the multi-agent system. The “cut” may prevent us from considering too much “microscopic” details of the MAS, while focussing on particular levels of description. The summed up stochastic influences might result from a more fine-grained deterministic description – but instead of taking this into detailed account, just some specific statistical (gross) properties are considered on the coarse-grained level. Noteworthy, the strength of the stochastic influences may also vary for different agents and may thus depend on local parameters or internal degrees of freedom.

The *deterministic* part  $f_i^{(k)}$  contains all specified influences that cause changes of the state variable  $u_i^{(k)}$ . This could be non-linear interactions with other agents  $j \in N$ , external conditions - such as forces resulting from external potentials, or the in/outflux of resources etc. or even an *eigendynamics* of the system that does not depend on the action of the agents. In the example of an ecosystem this eigendynamics may describe day/night or seasonal cycles, or the agent-independent diffusion of resources within the system.

### 3 Basic Agent Model of Network Formation

In order to set up a Brownian multi-agent model for the formation of networks we need to specify (i) the relevant state variables  $u_i^{(k)}$  and (ii) the dynamics for changing them, i.e.  $\dot{u}_i^{(k)}$ . For the

application considered, we may assume that each Brownian agent is characterized by three state variables: spatial position  $\mathbf{r}_i$ , (discrete) internal state  $\theta_i$  and (continuous) internal energy depot  $e_i$ . For the change of the agent's position we may assume an overdamped Langevin equation:

$$\frac{d\mathbf{r}_i}{dt} = f_i + \sqrt{2\varepsilon_i} \xi_i(t) \quad (2)$$

The second term denotes the stochastic influences, where  $\xi_i(t)$  is white noise with  $\langle \xi_i(t) \rangle = 0$  and  $\langle \xi_i(t) \xi_j(t') \rangle = \delta_{ij} \delta(t - t')$ . The strength of the stochastic force  $\varepsilon_i$  could be in general an individual parameter to weight the stochastic influences, this way it can for example measure the individual *sensitivity*  $\eta_i \propto 1/\varepsilon_i$  of the agent. The deterministic influences in the first term,  $f_i$ , are in the considered case assumed to result from the the (direct or indirect) interaction between the agents, as specified below.

The internal degree of freedom  $\theta_i(t)$  should have one of the following values:  $\theta_i \in \{0, -1, +1\}$ . Initially,  $\theta_i(t_0) = 0$  holds for every agent. The variable  $\theta_i$  can be changed in the course of time by an interaction between the moving agents and the nodes. To be specific, we consider a two-dimensional surface, where a number of  $j = 1, \dots, z$  nodes are located at the positions  $\mathbf{r}_j^z$  (cf. Fig. 1). A number of  $z_+$  nodes should be characterized by a positive potential,  $V_j = +1$ , while  $z_- = z - z_+$  nodes have a negative potential,  $V_j = -1$ . We note explicitly, that the nodes do *not* have any *long-range effect* on the agents, such as attraction or repulsion. Their effect is restricted to their location,  $\mathbf{r}_j^z$ .

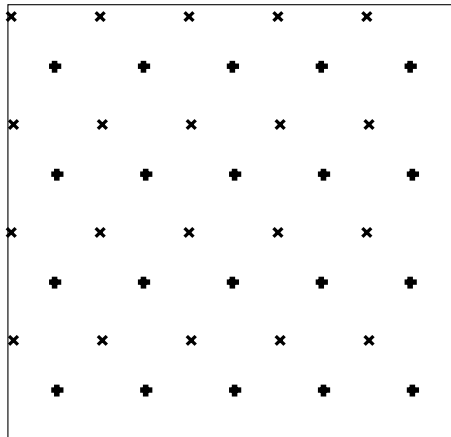


Figure 1: Example of a regular distribution of 40 nodes on a lattice of size  $A = 100 \times 100$ . For the computer simulations, periodic boundary conditions have been used.  $z_+ = 20$ ,  $z_- = 20$ .  $\{x\}$  indicates nodes with a potential  $V_j = -1$ ,  $\{+\}$  indicates nodes with a potential  $V_j = +1$ . (Schweitzer and Tilch, 2002)

It is the (twofold) task of the Brownian agents, first to *discover* the nodes and then to *link* nodes with an opposite potential, this way forming a self-organized network between the set of nodes.

If an agent hits one of the nodes, its internal degree of freedom is changed due to the following equation:

$$\Delta\theta_i(t) = \sum_{j=1}^z (V_j - \theta_i) \int_A \frac{1}{A} \delta(\mathbf{r}_j^z - \mathbf{r}_i(t)) d\mathbf{r} \quad (3)$$

The delta function is equal to 1 only for  $\mathbf{r}_j^z = \mathbf{r}_i$  and zero otherwise. So, eq. (3) indicates, that an agent can change its internal state,  $\theta_i$ , only if it hits one of the nodes. Then it takes over the value of the potential of the respective node,  $V_j$ , which means  $\theta_i$  remains constant if  $V_j = \theta_i$ , and  $\theta_i \rightarrow V_j$ , if  $V_j \neq \theta_i$ . We note that the probability for a (pointlike) agent to hit a (pointlike) node is almost vanishing. However, the computer simulations discussed in the following section are carried out on a discrete lattice, so the agent and the node both have a finite extension, in which case Eq. 3 makes sense.

Eventually, we have to describe the dynamics of the internal energy depot,  $e_i$ , of the agent. In this specific application, it is assumed that every time  $t_{n+}^i$ ,  $t_{n-}^i$  when Brownian agent  $i$  hits one of the nodes  $n+$  or  $n-$  its internal energy depot  $e_i$  is filled up to a maximum value  $e_{\max}$ . Then, the agent is in an active state that allows for certain actions. In our case, the agent is able to produce a chemical, either component  $(-1)$  or  $(+1)$ . Which *kind* of chemical is produced depends on the actual value of the internal parameter  $\theta_i(t)$  of the agent as specified below, whereas the *amount* of chemical produced depends on the actual value of the internal energy depot  $e_i(t)$ . For simplicity, we assume for the amount of the production rate

$$s_i \propto e_i \rightarrow s_i(\theta_i, t) = \beta e_i(t) \quad (4)$$

The internal energy depot can only be charged when hitting one of the nodes and can be filled only to a maximum  $e_{\max}$ . On the other hand, it is decreased by the production of the chemical. Hence, in the general balance equation

$$\frac{de_i}{dt} = q_i(\mathbf{r}_i, t) - p_i(\mathbf{r}_i, t) \quad (5)$$

the space and time dependent influx of energy into the internal depot is given by:

$$q_i(\mathbf{r}_i, t) = (e_{\max} - e_i(t)) \delta(\mathbf{r}_j^z - \mathbf{r}_i) \delta(t - t_{n\pm}^i) \quad (6)$$

whereas the “outflux” of energy from the agent’s depot,  $p_i(\mathbf{r}_i, t)$ , reads accordingly:

$$p_i(\mathbf{r}_i, t) = s_i(\theta_i, t) \quad (7)$$

We note that in previous investigations (Ebeling and Schweitzer, 2003; Ebeling *et al.*, 1999; Schweitzer *et al.*, 1998) an additional term for the active (accelerated) motion of the agent was included in Eq. 7. Further, a dissipative loss,  $-ce_i$ , of the internal energy depot was considered. Both terms are neglected here.



Considering Eq. (4), we can deduce from Eq. (5) the time-dependence of the production rate as  $s_i(\theta_i, t) \sim \exp\{-\beta t\}$ . The dependence on  $\theta_i$ , however, involves specific assumptions of our network model:

$$s_i(\theta_i, t) = s_{\max} \frac{\theta_i}{2} \left[ (1 + \theta_i) \exp\{-\beta(t - t_{n+}^i)\} - (1 - \theta_i) \exp\{-\beta(t - t_{n-}^i)\} \right] \quad (8)$$

Eq. (8) means that the agent is not active, as long as  $\theta_i = 0$ , which means before it hits one of the nodes the first time. After that event, the agent begins to produce either component (+1) if  $\theta_i = +1$ , or component (-1) if  $\theta_i = -1$ , at a rate that exponentially decreases with time. The maximum production rate  $s_{\max}$  depends on the maximum amount  $e_{\max}$ , after the internal energy depot was charged at one of the nodes.

The spatio-temporal concentration of the chemicals shall be described by a *chemical field*  $h_\theta(\mathbf{r}, t)$  consisting of the two components (+1) or (-1), which obeys the following dynamics:

$$\frac{\partial h_\theta(\mathbf{r}, t)}{\partial t} = -k_h h_\theta(\mathbf{r}, t) + \sum_{i=1}^N s_i(\theta_i, t) \delta_{\theta; \theta_i} \delta(\mathbf{r} - \mathbf{r}_i(t)) \quad (9)$$

The first term describes the exponential decay of the existing concentration due to spontaneous decomposition of the chemical, where  $k_h$  is the decomposition rate. The second term denotes the production of the field by the agents. Here,  $\delta_{\theta; \theta_i}$  means the Kronecker Delta used for discrete variables, indicating that the agents only contribute to the field component that matches their internal parameter  $\theta_i$ . The Delta function  $\delta(\mathbf{r} - \mathbf{r}_i(t))$  means that the agents contribute to the field only *locally*, at their current position,  $\mathbf{r}_i$ . Diffusion of the chemical substances is not considered here.

As a last step, we have to specify how the existence of the two different chemicals affect the motion of a Brownian agent, described by Eq. (2). We assume that the interaction term  $f_i$  results from an *indirect communication* among the agents. By producing one out of two chemicals at its current position, each agent generates *local information* that is stored outside the agent, i.e. the environment acts as a kind of external memory. This information should be accessible to other agents in the vicinity. Specifically, we assume that the agent is sensitive to local *gradients* in these two chemical components dependent on its current internal state  $\theta_i$ :

$$f_i = \alpha \frac{\theta_i}{2} \left[ (1 + \theta_i) \frac{\partial h_{-1}(\mathbf{r}, t)}{\partial \mathbf{r}} \Big|_{\mathbf{r}_i} - (1 - \theta_i) \frac{\partial h_{+1}(\mathbf{r}, t)}{\partial \mathbf{r}} \Big|_{\mathbf{r}_i} \right] \quad (10)$$

Here,  $\alpha$  is a dimensional constant only, but it could also act as an individual response parameter weighting the importance of the information received (Schweitzer, 1997).

This way, by producing and responding to two different local information in the system, the Brownian agents communicate with each other. The chemical field plays the role of an order parameter. It does not exist from the outset, but is generated by the agents. Once it is established,

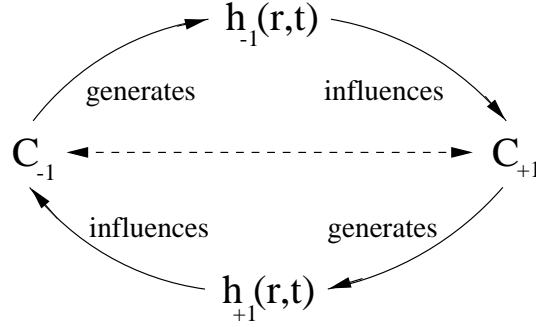


Figure 2: Circular causation between the Brownian agents in the different internal states,  $C_{-1}$ ,  $C_{+1}$ , and the two-component chemical field,  $h_{\theta}(\mathbf{r}, t)$

it reduces the “freedom” of the moving agents by guiding their motion using the information generated by other agents.

Fig. 2 summarizes the non-linear feedback between the two-component chemical field and the agents, as given by the Eqs. (8), (10). Our model assumes, that agents with an internal state  $\theta_i = 0$  do not contribute to the field and are not affected by the field. They simply move like Brownian particles. Agents with an internal state  $\theta_i = +1$  contribute to the field by producing component  $+1$ , while they are affected by the part of the field that is determined by component  $-1$ . On the other hand, agents with an internal state  $\theta_i = -1$  contribute to the field by producing component  $-1$  and are affected by the part of the field, which is determined by component  $+1$ . Moreover, if the agent hits one of the nodes, the internal state can be switched due to Eq. (3). Hence, the agent begins to produce a different chemical, while affected by the opposite potential. Precisely, at one time the agent does *not* respond to the gradient of the same field, which it contributes to via producing a chemical.

As the result of this nonlinear feedback between the Brownian agents and the chemical field generated by them, we can observe the selforganized formation of a network shown in the following section.

## 4 Computer Simulations of Network Formation

For the computer simulations, a triangular lattice with periodic boundary conditions has been used. The agents start initially at random positions. For the evolution of the network, we evaluate the sum  $\hat{h}(\mathbf{r}, t)$  of the two field components generated by the agents. For the plots, however, we have to match these values with a *grey scale* of 256 values, which is defined as follows:

$$\begin{aligned}
 c(\mathbf{r}, t) &= 255 \left[ 1 - \log \left( 1 + 9 \frac{\hat{h}(\mathbf{r}, t) - \hat{h}_{\min}(t)}{\hat{h}_{\max}(t) - \hat{h}_{\min}(t)} \right) \right] \\
 \hat{h}(\mathbf{r}, t) &= h_{+1}(\mathbf{r}, t) + h_{-1}(\mathbf{r}, t)
 \end{aligned} \tag{11}$$



This means that the highest actual value,  $\hat{h}_{\max}(t)$ , always refers to *black* ( $c = 0$ ), whereas the actual minimum value,  $\hat{h}_{\min}(t)$  encodes *white* ( $c = 255$ ). Both extreme values change of course in time, therefore each snapshot of the time series presented has its own value mapping.

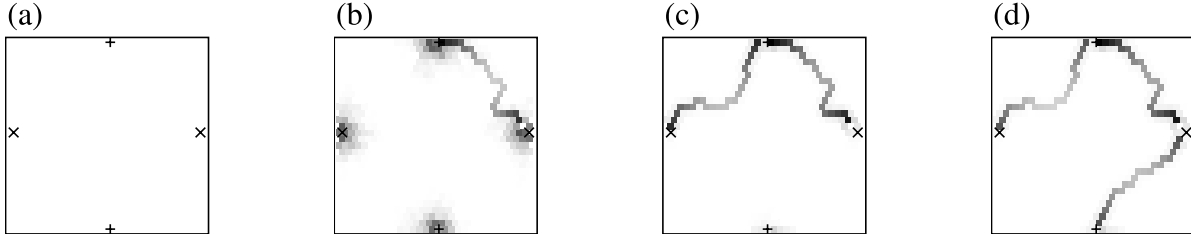


Figure 3: Formation of links between 4 nodes: (a) initial state, (b) after 100, (c) after 1.000, (d) after 4.500 simulation steps. Lattice size  $30 \times 30$ , 450 agents. Parameters:  $s_{\max} = 25.000$ ,  $k_h = 0.01$ ,  $\beta = 0.2$ ,  $k_h = 0.01$ . (Schweitzer and Tilch, 2002)

As a first example, we show the evolution of the connections between four nodes. Fig. 3 would suggest that in the course of time all nodes with an opposite potential should be connected. This however is not true because the existing connections cause a *screening effect*, which forces the agents to move along existing connections rather than making up new ones. This screening effect becomes more obvious, when the number of nodes is increased. Fig. 4 shows the time evolution of a network which should connect 40 nodes, as shown in the setup of Fig. 1. Here, we see that in the course of time the agents aggregate along the connections, which results in higher agent concentrations and in higher fields along the connections.<sup>†</sup>

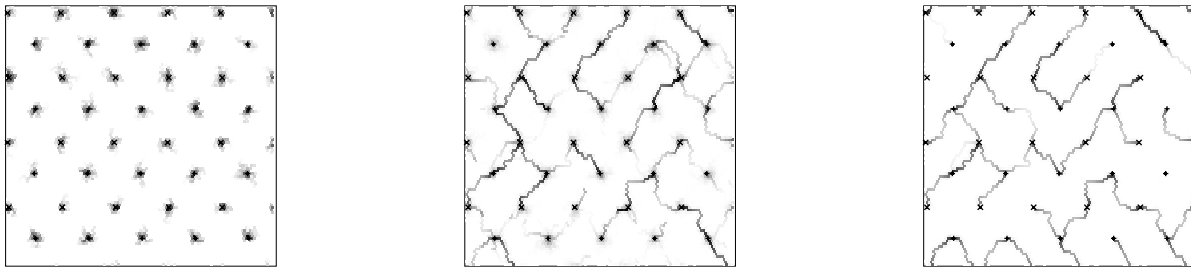


Figure 4: Time series of the evolution of a network after 10, 100, 1.000 simulation steps. Lattice size  $100 \times 100$ , 5.000 agents, 40 nodes,  $z_+ = 20$ ,  $z_- = 20$ . Parameters:  $s_{\max} = 10.000$ ,  $k_h = 0.03$ ,  $\beta = 0.2$ . (Schweitzer and Tilch, 2002)

In Figs. 3,4, the connections between the nodes exist as a two-component chemical field generated by the Brownian agents. This self-assembling network is created very fast and remains stable after the initial period. We note that the network formation is not restricted to regular or symmetric distributions of nodes. Fig. 5 shows a simulation, where different nodes are connected with a

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<sup>†</sup> A video of these computer simulations can be found at <http://intern.sg.ethz.ch/fschweitzer/until2005/network.html>.

center. An extension of the model has been applied to simulate the trunk trail formation in ants connecting a nest to different food sources (Schweitzer *et al.*, 1997) and will be discussed in Secs. 6, 7.

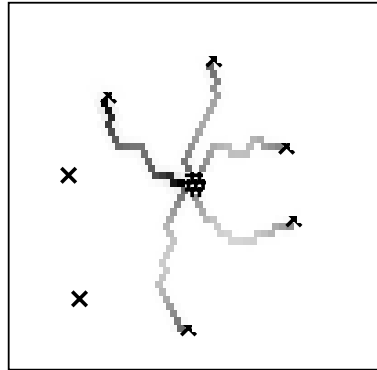


Figure 5: Formation of links between a center ( $z_- = 1$ ) and surrounding nodes ( $z_+ = 7$ ) after 10.000 simulation steps. Lattice size:  $50 \times 50$ , 2.000 agents. Parameters:  $s_{\max} = 20.000$ ,  $k_h = 0.02$ ,  $\beta = 0.2$ . (Schweitzer and Tilch, 2002)

Patterns like the networks shown in Figs. 3,4,5 are intrinsically determined by the history of their creation. It means that irreversibility and early symmetry breaks play a considerable role in the determination of the final structure. These structures are unique due to their evolution, and therefore can be understood only within a stochastic approach, as provided for example by the Brownian agent approach.

## 5 Estimation of the Network Connectivity

In order to characterize a network, one of the most important questions is, whether two nodes  $k$  and  $l$  are connected or not. In the model considered, a connection is defined in terms of the chemical field  $\hat{h}(\mathbf{r}, t)$  produced by the agents. In the beginning, however, the agents randomly visited almost every lattice site before their motion turned into a bound motion between the nodes. Therefore, the field  $\hat{h}(\mathbf{r}, t)$  has a non-zero value for almost every  $\mathbf{r}$ , which exponentially decays, but never vanishes. Hence, in order to define a connection in terms of  $\hat{h}(\mathbf{r}, t)$ , we have to introduce a *threshold value*  $h_{\text{thr}}$ , which is the minimum value considered for a link. More precisely, a *connection* between two nodes  $k$  and  $l$  should only exist, if there is a path  $s \in A$  between  $k$  and  $l$  on which the actual value of the field is larger than the threshold value:

$$\hat{h}(s, t) > h_{\text{thr}} \quad \text{for } s \in A \quad (12)$$

Such a definition does not necessarily assume that the connection has to be a *direct link*. Instead, it could be any path  $s$ , which may also include other nodes, as long as the value  $\hat{h}(s, t)$  along the path is above the threshold.

We want to define the *local connectivity*  $E_{lk}$  as follows:

$$E_{lk} = \begin{cases} 1 & \text{if } k \text{ and } l \text{ are connected by a path } s \in A, \\ & \text{along which } \hat{h}(s, t) > h_{\text{thr}} \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

We note that the connectivity  $E_{lk}$  does not change, if two nodes  $k$  and  $l$  are connected by more than one path.

If we consider a number of  $z$  nodes, then the *global connectivity*  $E$  which refers to the whole network, is defined as follows:

$$E = \frac{\sum_{k=1}^z \sum_{l>k}^z E_{lk}}{\sum_{k=1}^z \sum_{l>k}^z 1} = \frac{2}{z(z-1)} \sum_{k=1}^z \sum_{l>k}^z E_{lk} \quad (14)$$

Dependent on the configuration of nodes, there may be numerous different realizations for the connections, which result in the same connectivity  $E$ .

In order to use the definition for the connectivity to evaluate the simulated networks, we first have to define the threshold value  $h_{\text{thr}}$ . This should be the *minimum value* of  $\hat{h}(r, t)$  along a *stable connection* between two nodes. Based on this definition, we have derived in a recent paper Schweitzer and Tilch (2002) an estimate for  $h_{\text{thr}}$ :

$$h_{\text{thr}} = \bar{n} \frac{s_{\text{max}}}{k_h} \left( \frac{s_{\text{min}}}{s_{\text{max}}} \right)^{1/4} \quad (15)$$

Provided the set of parameters, used for the simulations, we find for the threshold the value  $h_{\text{thr}} = 1.7 \times 10^4$ , which is approximately  $h_{\text{thr}} \approx 2s_{\text{max}}$ . We note, that this is an estimate, which might give a rather high value and thus ensures, that values for  $\hat{h}(s, t)$  above the threshold *really* represent a *stable connection*  $s$ .

In a recent paper Schweitzer and Tilch (2002), we have investigated how the global connectivity  $E$  evolves in the course of time and how it changes in dependence on the density of agents,  $\bar{n}$ . In agreement with the visible evolution of the network presented in Fig. 4, three different stages can be clearly distinguished: (i) an *initial* period ( $t < 10^2$ ), where no connections yet exist, (ii) a *transient* period ( $10^2 < t < 10^4$ ), where the network establishes, (iii) a *saturation* period ( $t > 10^4$ ), where almost all nodes are connected, and only small fluctuations in the connectivity occur.

Here, we focus on the question, how the connectivity changes in dependence on stochastic influences. According to Eq. (1), in the Brownian agent approach the different state variables of

the agent are changed both by deterministic and stochastic influences. In our agent-based model of network formation, stochasticity influences are explicitly considered only in the equation of motion of the Brownian agents, Eq. (2). There, it was assumed that the strength of the stochastic force  $\varepsilon_i$  could be in general an individual parameter. This idea will be used in Sec. 6, here we want to assume for the moment that  $\varepsilon_i = \varepsilon$  is equal for all agents and can be further related to the *temperature*  $T$ , which is a physical measure of fluctuations in the system, independent of the agents. It was Einstein, who derived in 1905 a relation between the temperature and the diffusion of simple Brownian particles,  $D = (k_B/\gamma_0)T$ , where the proportionality is given by the Boltzmann constant  $k_B$  and the friction coefficient  $\gamma_0$ . Identifying  $\varepsilon = D$ , which is correct for the physical interpretation of the overdamped Langevin Eq. (2), we can now express the strength of the stochastic force by the temperature  $T$ , which as a global parameter acts as a boundary condition for the dynamics of the multi-agent system.

From the physics perspective, network formation – as other processes of dissipative structure formation in non-equilibrium systems (Biebricher *et al.*, 1995) – should become possible only below a certain critical temperature  $T_c$ . Above  $T_c$ , fluctuations in the system rather destroy the structures emerging. With respect to Eq. (2), this means that for  $T > T_c$ , the agents cannot pay sufficient attention to the deterministic influences given by  $f_i$  and thus behave more or less as simple Brownian particles. The delicate balance between the stochastic and deterministic influences of course depend on the current value of  $f_i$ , which in turn results from the history of the indirect communication process described by Eqs. (9), (10). However, in a so-called mean-field approximation for the agent density and the dynamics of the field components (Schweitzer, 2003) we were able to derive an expression for the critical temperature  $T_c$  below which the network formation becomes feasible:

$$T_c = \frac{\alpha}{2} \frac{\bar{s} \bar{n}}{k_B k_h} \quad (16)$$

This equation relates  $T_c$  to the mean density of agents,  $\bar{n} = N/A$ , and an average production rate of the chemicals  $\bar{s}$ , derived in (Schweitzer, 2003) as:

$$\bar{s} = \frac{s_{\max} - s_{\min}}{\ln s_{\max} - \ln s_{\min}} \quad (17)$$

By varying the stochasticity, expressed by the parameter  $T$  relative to the reference value  $T_c$ , we are now able to investigate the connectivity by means of computer simulations, carried out for the same setup as used for Fig. 4. Fig. 5 shows the connectivity, averaged over 200 simulations, as a function of the reduced temperature  $T/T_c$ , whereas Fig. 7 shows some simulated realizations of the networks for different reduced temperatures.

Fig. 5 clearly indicates that an optimal range of temperatures,  $0.3 \leq T/T_c \leq 0.5$ , exists where the average connectivity reaches a maximum. For the given setup of nodes and the given set of parameters, within this optimal range in the average more than 90 percent of the nodes are connected. For  $T > 0.5 T_c$ , the connectivity breaks down drastically, which means that the motion of the agents is mainly determined by the stochastic forces, and no attention to the gradient of the field is paid.

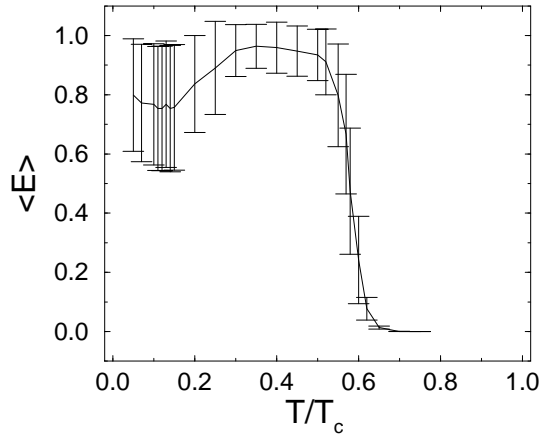


Figure 6: Network connectivity  $E$ , Eq. (14), averaged over 200 simulations vs. reduced temperature  $T/T_c$  Eq.( 16). For the setup see Fig. 4. (Schweitzer, 2003)

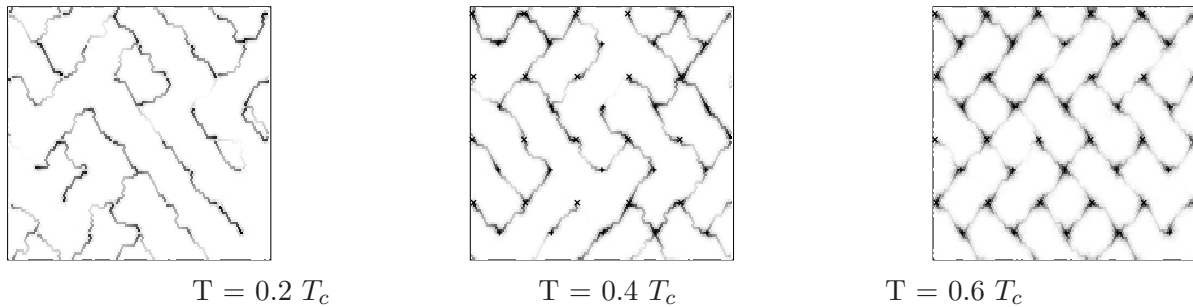


Figure 7: Network formation dependent on the reduced temperature  $T/T_c$ , Eq. (16). Shown is the value  $c(\mathbf{r}, t)$ , Eq. (11) after 5.000 simulation steps. Other parameters see Fig. 4. (Schweitzer, 2003)

On the other hand, for  $T < 0.3 T_c$ , the connectivity decreases because during the transient period of establishing the network, the agents have payed too much attention to the gradient, and therefore are trapped in the first established links, instead of moving around. In this range of temperatures, the average connectivity  $\langle E \rangle$ , Fig. 5, displays large fluctuations, so almost every realization between  $E = 0.5$  and  $E = 1$  has a certain likelihood. Here, the early history in creating the network plays an important role for the eventual pattern and the connectivity. However, in the optimal range of temperature, the effect of the early symmetry breaks is weakened to a certain extent, thus leading to smaller fluctuations in  $E$ .

We also want to note, that in the range of smaller temperatures, of course less connections between the nodes occur, but these connections provide a much stronger link in terms of the field  $\hat{h}(\mathbf{r}, t)$ . One of the reasons for the breakdown of the connectivity for  $T > 0.5 T_c$  in Fig. 5 results from the fact, that many of the links established have a value below the threshold,  $h_{\text{thr}}$ ,

and therefore are not considered as stable connections. This can be seen by comparing the pictures in Fig. 7. Here, the nodes still seem to be connected for  $T = 0.6 T_c$ , the value of the related field however is below the threshold,  $h_{\text{thr}}$ .

## 6 Modeling Trail Formation in Ants

In this section, the network model introduced above is extended to simulate the formation of directed trails, as observed in group-raiding ants. So, the nodes are given now by a nest and different food sources, and the links are given by the directed trails between nest and food. These connections do not exist from the very beginning; they have to be generated by the agent community in a process of self-organization. Hence, the agents have to perform two quite different tasks, which are referred to each other: first, they have to detect food places - unknown to them, and then they have to link these places to their original starting point by forming a trail, with local chemical orientation as the only tool provided.

For our model, we assume that initially all agents are concentrated in one place, the “nest”. The internal parameter,  $\theta_i = \pm 1$ , describes whether an agent has found food yet or not. If the agent starts from the nest, the internal parameter is always set to  $\theta_i = +1$ . Only if the agent has detected a food source,  $\theta_i$  is changed to  $\theta_i = -1$ . Dependent on the value of  $\theta_i$ , the agent is able to produce one of two different chemicals with a rate  $s_i(\theta_i, t)$ , which was already described in Eq. (8). I.e., the agent produces chemical of component (+1), when it leaves the nest ( $\theta_i = +1$ ) at time  $t_n^i$ , and starts to produce chemical of component (-1) instead, after it has found food ( $\theta_i = -1$ ) at time  $t_f^i$ . The dynamics of two chemical components is again given by Eq. (9). Further, the agents are affected by the different kind of chemicals as described by Eqs. (2)(10).

One could argue that for the simulation of foraging trails of ants, the most simple assumption could be that *only* individuals which found food create a pheromone trail, while turning straight back to the nest. These trails could then be used by other ants to find the food sources, once they found the trails. However, those simulations have to overcome the problem where the first ant which found food got the information from to find its way back home. In biological systems, geocentric or egocentric navigation could provide this additional information needed for successfully returning to the nest. Since our model simulates the formation of trunk trails without counting on navigation and internal storage of information, our main assumption is that a simple chemotactic response would be sufficient to generate trails between the nest and the food sources. However, we have to distinguish trails which have been created during the (unsuccessful) search for food from trails which really lead to a food source. In our model, this additional information is encoded by assuming that the agents produce, and respond to, two different chemical fields.

For the computer simulations, we have to address an additional problem that results from obvious differences in complexity between biological entities and Brownian agents. It is known

from central foragers like ants that they are able to leave a place where they don't find food and reach out for other areas. This indicates that they have at least a certain ability to increase their mobility since they have a certain aim, finding food. Agents in our model however do not have aims and do not reflect a situation. So, they stick on their local markings even if they they did not find any food source. In order to increase their mobility in those cases, we assume that every agent is described by an additional internal variable, its individual sensitivity  $\eta_i$  to follow a chemical gradient. As long as the agent does not hit a food source, this sensitivity is continuously decreasing via an increase of the stochasticity  $\varepsilon_i$  in Eq. (2), i.e. we assume here  $\eta_i = \alpha/\varepsilon_i$ , where  $\alpha$  is the response parameter to weight the influence of the gradients. Increasing the individual stochasticity then means that the agent more and more ignores the chemical detected and thus becomes able to choose also sites not visited so far.

However, if the agent does not find any food source after a certain number of steps, the sensitivity  $\eta_i$  becomes negligible, and the agent only behaves like a Brownian particle. From the utilitarian perspective, such a agent becomes "useless" for the exploitation of food sources, if its individual noise level,  $\varepsilon_i$ , exceeds a maximum value,  $\varepsilon_{\max}$  (which could be related to the critical temperature, discussed above). So we simply assume that the agent "dies" at this level and is removed from the system. On the other hand, if the agent hits a food source, its sensitivity is set back to the initial high value and is kept constant to increase the chance that the agent finds its way back along the gradient. The switch of the level of sensitivity is accompanied with the use of the two different chemical markers, and in this particular sense reminds on *chemokinesis* (Dunn, 1983), where the level of activity can be changed due to chemical conditions in the surrounding.

Using the internal parameter  $\theta_i$  again, we may assume that the sensitivity  $\eta_i$  of the agent changes according to:

$$\frac{\alpha}{\eta_i(t)} = \varepsilon_i(t) = \frac{\theta_i}{2} \left\{ (1 + \theta_i) \left[ \varepsilon_0 + \delta\varepsilon (t - t_n^i) \right] - (1 - \theta_i) \varepsilon_0 \right\} \quad (18)$$

Here,  $\delta\varepsilon$  is the increase of the individual noise level,  $\varepsilon_i$ , per time step if the agent has not found any food source.

This way, we have completed our model for the trail formation in ants. The basic ingredient is the chemotactic response to a chemical gradient which has been produced by the agents themselves. Hence, the basic equations are given by the equation of motion for the agents, Eq. (2), which is coupled with the equations for the two chemical fields, Eq. (9). We note that, with respect to biology, there are different parameters which may influence trail following in addition to sensitivity, such as trail fidelity, traffic density, detection distance, endurance of the trail, navigation capabilities etc. (Edelstein-Keshet *et al.*, 1995; Haefner and Crist, 1994). Contrary, our model considers only minimal assumptions for the trail formation. Here, the formation of trail patterns is solely based on simple *local* chemical communication between the agents, with no additional capabilities of orientation or navigation. The computer simulations outlined below should prove that the simple local rules assumed for the action of the agents are sufficient enough to solve such a complex problem.



## 7 Computer Simulations of Trail Formation in Ants

Different from arbitrary *non-directed* tracks which are commonly used by the agents but lead to nowhere, *directed trails* should link a starting point (e.g. a nest) to a destination point (e.g. a food source) and back. In order to define start and destination, we assume on the two-dimensional surface a center (nest) where the agents are initially concentrated, and random distribution of five separated food sources unknown to the agents. As in the network model described before, both the nest and the food sources, *do not attract* the agents by a certain long-range attraction potential, they are just particular sites of a certain size (defined in lattice sites). It is assumed that the food sources can be exhausted by the visiting agents, which are assumed to carry part of them back to the nest.

Fig. 8 shows a time series of the trail formation in terms of the spatial concentration of chemical component ( $-1$ ), which is coded again in a grey scale. In Fig. 8a, we see that two of the food sources randomly placed on the lattice, have about the same distance to the nest, but the one, which - by chance - has been discovered first, will also be the one linked first to the nest. This reflects the influence of *initial symmetry breaking effects*.

If the food sources are exhausted by the agents carrying food to the nest, they vanish. Therefore, the individual sensitivity of the agents coming from the nest to the food cannot be set back to the high initial value and the agents further increase their mobility by ignoring the trail, they reach out again, and by chance discover new food sources. But since a trail already exists, those sources in a close vicinity to the one that disappeared have a larger probability to be discovered. Part of the “old” trail is re-used then to link the new food source to the nest (compare Fig. 8a,b,c).

Fig. 8b shows that at the same time also more than one sources could be linked to the nest by trails. However, all trails compete for the agents to be maintained, and the trails could survive only if the concentration of the chemicals is above a certain critical value. Therefore, the coexistence of different trails only lasts for a short time. Hence, the trails to the different food sources will appear one after the other, with the old trails disappearing again by decomposition because they are no longer maintained. After a source is exhausted, we usually observe a time lag before the next one is exploited. This is needed for the system to “forget” about the useless trail and to establish a new trail to the next source. Thus, we can distinguish between two alternating dynamical stages: (i) a stage of rather random motion of the agents (*exploration* of the sources), and (ii) a stage of directed motion of the agents along the trail (*exploitation* of the sources).

The flexibility of the model is indicated by the fact, that even after a long period of trail formation in the lower part of the lattice, the agents are able to link the food source on the upper left side of the lattice to the nest (Fig. 8d) and therefore finally have detected and linked all five sources to the nest. The desorientation stage before exploiting the last source is especially long, because the agents could not build on any previous part of a trail into that direction, whereas

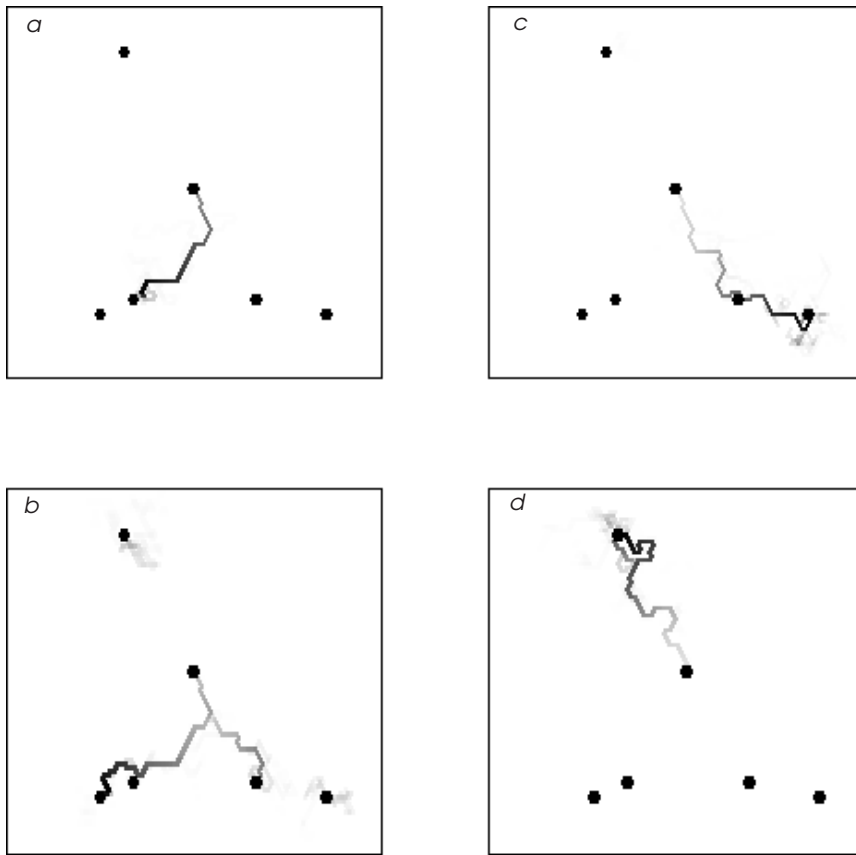


Figure 8: Formation of trails from a nest (middle) to five randomly placed food clusters. The distribution of chemical component  $(-1)$  (see text) is shown after (a) 2000, (b) 4000, (c) 8500, and (d) 15000 simulation time steps, respectively. (Schweitzer *et al.*, 1997)

the desorientation stage before exploiting the third and the fourth source is very short, because of the trails which already exist into that area.

## 8 Conclusions

In this chapter, we have proposed a multi-agent approach to model the self-organization of networks. This approach is based on the interactions of the agents on a local or “microscopic” level, which could lead to the emergence of the structure as a whole on the global or “macroscopic” level. In our model, the *emergence* of network structures, i.e., the *self-organized* formation of links results from the dynamic interaction of the agents, which is different from drawing lines between a set of known nodes. The agents have to solve two different tasks in parallel, namely

to discover new nodes and to link them to the network *without* external guidance. They also have to ensure the adaptivity of the network.

In our model, the self-assembling of networks between arbitrary nodes is based on the nonlinear interaction of Brownian agents. Different from a circuit diagram, for instance, which determines the different links between the nodes in a *top-down* approach of hierarchical planning, the connections here are created by the agents *bottom-up*. The spontaneous emergence of different network structures was characterized as a self-organizing process. As in every evolutionary game, also for the emergence of networks critical parameters exist. As an example, we have discussed the strength of fluctuations compared to the influence of the chemical gradients, but this also holds for other parameters in the model. If, for instance, the decay rate for the decomposition of the chemicals is too high, or the sensitivity of the agents is too low, they lose their local orientation and get lost, so that no links appear. If, on the other hand, the decay rate is too low, it takes a much longer time, before distinct links appear, because the selection pressure is too low.

The model turned out to be very flexible regarding the geometry of the nodes to be connected. The locations of the different nodes act as a boundary condition for the structure formation, but do not determine the way of connecting the different nodes. If, for example, a particular link breaks down, the agents would be able to *repair* it by re-establishing the field, again, or by creating a new one. It was shown (Schweitzer, 2003) that the switching behavior between a connected and a disconnected state can be very short, which would allow the construction of a dynamic switch.

To maintain a network, to keep it updated, to repair links, are ambitious aims for which natural paradigms exist, e.g. in social insects or neural networks. These have also inspired the design of our multi-agent model. Compared to the complex “individual-based” models in ecology (DeAngelis and Gross, 1992; Huston *et al.*, 1988) the agent model proposed here provides a very simple but efficient tool to simulate a specific structure with only a few adjustable parameters. The major difference to biology is denoted by the fact, that the agents used in the simulations do not have individual capabilities to store and process informations about the locations of the nodes, or to count on navigational systems or additional guidance. They rather behave like physical particles which respond to local forces in a quite simple manner, without “implicit and explicit intelligence” (Haefner and Crist, 1994). Noteworthy, these local forces only result from the chemical markings produced by themselves. This way, we propose a kind of minimalistic approach for the formation of networks which is built on the dynamical essence of the feedback processes rather than on the internal complexity of the agents.

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