Opinion Spreading and Neighbourhood Models

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Outlook

I.- Neighbourhood models in minority spreading

(a) Introduction

(b) Galam's model for minority opinion spreading

(c) Effects of locality: Neighbourhood models

(d) Results

II.- System size stochastic resonance

(a) Simple majority model for opinion formation

(b) Results

III.- Conclusions and Prospectives

Galam Model: Introduction

Question: "*How come that an initially minority opinion can, in a truly democratic process, become majority?*"

S. Galam, Eur. Phys. J. ***B*** **25**, 403(2002), *Physica* ***A*** **320**, 571 (2003)

Galam Model: Introduction

Question: "*How come that an initially minority opinion can, in a truly democratic process, become majority?*"

Examples:

- The September-11th no-plane Pentagon hoax (***the spread of such rumour in France and UK***).
- Minority opinion against an structural change in society finally becomes a majority (***Maastritch related Ireland & EU Constitution France voting).***
- Authorship of 2004 terrorist attacks in Madrid (***Eta versus Al-Qaeda)***

•**Definition of opinion:** Binary value each agent adopts on an issue.

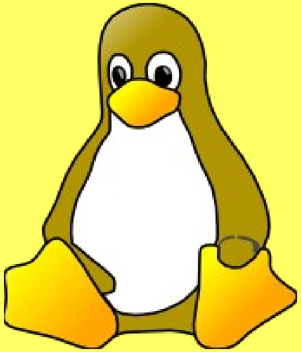
•**Basic premise:** The issue is discussed is small groups and each group adopts the position of the majority with a bias in case of a tie.

Galam Model: Description

There is a **binary opinion**:

Each agent has one of two opinions (**blue** or **yellow**)

In favor (+)



Against (-)

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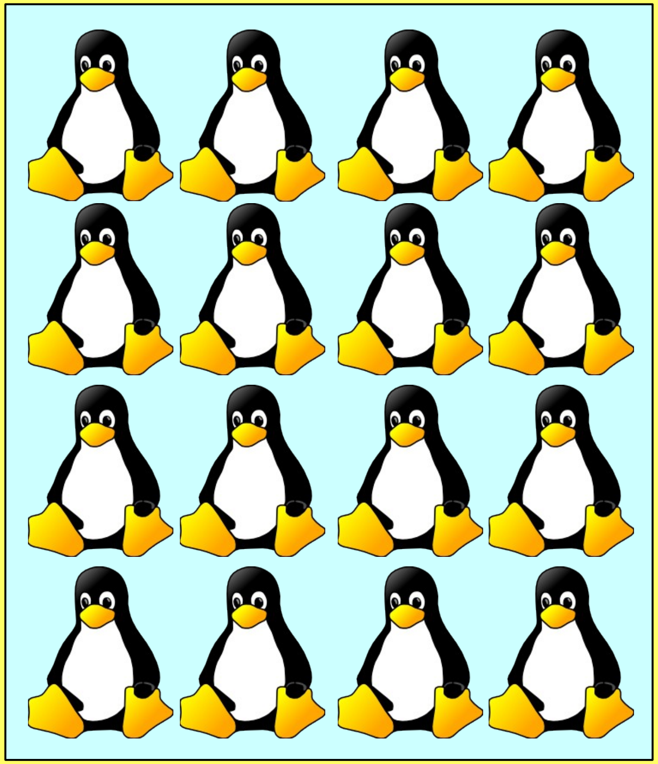
Galam Model: Description – opinion formation

Agents meet in ***decision cells***, to discuss on the topic

These cells are defined only by their size k

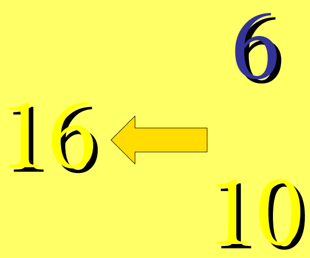
k=16

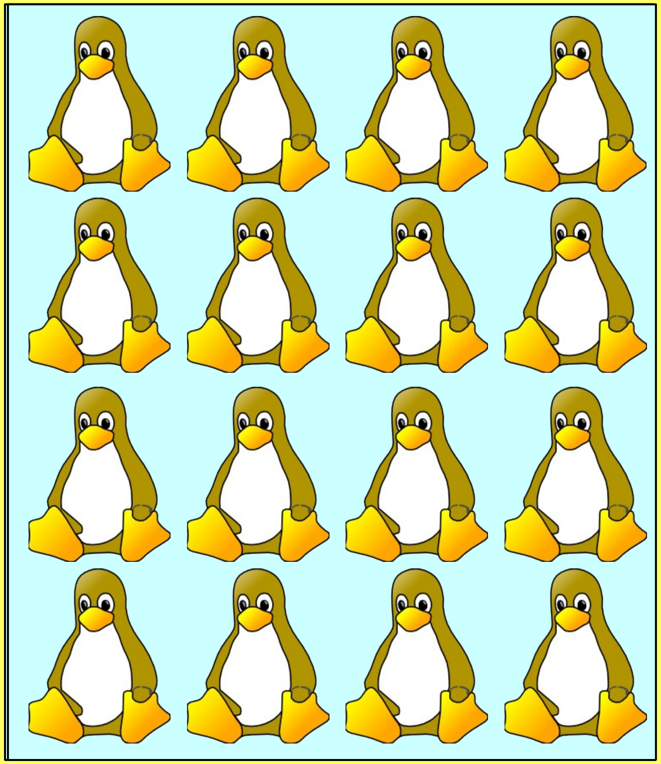


...within the decision cell, all the individuals adopt an opinion...

Galam Model: Description – opinion formation

If there is a majority of ***yellow*** opinion





ALL the agents in the cell adopt the

yellow opinion

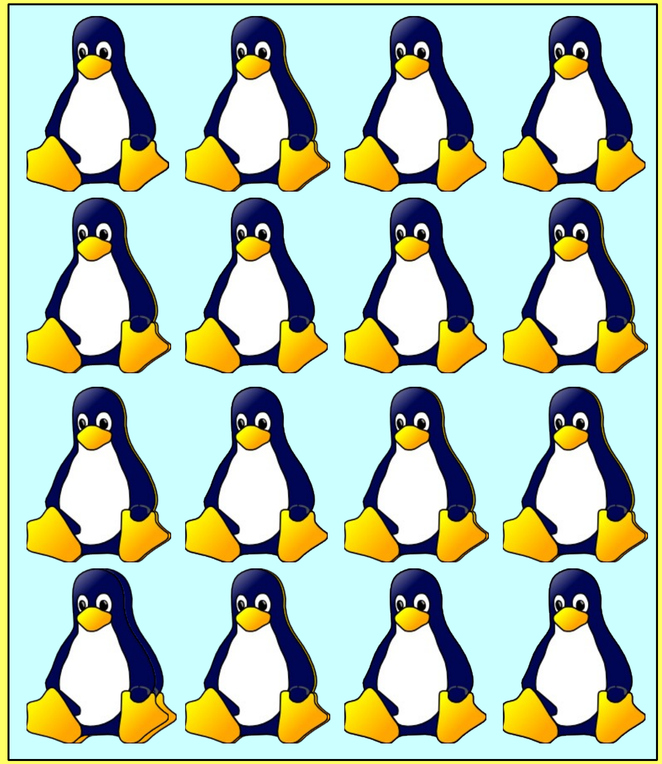
Galam Model: Description – opinion formation

If there is a majority of ***blue*** opinion

9

16 ⟵

7

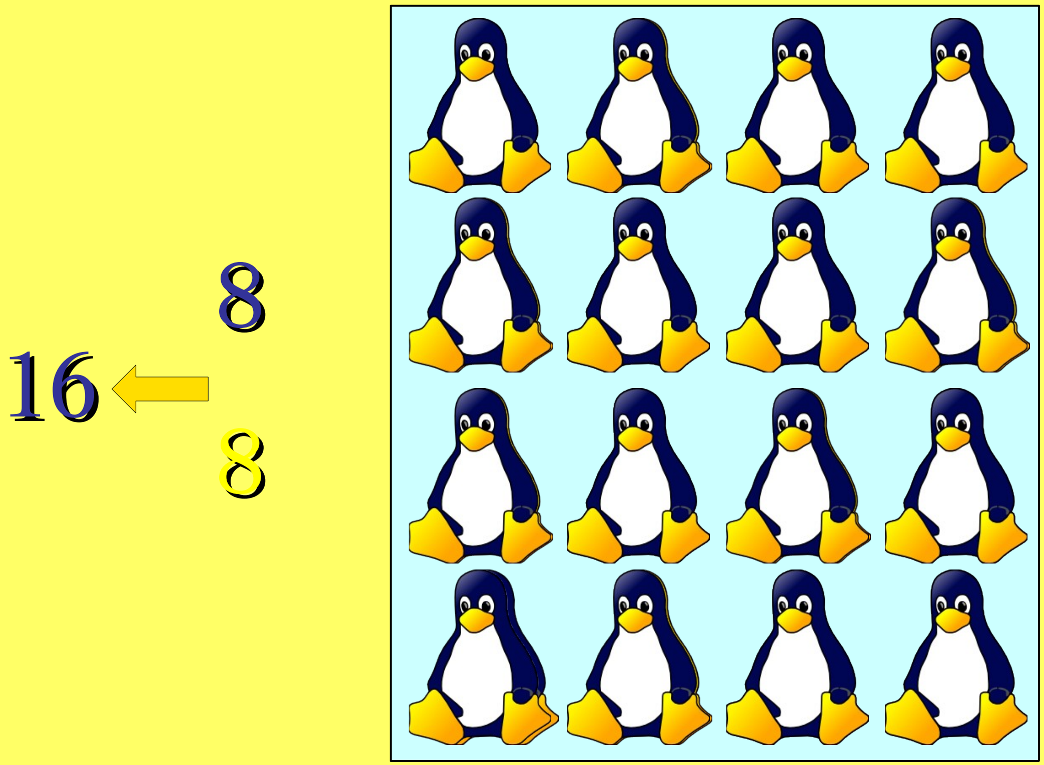


ALL the agents in the cell adopt the

blue opinion

Galam Model: Description – opinion formation

If there is a tie between ***blue*** and ***yellow*** opinions

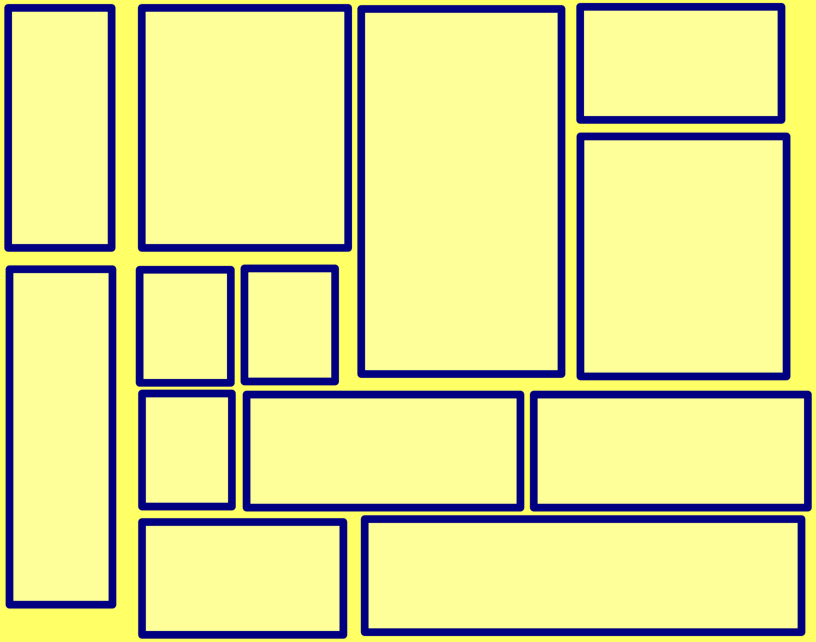


ALL the agents within the cell adopt the **blue** opinion: favoured opinion

Galam Model: Description – dynamic rule

The system composed by N agents and decision cells with a given size distribution

At time t, each agent chooses at random a decision cell



Galam Model: Description – dynamic rule

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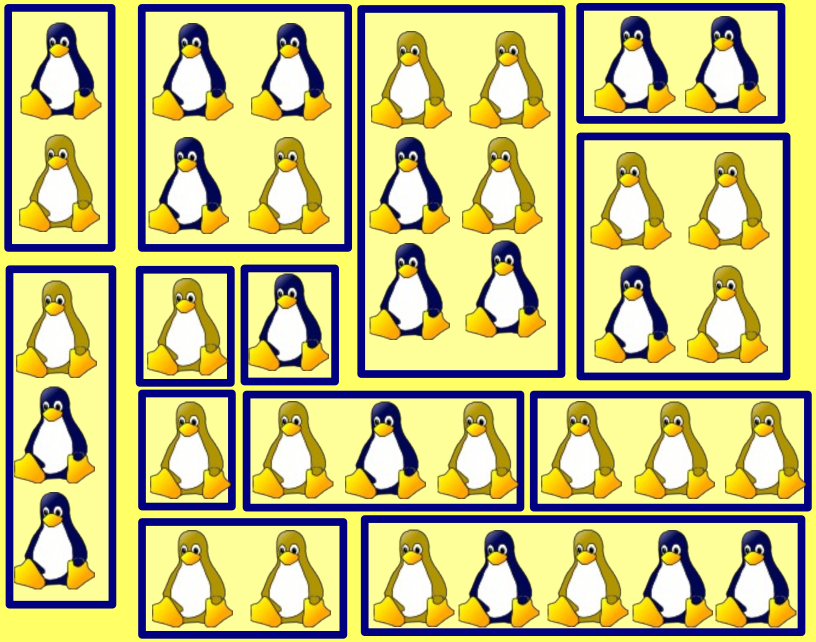

$P_+ = 17/37$ ------ $P_- = 20/37$

Galam Model: Description – dynamic rule

The system composed by *N* agents and decision cells with a given size distribution

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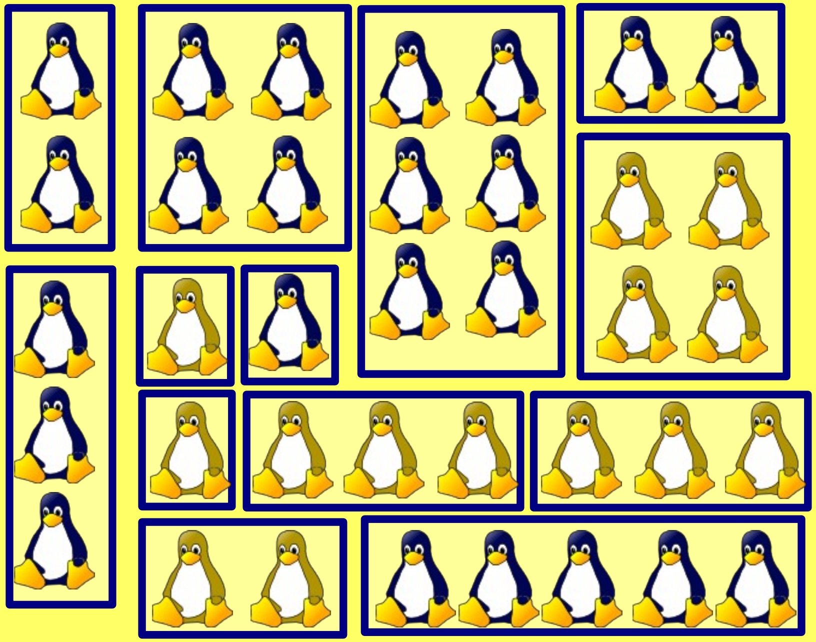
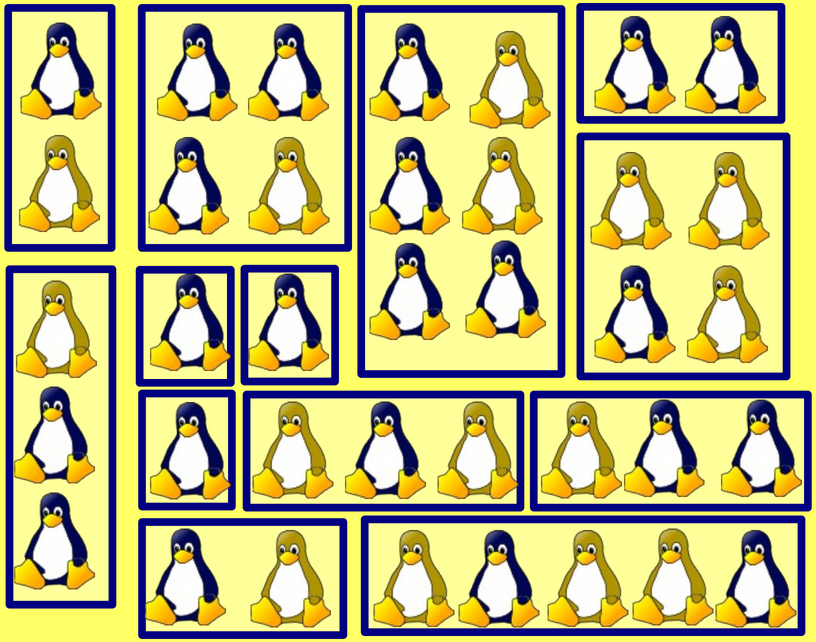


$$P_+ = 23/37 \text{ ------ } P_- = 14/37$$

Galam Model: Description – dynamic rule

At time t+1, the agents choose at random another decision cell → **Non-local model**



$P_+ = 23/37$ ------ $P_- = 14/37$

Galam Model: Description – dynamic rule

At time $t+1$, the agents choose at random another decision cell $\rightarrow$ **Non-local model**

There are two parameters in the model:

- the initial density of supporters of the favoured opinion $P_+(t=0) = p$
- the size distribution of decision cells

We considered decision cells whose size is uniformly distributed in the interval $[1, M]$

Galam Model: Mean-field analysis

There is a polynomial function $F(t)$, such that $P_+(t+1) = F[P_+(t)]$

There are three fixed points:

two trivial and the **faith (unstable fixed) point,** p_c

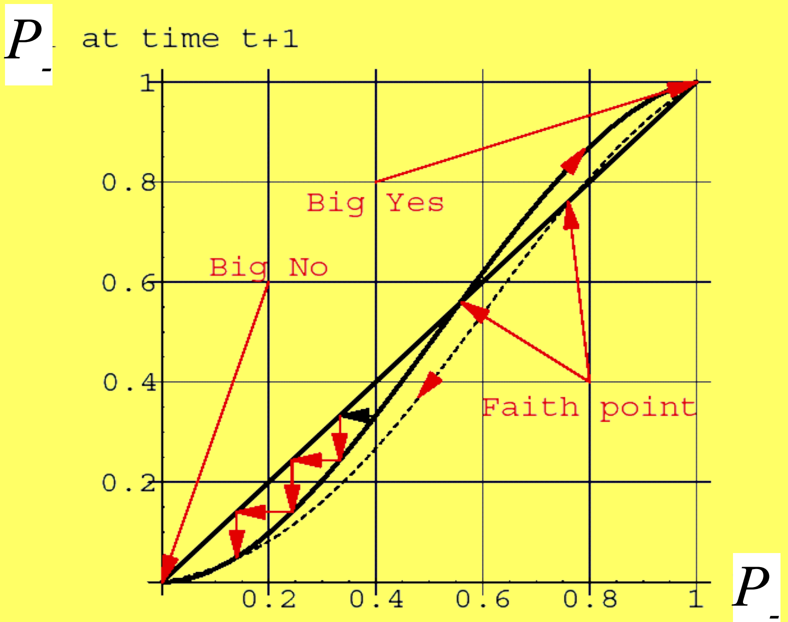


Fig. 1. Variation of $P_+(t+1)$ as function of $P_+(t)$. The dashed line is for the set $a_1 = a_2 = a_3 = a_4 = 0.2$, $a_5 = a_6 = 0.1$, $L = 6$ and $P_{+F} = 0.74$. The plain line is for the set $a_1 = 0$, $a_2 = 0.1$ and $a_3 = 0.9$ with $L = 3$ and $P_{+F} = 0.56$. Arrows show the direction of the flow.

There is a critical amount of initial supporters $p_c < 1/2$ such that the minority opinion finally becomes majority

S.Galam, *Physica* ***A*** **320**, 571 (2003)

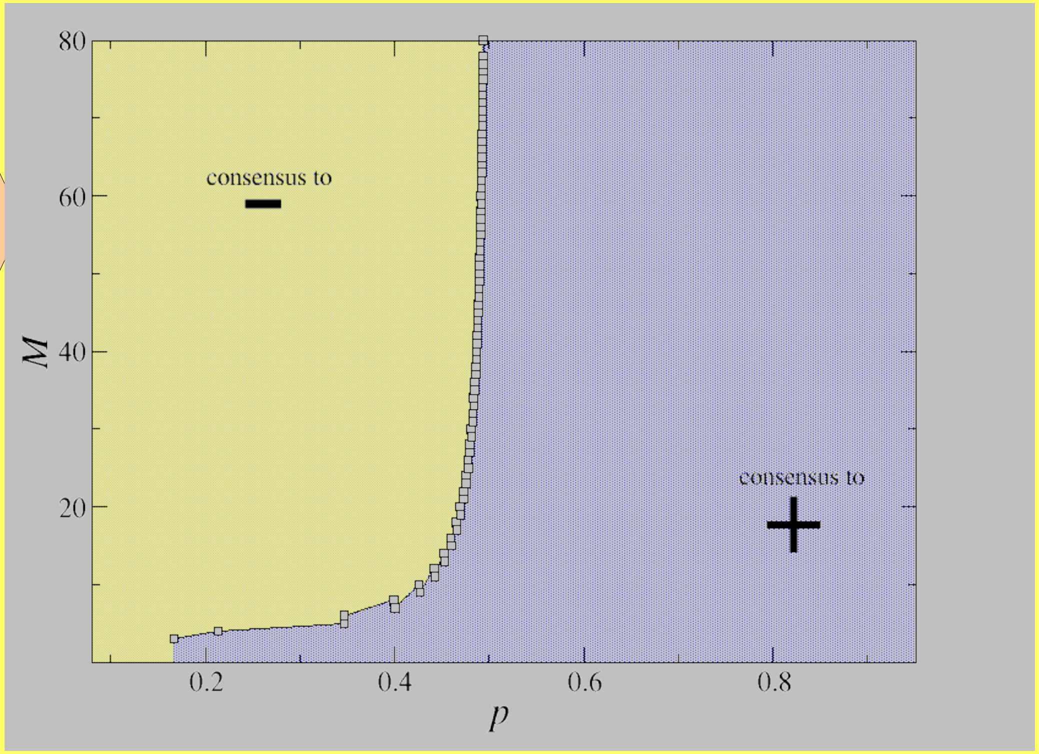
Galam Model: Order parameter

ρ probability that favoured opinion wins

Thermodynamic limit result:

$$\rho = 1 \text{ if } p > p_c$$

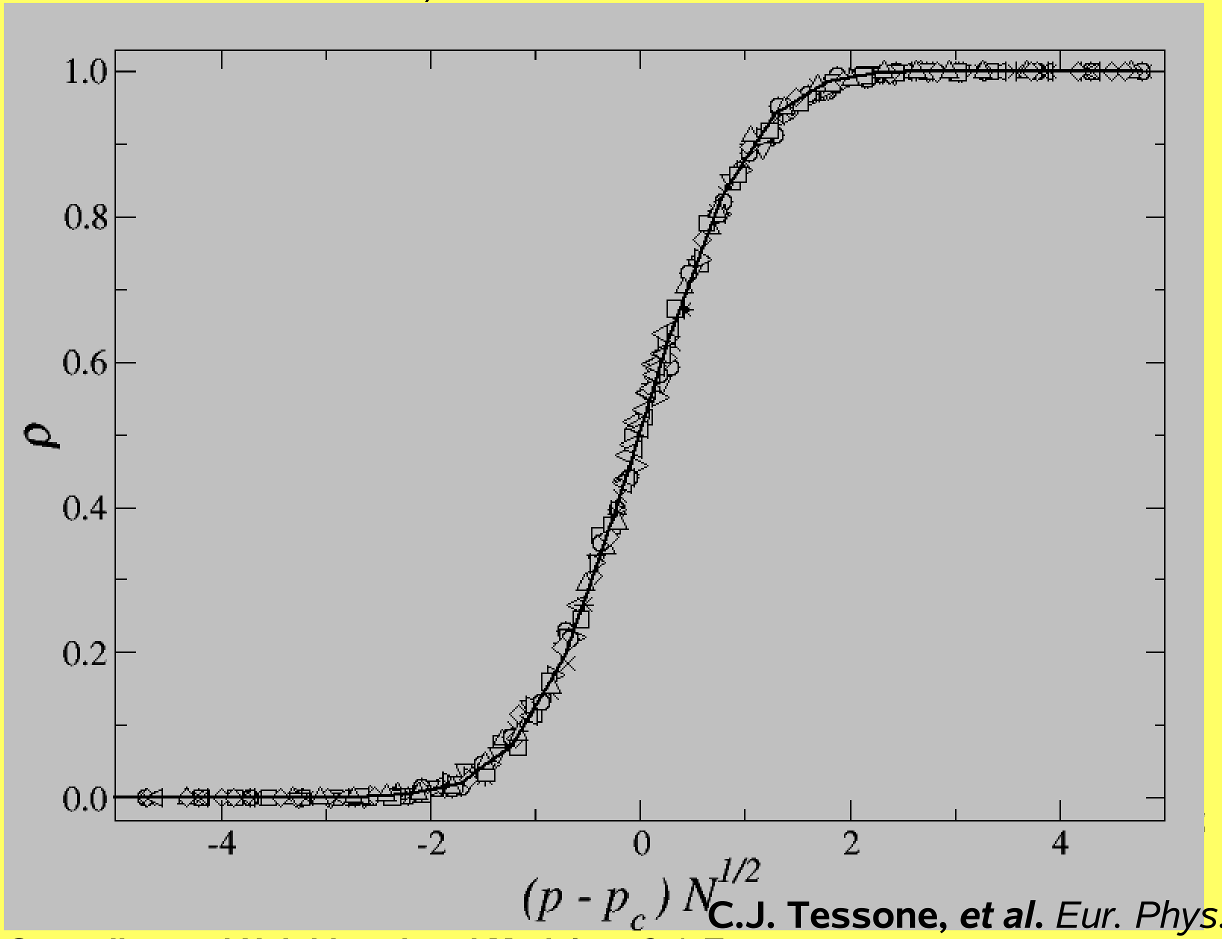
$$\rho = 0 \text{ if } p > p_c$$



Galam Model: Order parameter – size effects

Finite N: Smoothed 1st-order transition

There is a region of width $N^{-1/2}$ in which the outcome of a run is not well known (standard finite size effect)

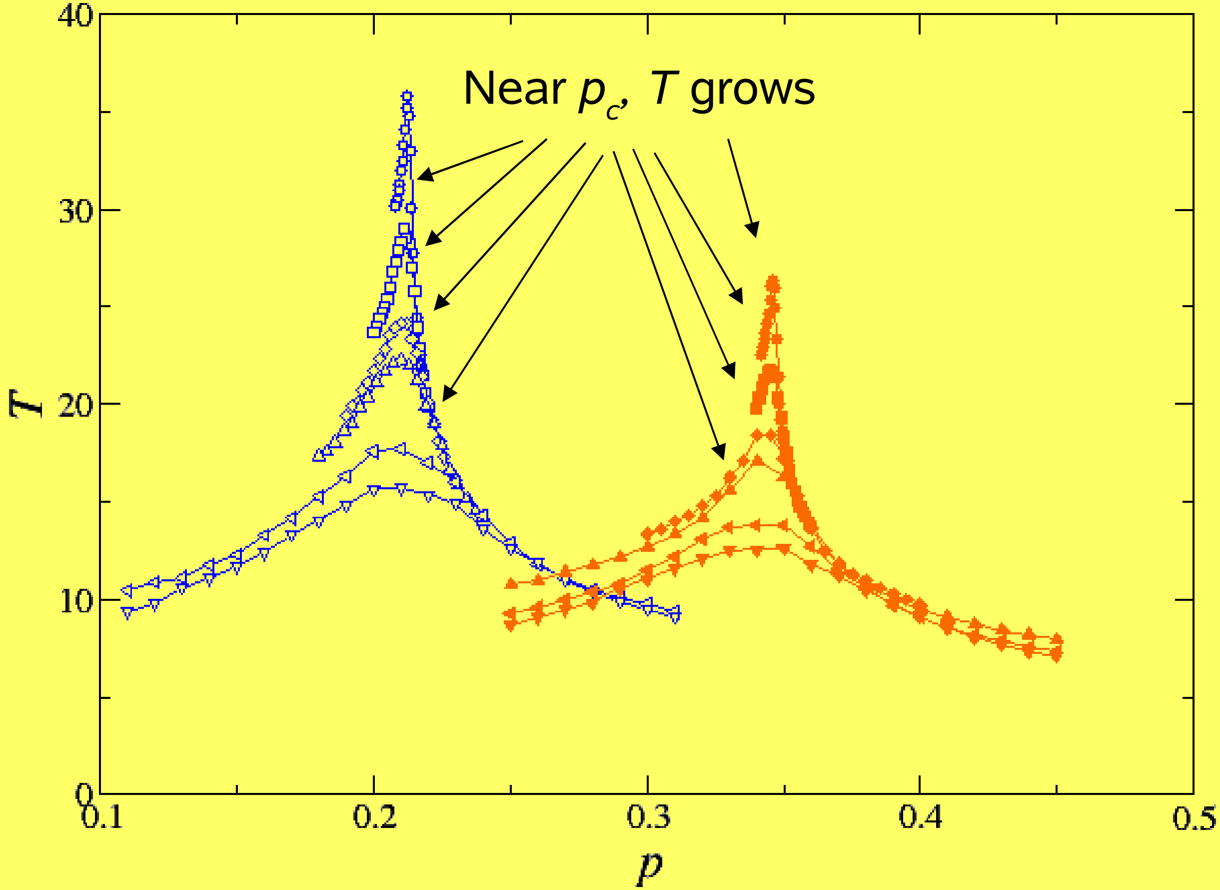


C.J. Tessone, ***et al.*** *Eur. Phys. Jour.* **B39,** 535 (2004).

Galam Model: Order parameter – size effects

Time to reach consensus, $T \sim \ln N$ *(theory says so)*

Linearisation around the fixed points $P_+(t+1) = F[\,P_+(t)\,]$



C.J. Tessone, *et al.* *Eur. Phys. Jour.* **B39,** 535 (2004).

Neighbourhood Model: definition

Introduced to study the effect of considering locality in the Galam Model

Naturally interpolates between local neighbourhood and *all-to-all* interaction

Acts on a regular lattice

In these local models, individuals are fixed at the sites of a regular lattice

The meeting cells are neighbourhoods defined by spatial location (local effects).

Meeting cells change with time

Neighbourhood Model: definition

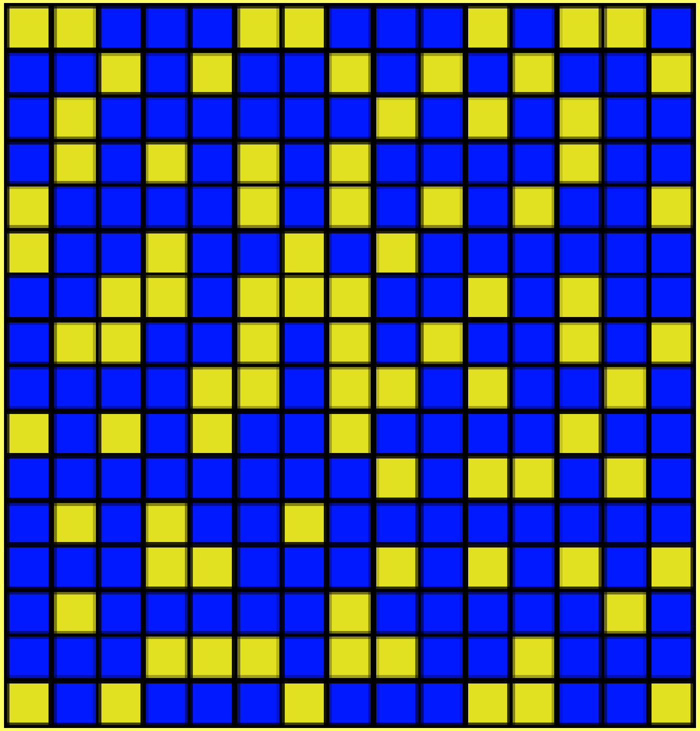
A site (x,y) of the system is randomly chosen

the lateral sizes are drawn from uniform distributions

$$1 \leq m_x \leq M$$

$$1 \leq m_y \leq M$$

For each update,
time increases as
$t \rightarrow \; t + m_x . m_y / N$



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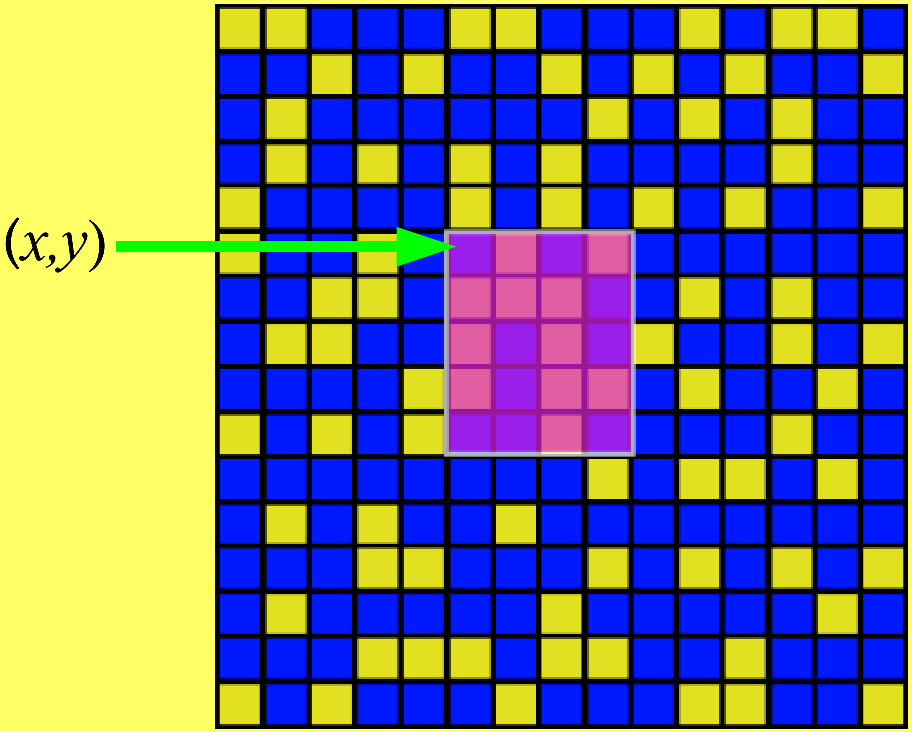

For each update, time increases as
$t \rightarrow t + m_x . m_y / N$

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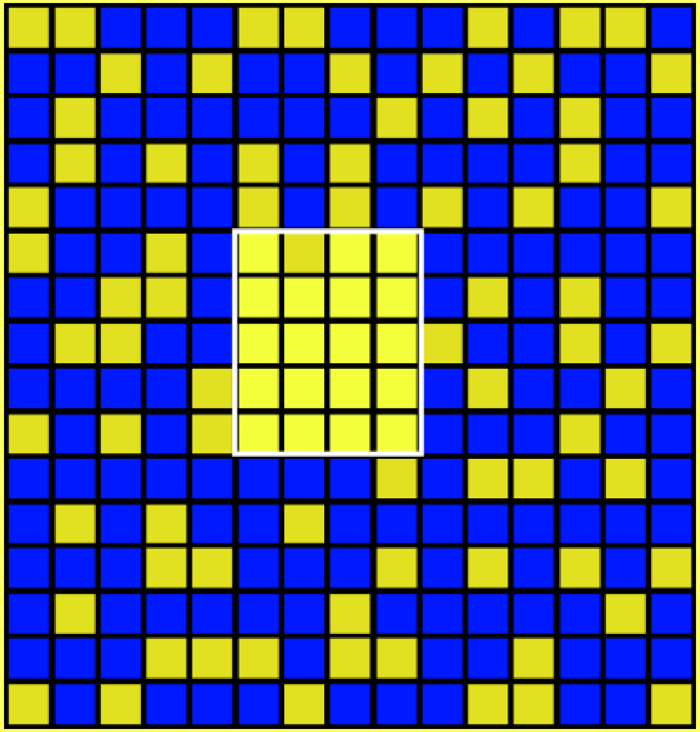

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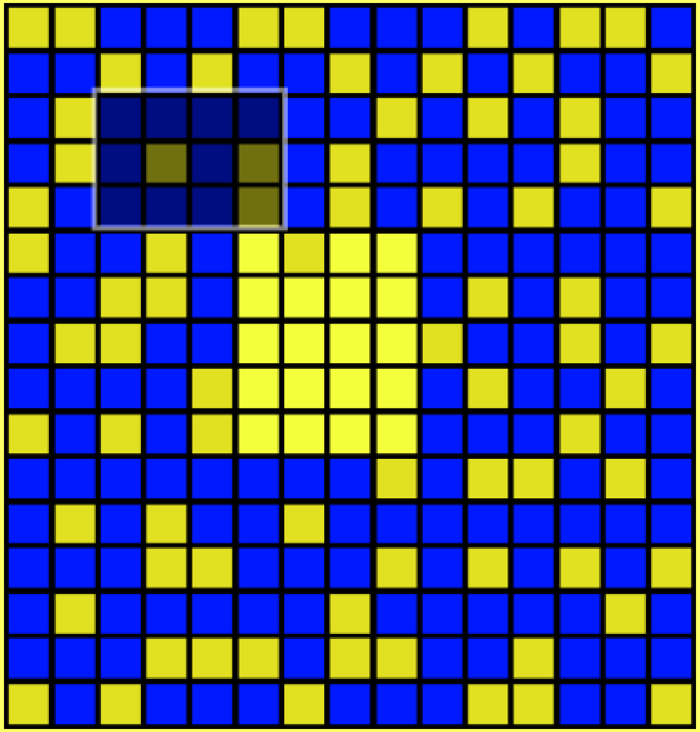


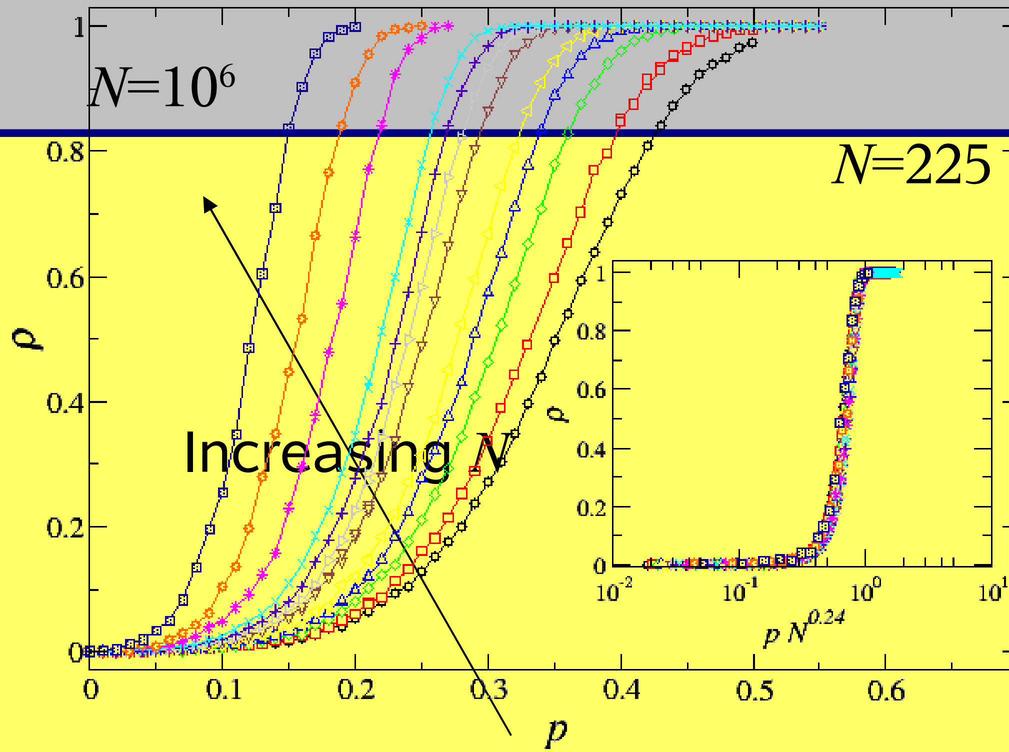
For each update,
time increases as
$t \rightarrow \; t + m_x . m_y / N$

play simulation

Neighbourhood Galam Model: apparent phase transition

Neighborhood Models: Steady State

There is an apparent phase transition: For finite N there is a critical value of p



$\rho\,(p,N) = f(pN^{\alpha})$

For $N \to \infty$, $p_c \to 0$

$p_c \sim N^{-\alpha}$

α = 0.24 (Asynchronous 2D model).

Neighbourhood Galam Model: apparent phase transition

$$\rho\,(p,N)=f(pN^{\alpha})$$

$$p_c \sim N^{-\alpha}$$

...This happens independently of the dimensionality of the lattice...

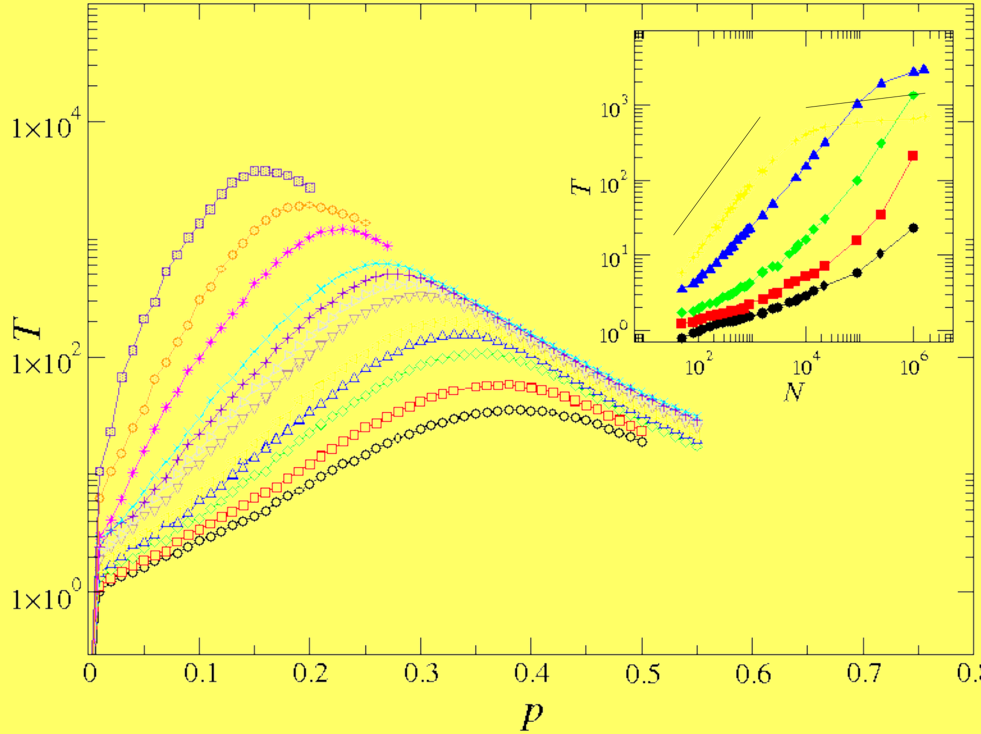
In the thermodynamic limit, the transition shifts toward

$$p=0$$

In an infinite system, the blue (favoured) opinion wins regardless the amount of initial supporters

Neighbourhood Galam Model: time to reach consensus

There is a "critical slowing-down" near the transition (typical footprint of phase transitions)



diff. exponents!!!

The time to reach consensus, scales as a power law of N

Neighbourhood Galam Model: critical radius

Depending on its initial size, the formed domains, may grow or shrink

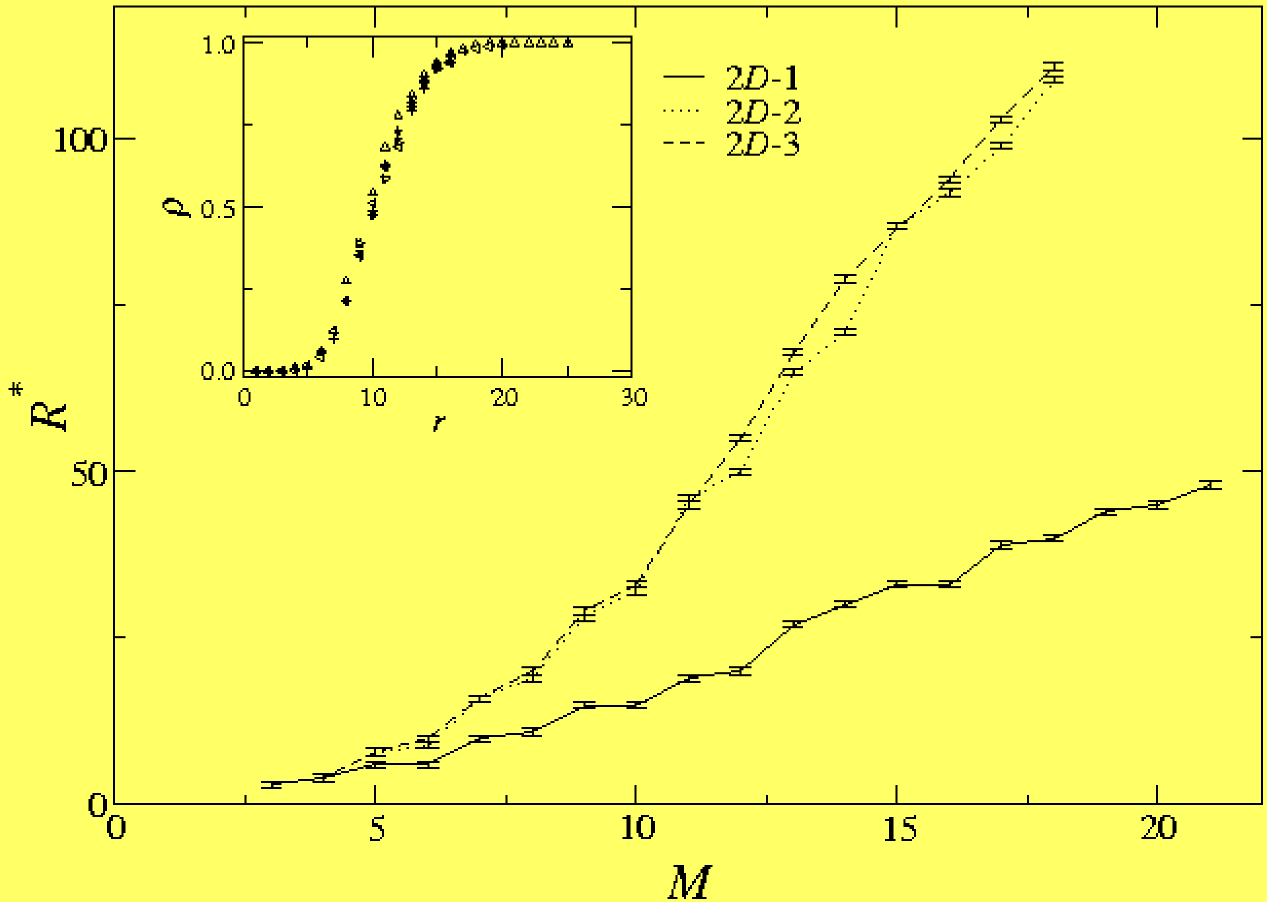
play simulation

We start with an initial circular island of radius r of favoured opinion and watch if it grows or shrink

We compute $\rho(r)$, and define R^* as the value of r such that $\rho(r)=0.5$

The size of the system must be such that $p = \pi R^{*\,2} / N < p_c$, i.e. a domain of blue opinion with typical radius equal to R^* should be plausible

Neighbourhood Galam Model: critical radius



Neighbourhood Galam Model: **summary**

Fluctuations (in the form of a finite number of agents) in **Galam's original model**:

-Smoothing of the transition of size $N^{-1/2}$.

-Time to reach consensus ~ $\ln N$.

In **neighbourhood models:**

Transition point shifts to $p_c=0$ in the thermodynamic limit (**apparent phase transition**).

Time to reach consensus grows as a power law of N.

Neighbourhood models are more effective to spread a minority opinion, although spreading takes a longer time.

Part II: Effects of system size in fashion spreading

Effects of system size in fashion spreading: introduction

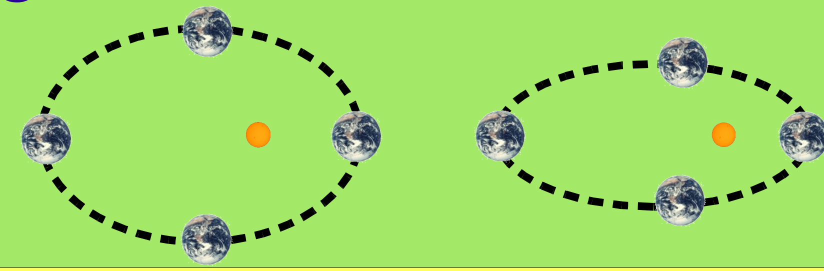
Stochastic resonance...

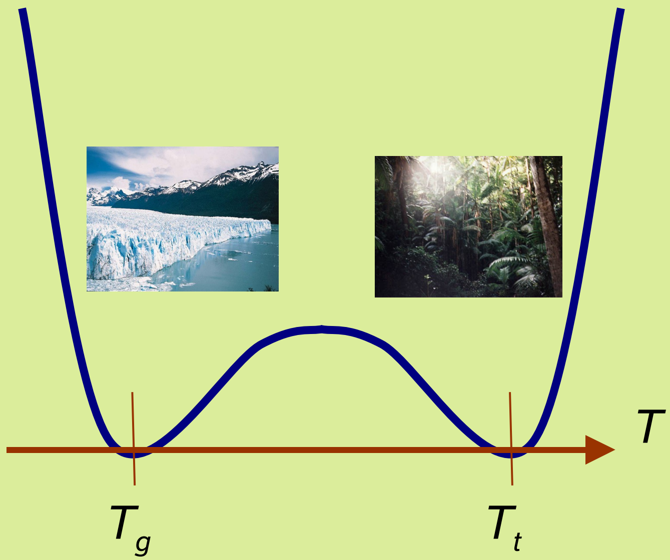
Given a non-linear system subject to noise and a (weak) external signal, the phenomenon of SR is one such that the signal gets amplified when the right amount of noise is applied to the system.

Effects of system size in fashion spreading: introduction

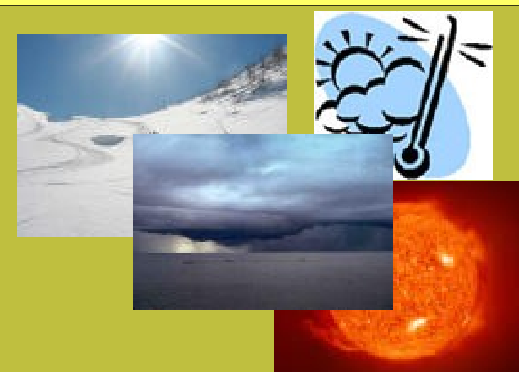
Stochastic resonance... ingredients

periodic modulation





bistable climate



fast changes in radiation

...An optimum level of noise exists, such that the weak modulation of excentricity change can be amplified...

Effects of system size in fashion spreading: introduction

Basic mechanism of stochastic resonance

$$\frac{d}{dt}x(t) = \underbrace{-x(t)^3 + b\,x(t)}_{\text{bistable system}} + \underbrace{A \sin\left(\frac{2\pi}{T_s}t\right)}_{\text{periodic signal}} + \xi(t)$$

bistable system

periodic signal
...weak...

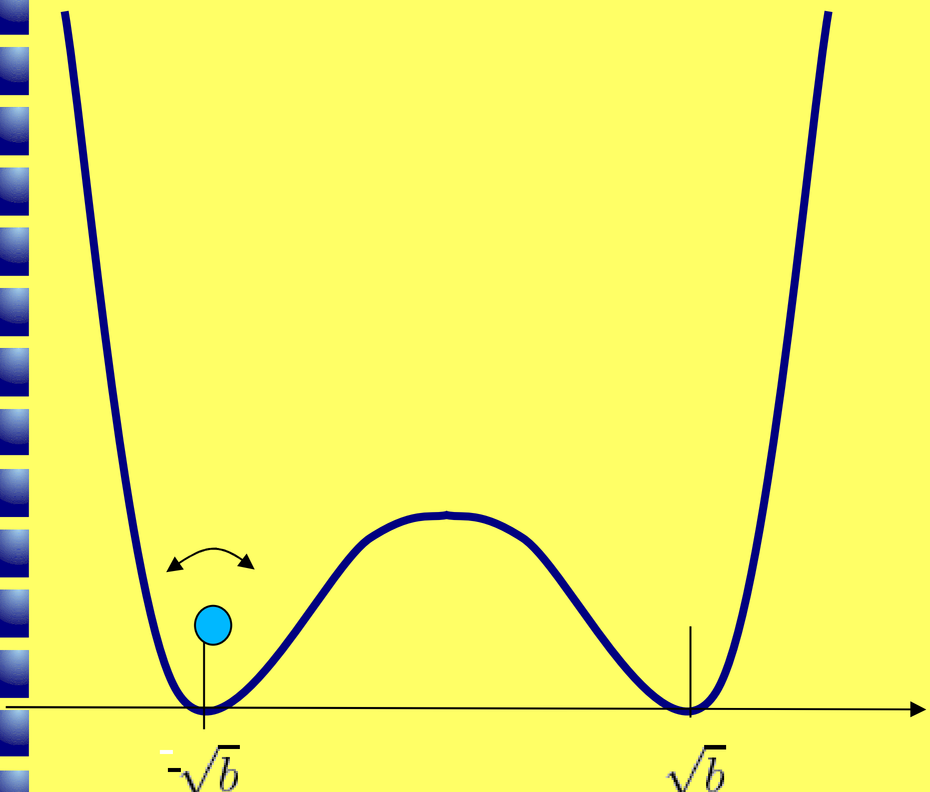
noise
...Gaussian white ...

$$\langle \xi(t) \rangle = 0,\ \langle \xi(t)\xi(t') \rangle = 2D\,\delta(t-t')$$

Effects of system size in fashion spreading: introduction

Basic mechanism of stochastic resonance

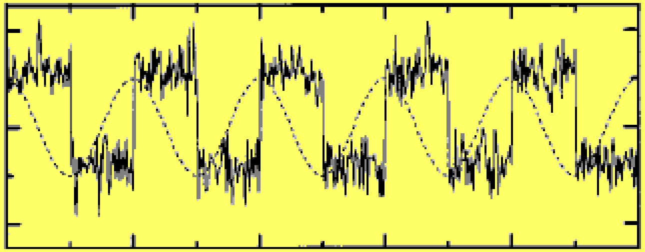
$$\frac{d}{dt}x(t) = -x(t)^3 + b\,x(t) + A\,\sin\left(\frac{2\pi}{T_s}t\right) + \xi(t)$$



When is satisfied the relation

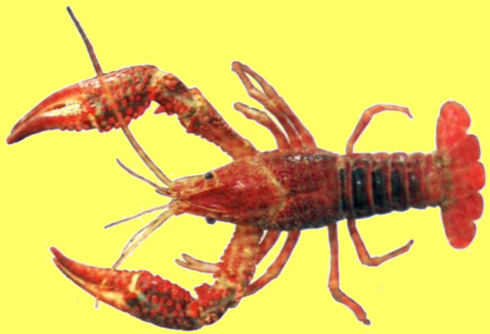
$$2\tau_K \cong T_s$$

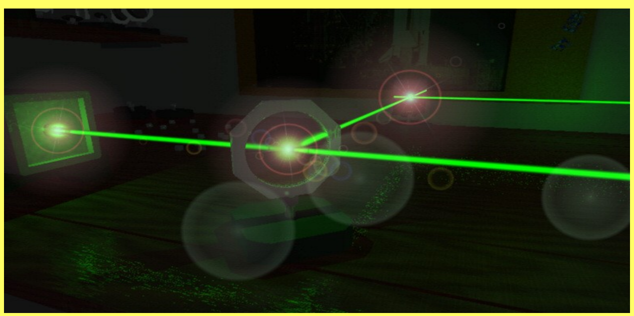
the synchonization between the external signal and system dynamics is optimal

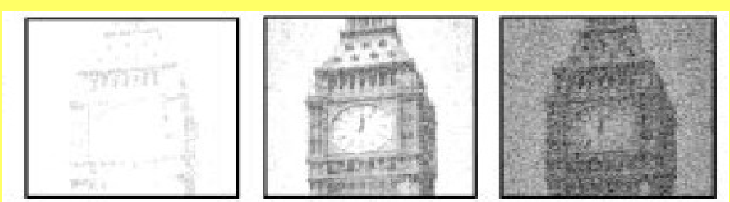


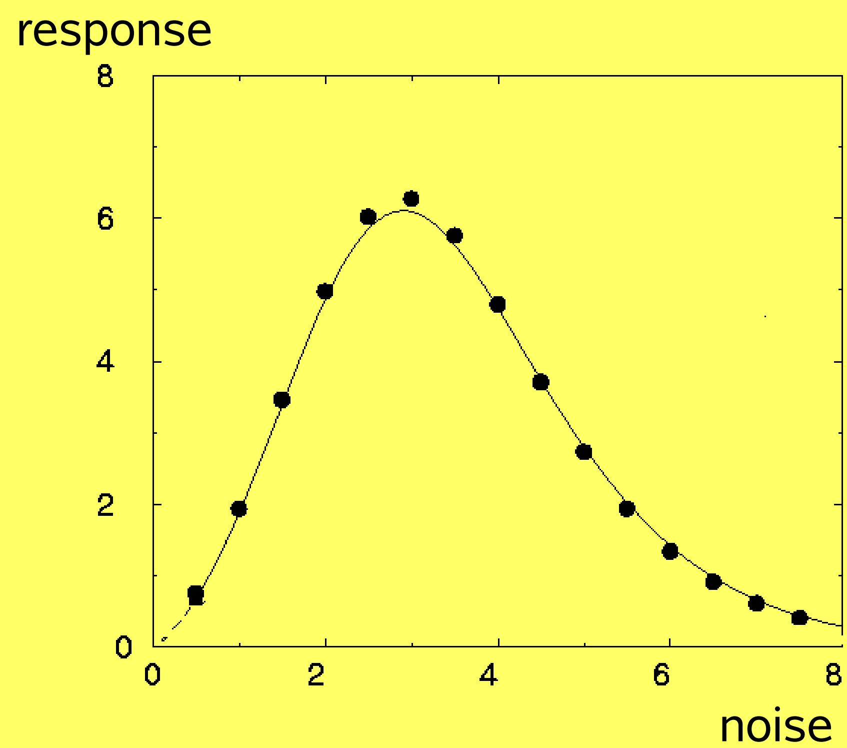
Effects of system size in fashion spreading: introduction

Typical result of stochastic resonance









Effects of system size in fashion spreading: introduction

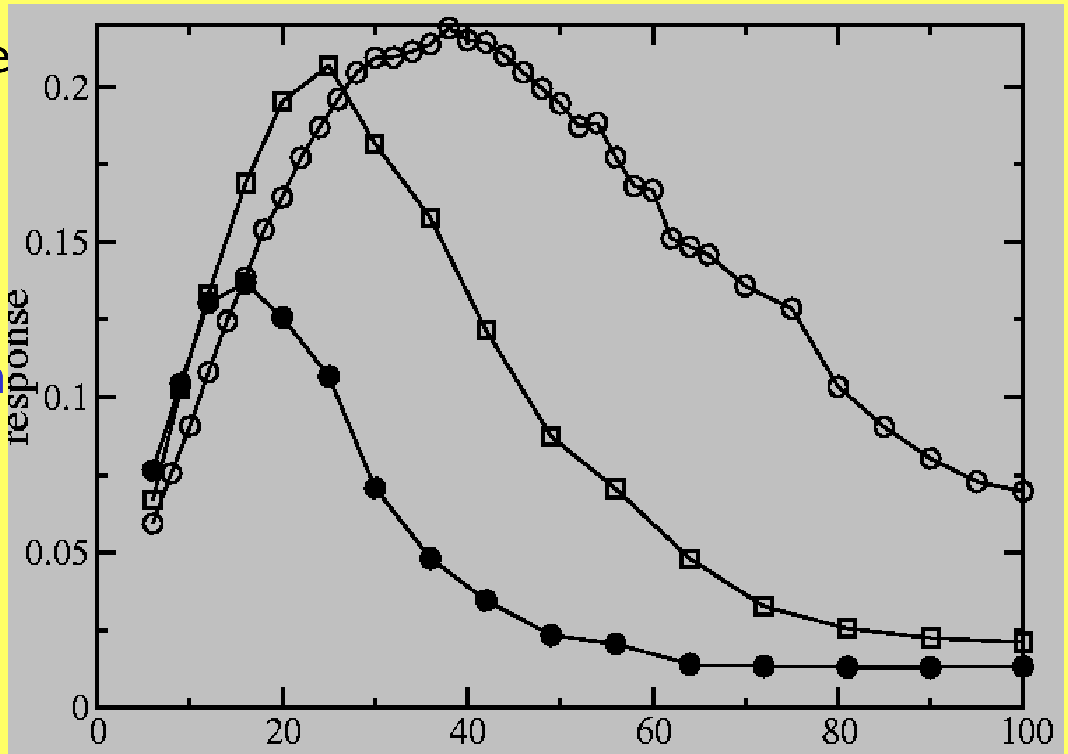
System-size stochastic resonance

If we conside

$\dot{x}_i =$ $A\cos(\omega t)$

And con $\frac{1}{N}\sum_{i=1}^{N} x_i$

$\dot{X} =$



… The effec… can be controled through *D* and system … ze *N*…

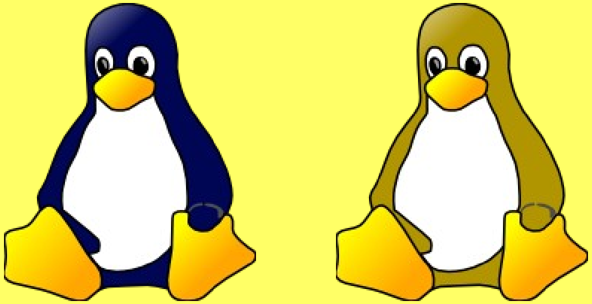
N

[Pikovsky, *et al.*, Phys. Rev. Lett. 88 (2002) 050601]

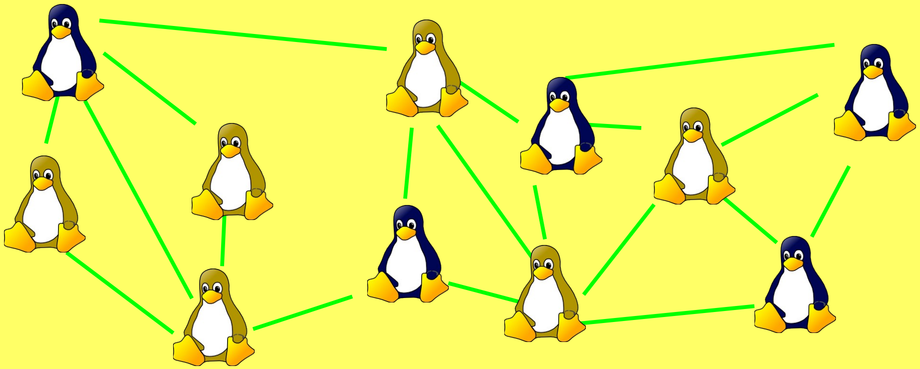
Effects of system size in fashion spreading: model description

[M. Kuperman, D.H. Zanette, *Eur. Phys. Jour. B*, **26** 387 (2002)]

- System formed by *N* individuals which have one of two opinions



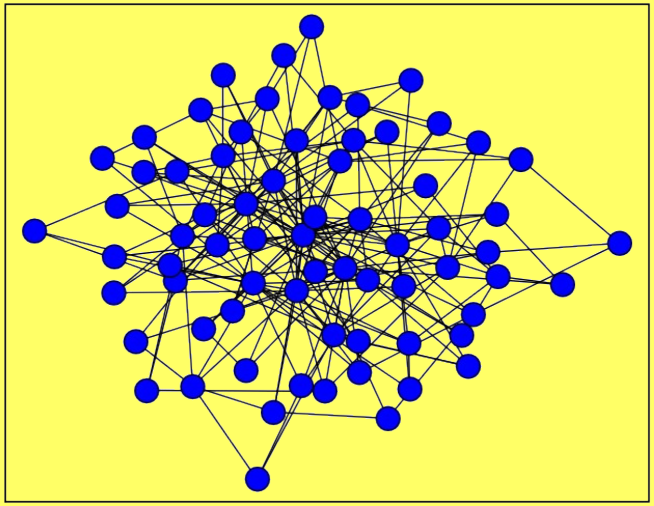
- Each individual has a set of neighbors

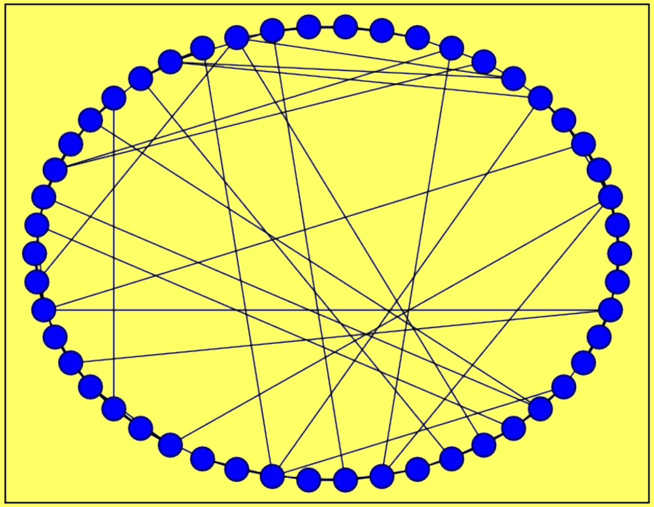


Effects of system size in fashion spreading: model description

As the network we considered either small world ones and scale-free

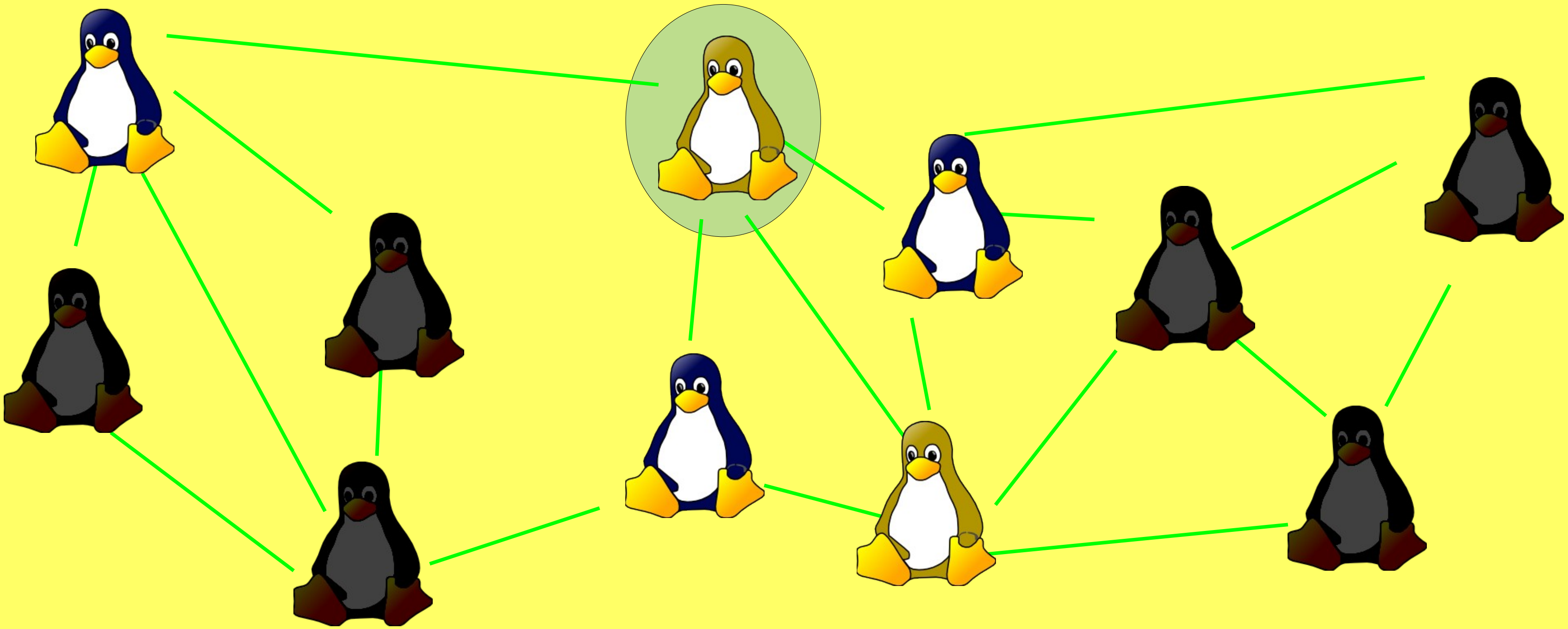
the existence of the phenomenon is independent on the network topology





Effects of system size in fashion spreading: model description

- **Opinion Update: 1 *imitation***
  An individual is randomly chosen and takes the majoritary opinion of his neighbors



Effects of system size in fashion spreading: model description

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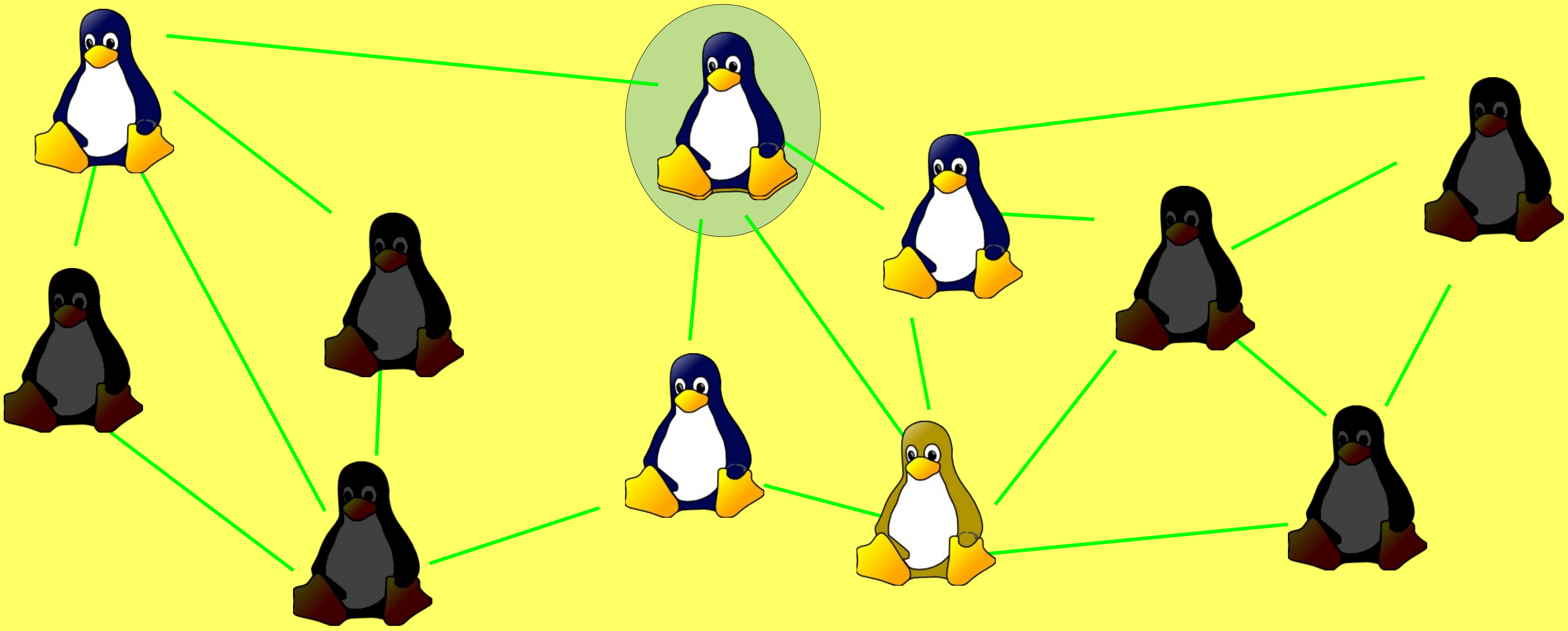
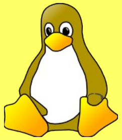


Effects of system size in fashion spreading: model description

- **Opinion Update:** 2 ***External influence***

The social preference for one of each opinions is assumed to change periodically in the form

$$\epsilon \cos\left(\omega\, t\right) \begin{cases} < 0 \\ > 0 \end{cases}$$

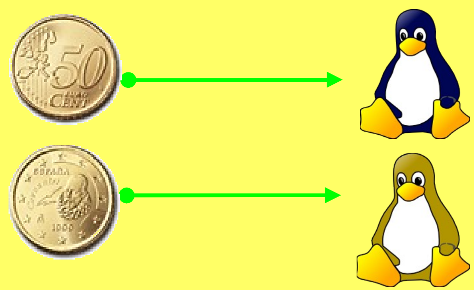


With probability $p_f(t) = \left|\epsilon \cos\left(\omega\, t\right)\right|$ the favored opinion is taken

Effects of system size in fashion spreading: model description

- **Opinion Update:** 3 ***random choice***

With probability **p** a random opinion is taken

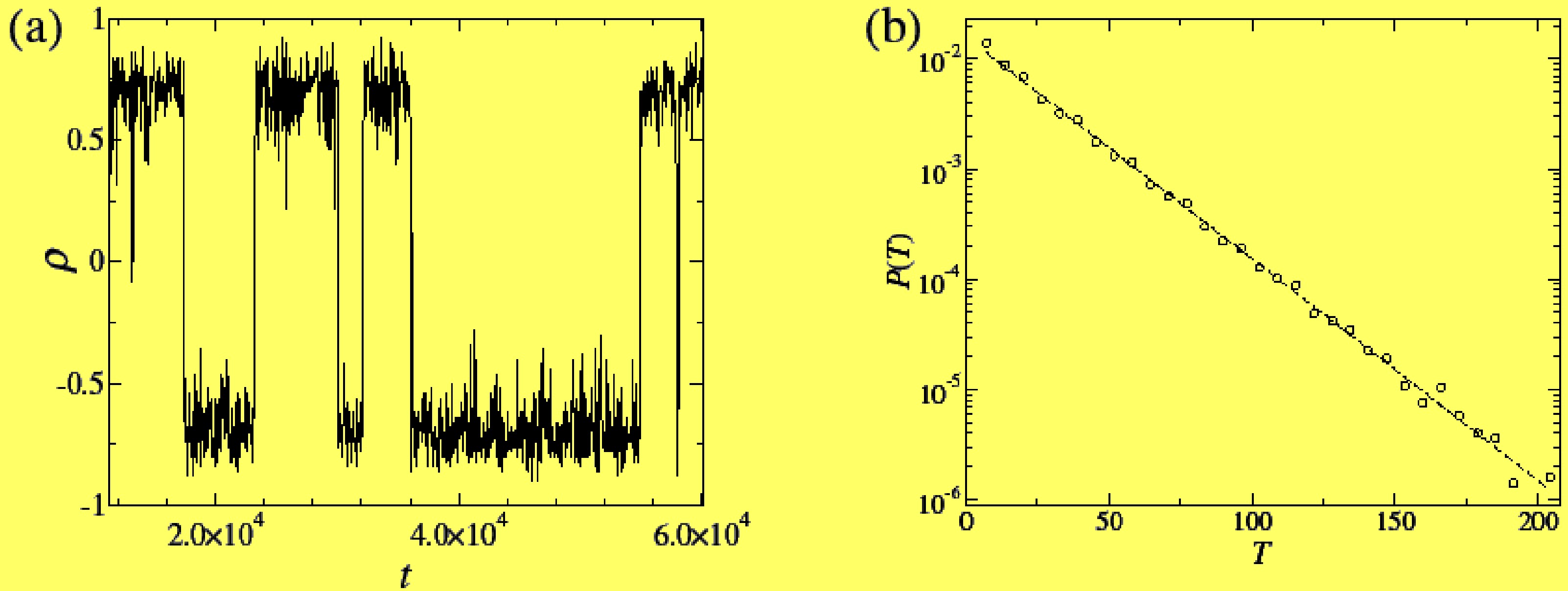


- Steps **1,2,3** are applied ***CONSEQUTIVELY***
- After each repetition, *t* increases by 1/N

Effects of system size in fashion spreading: results

Evolution of the system

… In absence of an periodic signal, the system behaves as a bistable one



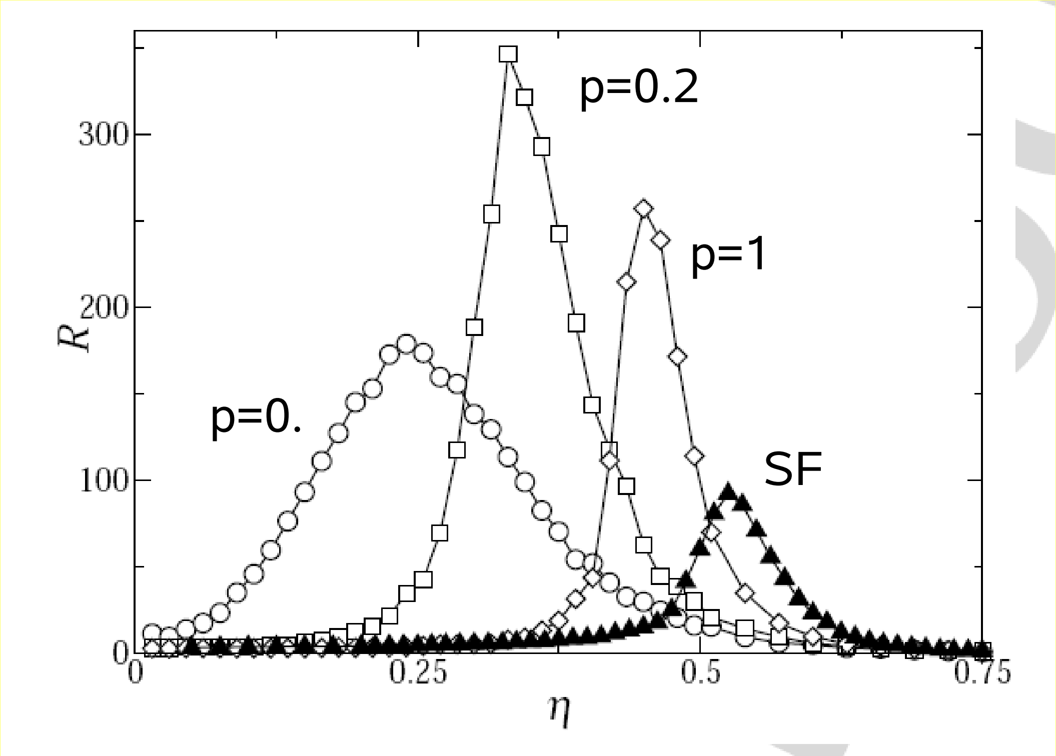
… η is a measure of noise intensity…

All the ingredients needed for the Stochastic Resonance phenomenon are present...

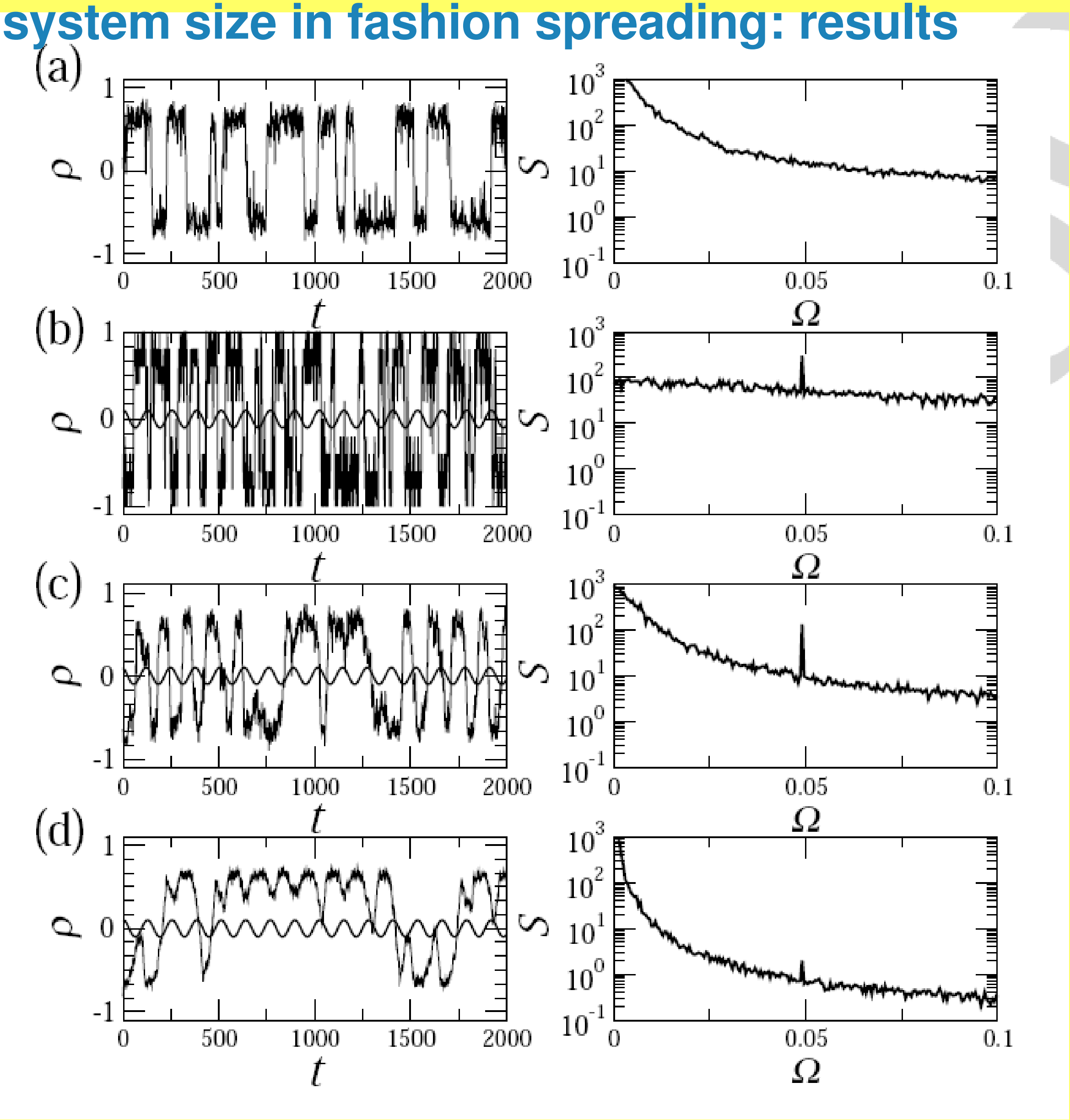
Effects of system size in fashion spreading: results

-We have all the ingredients for stochastic resonance:

-Bistable system, Coupling, Noise, External forcing



Effects of system size in fashion spreading: results



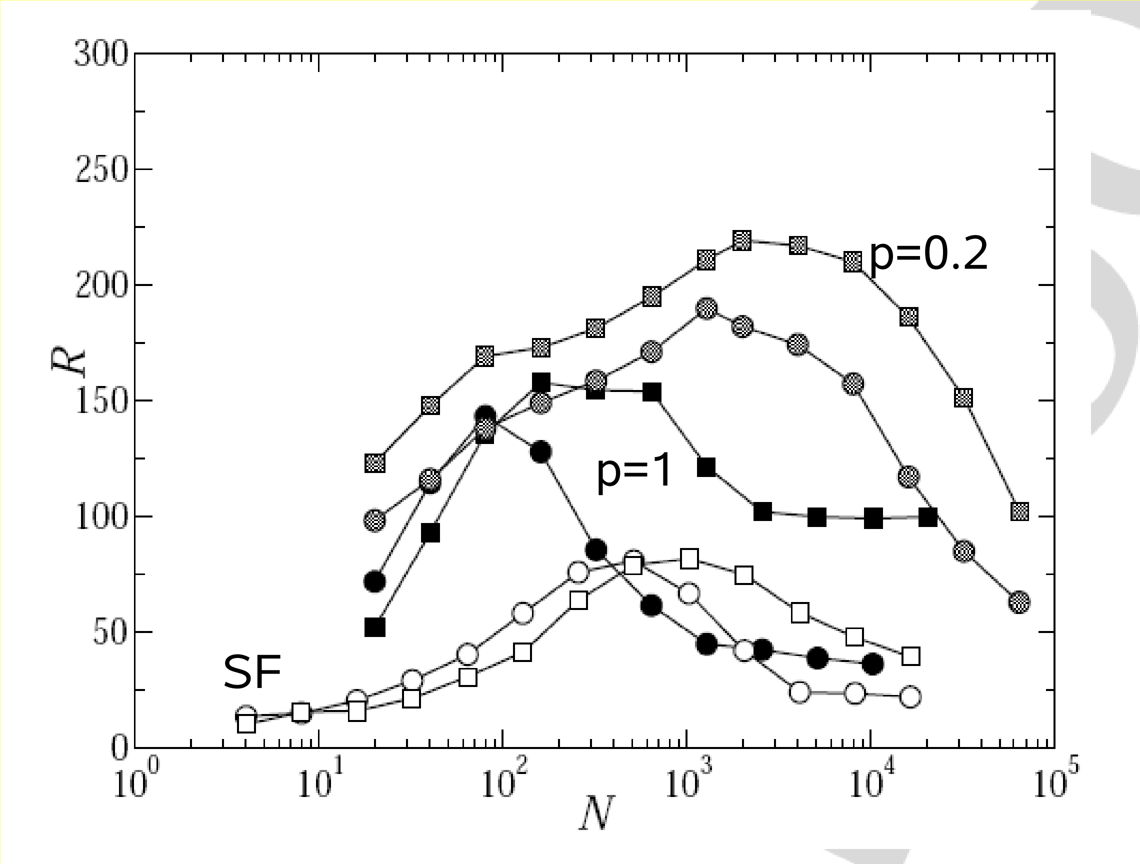
N=10

N=100

N=1000

Effects of system size in fashion spreading: results

Signal-to-noise ratio shows an optimum system size



Conclusions

In Statistical Physics we are used to the thermodynamic limit

$$N \to \infty$$

But in real physical systems, we should be happy with

$$N \gg 10^{23}$$

In computer simulations we struggle for larger and larger sizes and always try to extrapolate to infinite size.

Social systems are never that large and new phenomena can appear depending on the size or the number of individuals considered.

Conclusions

-System size can have a non-trivial role in some phase transitions in models of social interest.

-Apparent phase transitions appear in models of social interest (biased opinion).

-In noise driven systems, the "quality" of the output (synchronization with an external forcing or its regularity) depends on the system size.

- In a majority opinion formation model, an external influence works optimally in a society of the proper system size.

-This work stresses the non-trivial role that the system size has in the dynamics of social systems

-The thermodynamic limit should not be taken routinely in those models.

References

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