Probleme der Dynamik und stochastischen Theorie dissipativer Hamiltonscher Systeme
ETH Zürich 2006

Werner Ebeling
mit Jörn Dunkel, Udo Erdmann
Lutz Schimansky-Geier und Sergey Trigger
Humboldt-Universität Berlin
und
A. Chetverikov,V. Makarov,M. Velarde
Uni Complutense Madrid

Gliederung

- Grundlagen der Dynamik und Stochastik
- Beispiele aus Physik, Biologie
- Modelle dissipativer Bewegungen
- Aktive Bew in Potmulden-exakte Lösungen, Bifurkationsverhalten
- Aktive Bew in komplexen Potentialen
- Schwärme aktiver Teilchen mit WW,
- Geladene aktiveTeilchen

Skizze

- Als dissipative Hamiltonsysteme bezeichnet man in der Mechanik Systeme mit dissipativen Kräften.
- Bekannte Anwendungen: Selbsterregte Schwingungen/Wellen, Uhren, Motoren
- Neu: Brownsche Modelle der Schwarm- und Agentendynamik (Buch von FS)
- Hier: Neue Probleme & Lösungen für Modelle diss stochast Bewegung

1. Grundlagen der nichtlinearen dissipativen Mechanik

Von Helmholtz/Rayleigh/Barkhausen/ Van der Pol /Andronov zur Theorie der

Pionierarbeiten

- Helmholtz: “Die Lehre von den Tonempfindungen ...” (1863).
- Rayleigh: “Theory of Sound” (1883,1894).
- Poincare: Mechanique celeste (1892)
- Barkhausen: Dissertation NL Schwing (1907)
- Van der Pol: Theory of triode vibration (1920)
- Andronov: Cycles limites de Poincare (1929)
- Andronov/**Witt**/Chaikin: Th.d.Schwingungen (1939, 1959, 1965, 1966)

Rayleigh’s model of Brownian particles with energy support -> nonlinear friction

The idea: in the standard theory of linear oscillations

$$\frac{dx}{dt} = v\,;\ \frac{dv}{dt} = -\gamma v - \omega_0^2 x^2\,;\ \frac{dE}{dt} = -\gamma v^2 \qquad (1)$$

If $\gamma > 0$ (positive friction, energy loss) - damped oscillations,
if $\gamma < 0$ (negative friction, energy support) - amplification.
Rayleigh: need $\gamma < 0$ + nonlinearity to control amplitude.

$$\frac{dv}{dt} = \kappa v - \kappa' v^3 - \omega_0^2 x^2\,;\ \frac{dE}{dt} = v^2(\kappa - \kappa' v^2) \qquad (2)$$

Sufficient condition for active motion: $\kappa > 0, \kappa' > 0$.

Bewegungsgl. von Rayleigh

$$\frac{d}{dt}\mathrm{v} = -\gamma(v^2)\mathrm{v} - \omega_0{}^2 x,$$

Statt $\gamma = \gamma_0 = \mathrm{const},$

Funktion der Geschw. = negative Reibung :

$$\gamma(v^2) = -\gamma_1 + \gamma_2 v^2$$

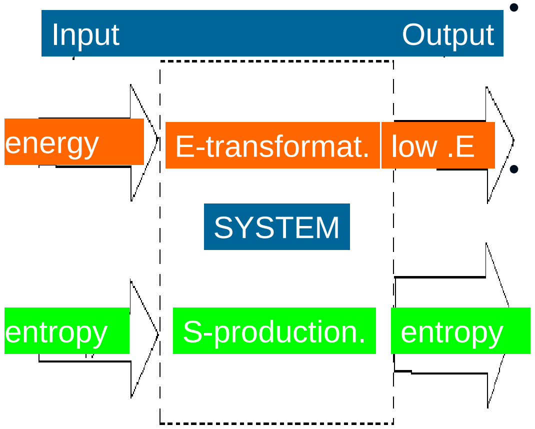
Verallgemeinerung: kanonisch-diss Systeme

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i} - g(H)\frac{\partial H}{\partial p_i}$$

The dissipative dynamics of this so-called canonical-dissipative system does not conserve the energy since

$$\frac{dH}{dt} = -g(H)\sum_i \left(\frac{\partial H}{\partial p_i}\right)^2$$

Thermodynamik offener Systeme: Barkhausen (1907), Prigogine (1947)



- **import of high-valued energy and export of low-valued energy = conditio sine qua non.**
- **That means: We need export of entropy, to compensate the unavoidable production of entropy by irreversible processes !!!**

Aktive Brownsche Teilchen mit nichtlinearer Reibung

$$\frac{d}{dt}v + \frac{1}{m}\frac{dU}{dr} = -\gamma(v^2)\,v + \sqrt{2D}\cdot\xi(t),$$

$$\gamma(v^2) = -\gamma_1 + \gamma_2 v^2$$

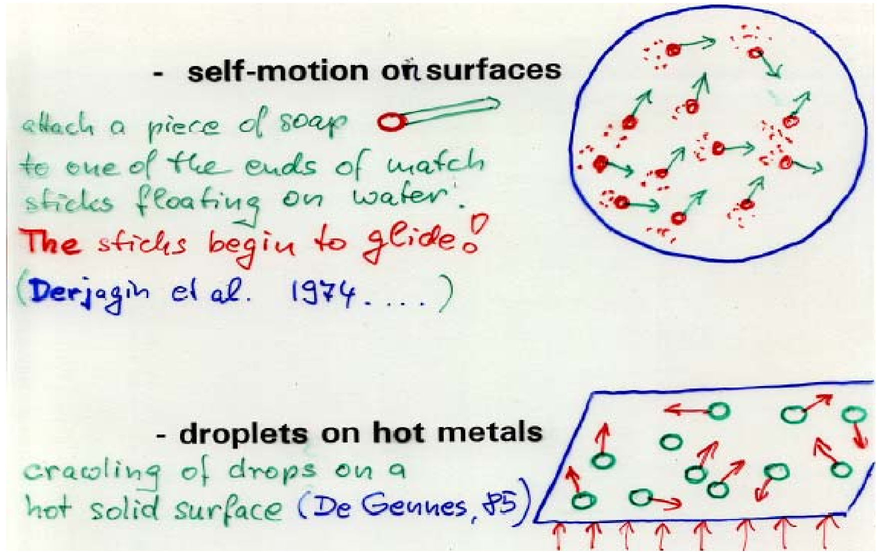
Fokker-Planck equation:

$$\frac{\partial P(\mathbf{r},\mathbf{v},t)}{\partial t} = \frac{\partial}{\partial \mathbf{v}}\left\{\gamma(\mathbf{r},\mathbf{v})\,\mathbf{v}\,P(\mathbf{r},\mathbf{v},t) + D\frac{\partial P(\mathbf{r},\mathbf{v},t)}{\partial \mathbf{v}}\right\} - \mathbf{v}\frac{\partial P(\mathbf{r},\mathbf{v},t)}{\partial \mathbf{r}} - \nabla U(\mathbf{r})\frac{\partial P(\mathbf{r},\mathbf{v},t)}{\partial \mathbf{v}}$$

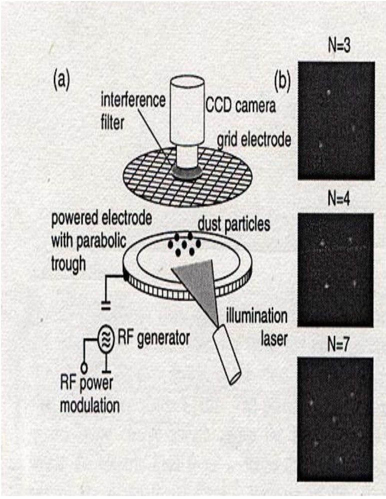
2. Grundlagen der stochastischen Theorie

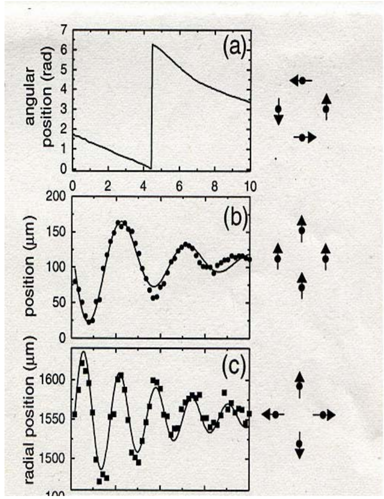
- Stratonovich: entwickelte ~ 1960 die statist Theorie der Rayleigh/van der Pol - Oszillatoren, Fokker-Planck-Gl.
- Klimontovich: Brownsche Teilchen mit aktiver Reibung, Lösungen der Fokker-Planck Gl., Fluktuationen, Korrelations-funktionen

3. Beispiele dissipativer Dynamik aus Physik und Biologie



2d-Staubplasmen: Melzer, Klingworth,Piel: PRL 2001

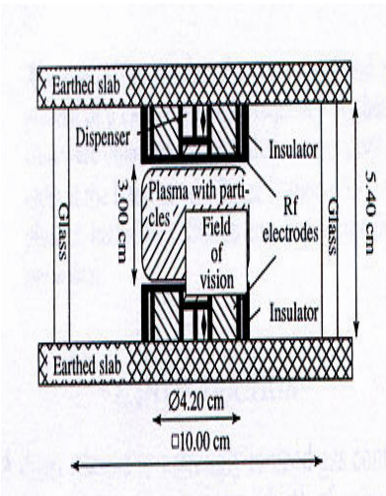


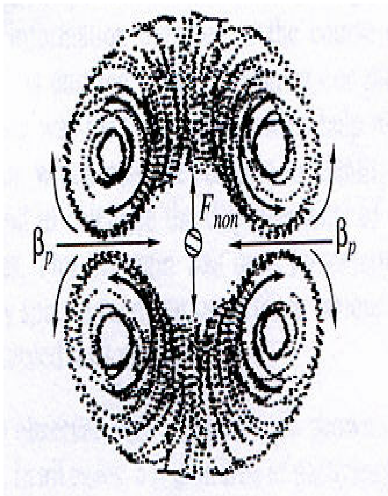


Dynamik biologischer Zellen

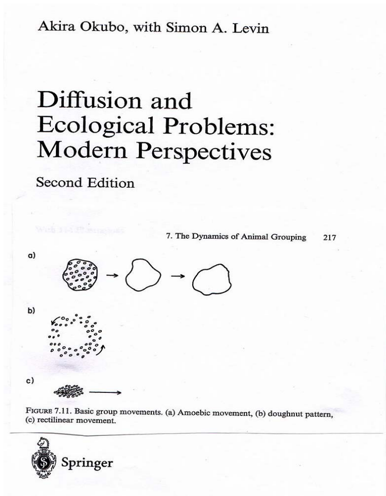
- For example granulocytes (white blood cells) can move actively on glass plates (experiments of Gruler, Schienbein et al.)
- Exist many other types of cell motion as taxis (with bias to a direction), as klinokinesis (bias of turning) etc.
- cells can show rotations, change of form and other complex motions ....
- This is important for their function !!!

ISS: 3d-Staubplasmen Fortov 2002





Typische Formen der Bewegung höherer Organismen (nach ökologischen Beobachtungen)



Akira Okubo, with Simon A. Levin

Diffusion and Ecological Problems: Modern Perspectives

Second Edition

7. The Dynamics of Animal Grouping 217

a)

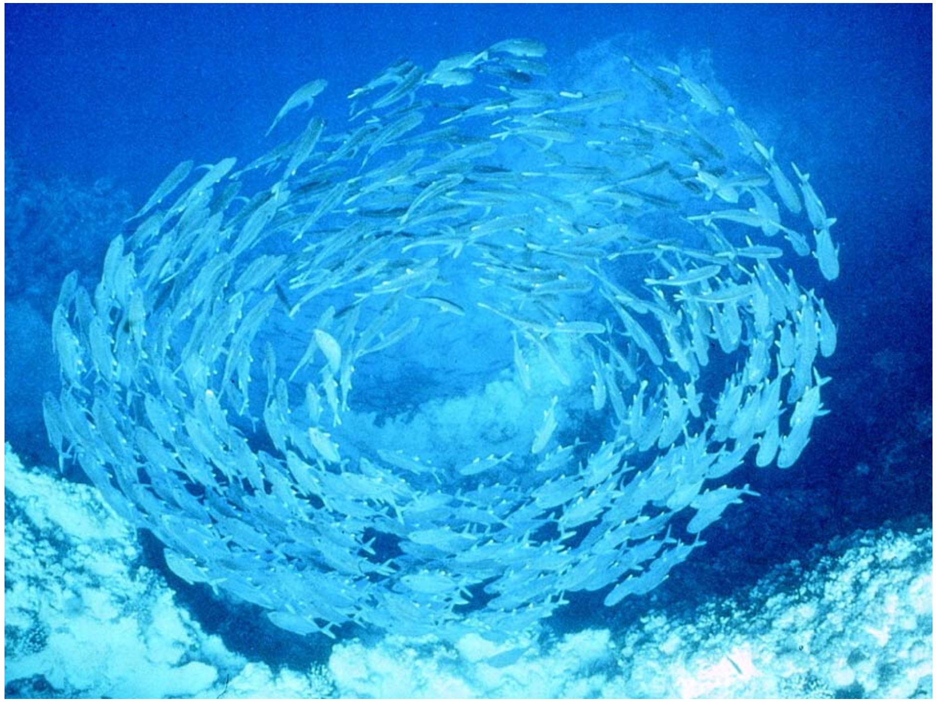
b)

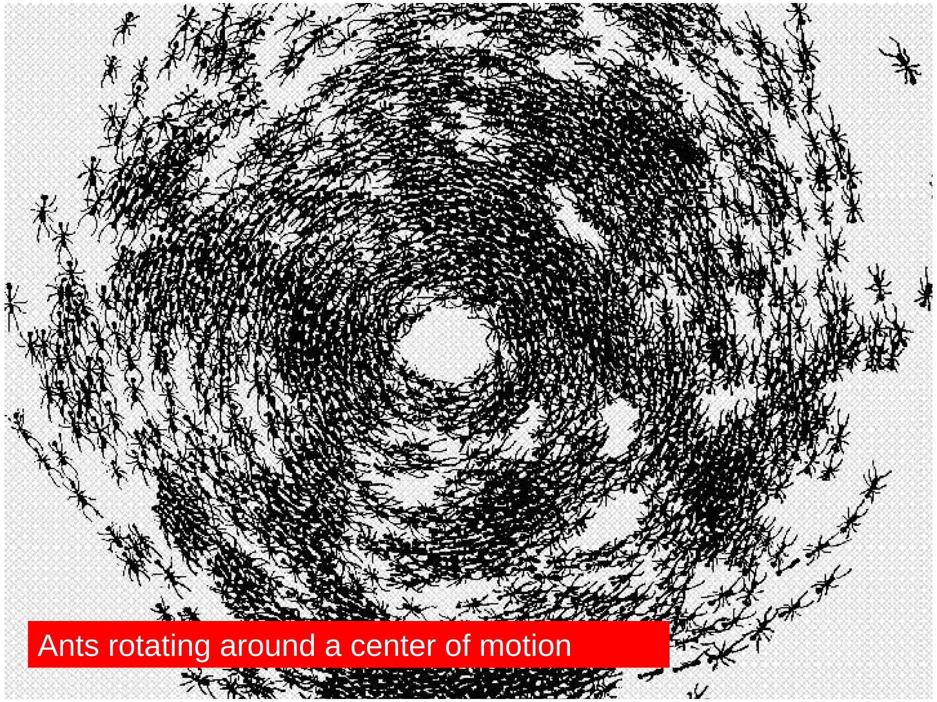
c)

FIGURE 7.11. Basic group movements. (a) Amoebic movement, (b) doughnut pattern, (c) rectilinear movement.

Springer

- generalization of many observations shows: Swarms have dynamical modes
- translational modes (rectilinear motion)
- rotational modes (swarm rotation)
- amoeba-mode (change of form)





Rotationsbewegungen von Fischen

4. Modelle dissipativer Bewegung mit Energiezufuhr

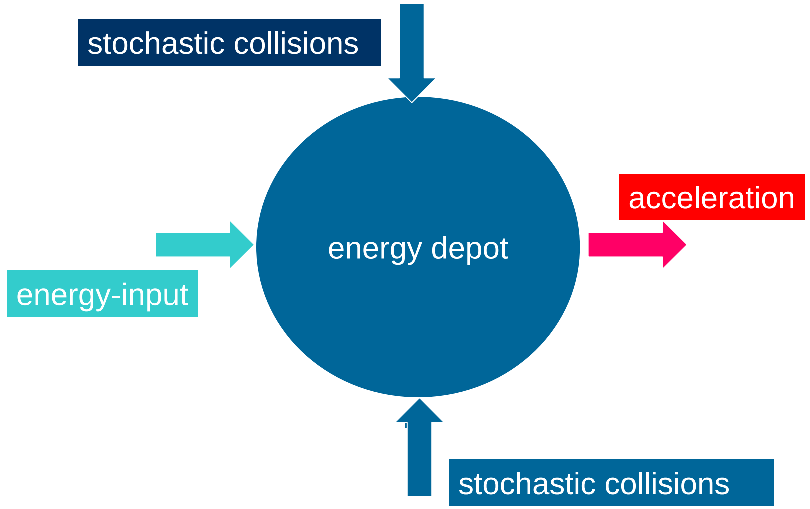
Energie wird aus der Umgebung mit Rate q aufgenommen, im Depot gespeichert und umgesetzt.

Energiebilanz:

$$\frac{d}{dt} e(t) = q(r) - ce(t) - dv^2 e(t),$$

adiabat Näherung: $q(r) = q_0, \quad e = \frac{q_0}{c + dv^2}$

Gespeicherte Energie → Beschleun

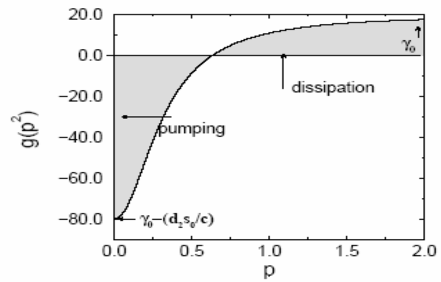


Friction: γ = velocity-dependent, possibly with a negative part !!! (pumping). Thermal equilibrium: $\gamma(\boldsymbol{v}) = \gamma_0 = \text{const.}$. General nonequilibrium case (SET-model):

Adiabatic appr.

$$\gamma(\boldsymbol{v}^2) = \left(\gamma_0 - \frac{dq}{c + dv^2}\right) \qquad (4)$$

where c, d, q = positive constants characterizing the energy flows from a depot to the particle.



Bewegungsgleichungen für Brownsche Teilchen mit E-Zufuhr Annahme eines Motors mit Tank e(t)

$$m\frac{d}{dt}v + \frac{dU}{dr} = mde(t)v - m\gamma_0 v + m\sqrt{2D}\cdot\xi(t)$$

Active term (an engine) | passive friction | noise

Löse die Fokker-Planck-Gl. für freie Teilchen

Stochastic force (assume that only the passive friction generates noise !!! $D = \gamma_0 kT$):

$$\langle \xi_i(t)\rangle = 0; \ \langle \xi_i(t)\xi_j(t')\rangle = \delta(t-t')\delta_{ij}$$

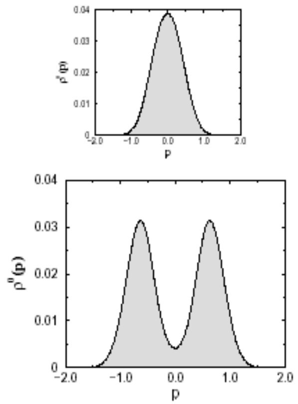
Free particles: v^2 = conserved quantity

Canonical-dissipative: → FPE has exact solutions .

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \boldsymbol{v}}\left(\gamma \boldsymbol{v} f + D\frac{\partial f}{\partial \boldsymbol{v}}\right)$$

$$f_0 = C\exp\left[-\frac{v^2}{2kT} + \frac{q}{2D}\log\left(1 + \frac{d}{c}v^2\right)\right]$$

Velocity distribution



Above undercritical, below overcritical

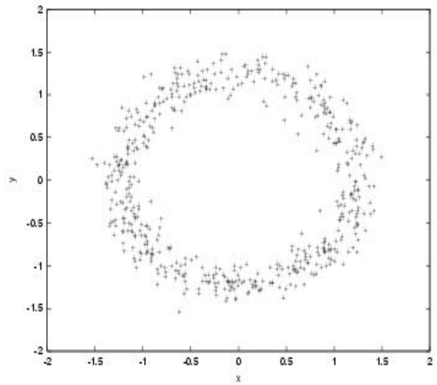
5. Aktive Bewegung in Potentialmulden

- Normal BM:
- Boltzmann df
- centered around the minimum of the potential
- Active BM: force equil.

$$m \frac{v_0^2}{r_0} = m \omega_0^2 r_0$$

$$v_0 = \omega_0 r_0$$

- Active BM: right + left limit cycle r_0



Kanonisch-diss Systeme (allgemein)

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -\frac{\partial H}{\partial q_i} - g(H)\frac{\partial H}{\partial p_i} + (2D(H))^{1/2}\xi(t). \tag{13}$$

Here $\xi(t)$ is a delta-correlated white noise. The essential assumption is, that noise and dissipation depend only on H. The following Fokker-Planck equation corresponds to the Langevin equation

$$\frac{\partial \rho}{\partial t} + \sum p_i \frac{\partial \rho}{\partial q_i} - \sum \frac{\partial H}{\partial p_i}\frac{\partial \rho}{\partial p_i} = \sum \frac{\partial}{\partial p_i}\left[g(H)\frac{\partial H}{\partial p_i}\rho + D\frac{\partial \rho}{\partial p_i}\right]. \tag{14}$$

The special structure of the dissipative and noise terms permits to find exact stationary solutions in the following form

$$\rho_0(q_1 ... q_f p_1 ... p_f) = Q^{-1} \exp\left(-\int_0^H \mathrm{d}H' \frac{g(H')}{D(H')}\right). \tag{15}$$

Dynamik ohne Rauschen

Betrachte Pottopf $U(x_1, x_2) = \frac{1}{2}(a_1 x_1^2 + a_2 x_2^2)$

$$m\frac{d}{dt}v_1 + a_1 x_1 = -m\gamma(v^2)v_1$$

$$m\frac{d}{dt}v_2 + a_2 x_2 = -m\gamma(v^2)v_2$$

Für $a_1 = a_2 = m\omega_0^2$ ist eine exakte Lösung der Grenzzyklus

$$x_1 = r_0 \cos(\omega_0 t + \psi)$$

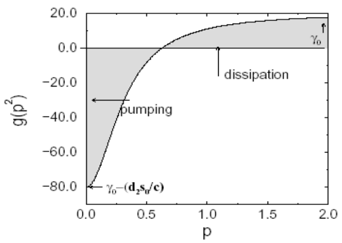
$$x_2 = r_0 \sin(\omega_0 t + \psi)$$

$$r_0 = v_0 / \omega_0$$

Für $a_1 \neq a_2$ komplizierte Lissajousfiguren the characteristic velocity v_0

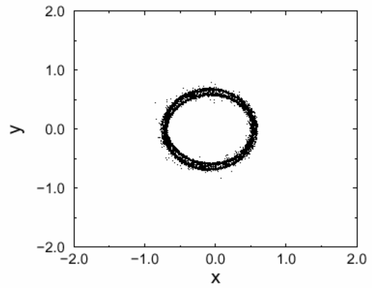
= zero of friction = attractor of motion

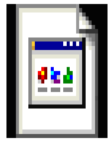
Depot model - SET



Active friction: Zero of the velocity $v_0^2 = \frac{d}{c}\mu; \quad \mu = \frac{qd}{c\gamma_0} - 1$

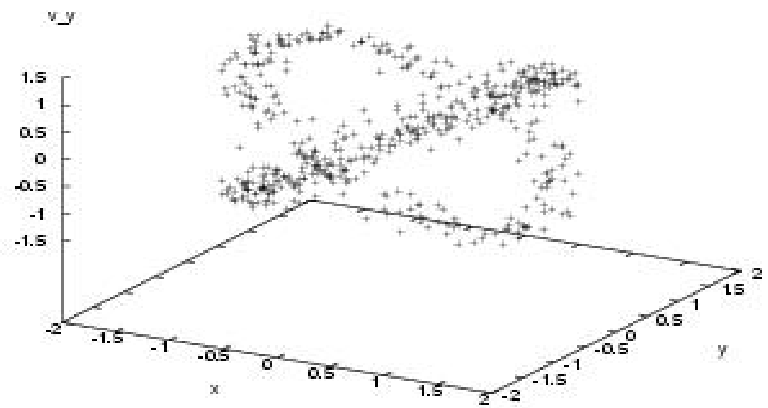
10000 aktive Teilchen um linear anzieh. Zentrum



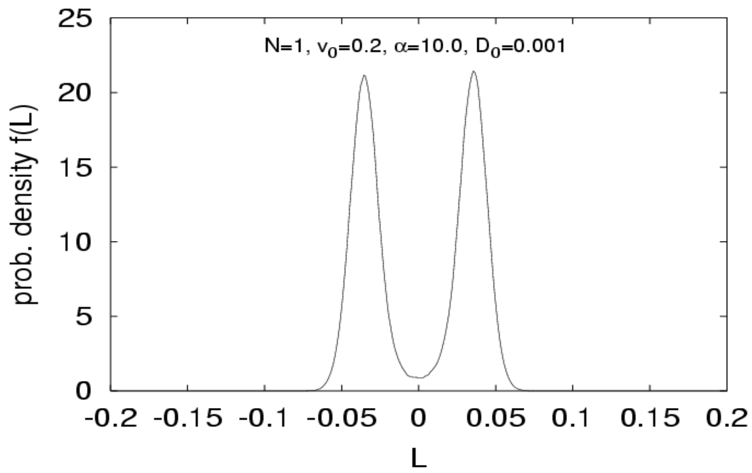


Swaest1.gif

Beobachte Grenzzyklen im Uhrzeiger und Anti-Uhrzeigersinn



Lösung der FPGl. f(H,L)

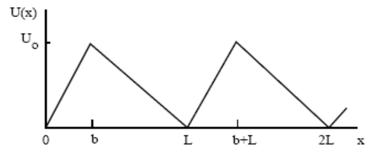


6.Aktive Bewegung in komplexen Potentialen

- 1. Studiere anharmonische, nicht radialsymmetrische Potentiale, stabile Lissajousfiguren, Arnold-Zungen
- aktive Bewegung auf Ratchets, geschlossene und offene stabile Trajektorien
- aktive Wellen auf Ketten, optische und solitonartige stabile Moden

Physica A 273 (1999) 294–314

Ratchets .....



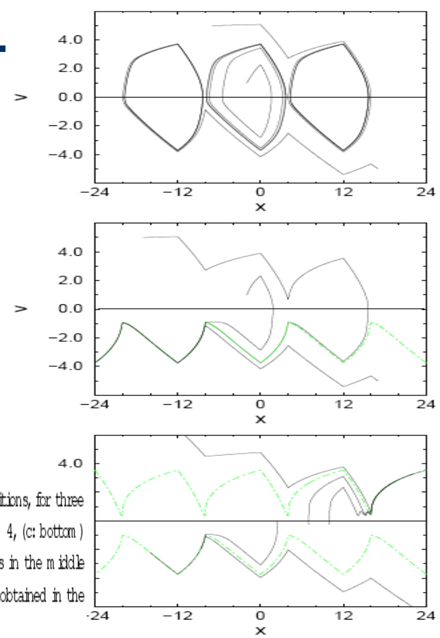


Figure 2: Phase-space trajectories of particles starting with different initial conditions, for three different values of the conversion parameter d_2: (a: top) $d_2 = 1$, (b: middle) $d_2 = 4$, (c: bottom) $d_2 = 14$. Other parameters: $q_0 = 1$, $c = 0.1$, ${}_0 = 0.2$. The dashed-dotted lines in the middle and bottom part show the unbound attractor of the delocalized motion which is obtained in the long-time limit.

Nicht radialsymm Pot: Frequ.verh n=2,3

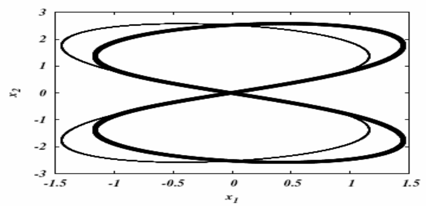


Fig. 2. Projections of the two limit cycles to the $\{x_1, x_2\}$-plane corresponding to $m:n = 2$ resonance obtained from simulations (Rayleigh law: $\alpha = 5$, $\beta = 1$).

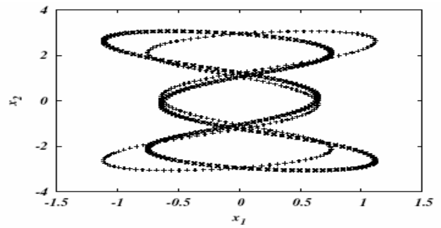


Fig. 6. Projections of the two limit cycles to the $\{x_1, x_2\}$-plane corresponding to the $m:n = 3$-resonance obtained from simulations ($\omega_1 = 2.7$, $\omega_2 = 1$, all other parameters as in Fig. 2).

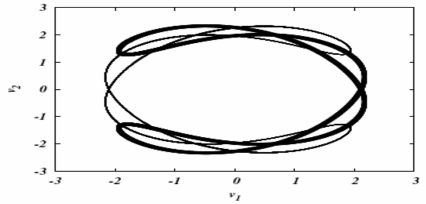


Fig. 3. Projections of the two limit cycles to the $\{v_1, v_2\}$-plane corresponding to $m:n = 2$ to the $\{v_1, v_2\}$-plane obtained from simulations (same parameters as in Fig. 2).

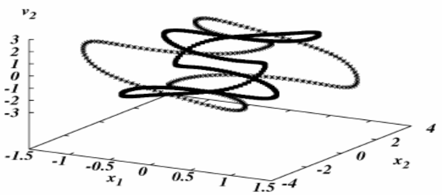
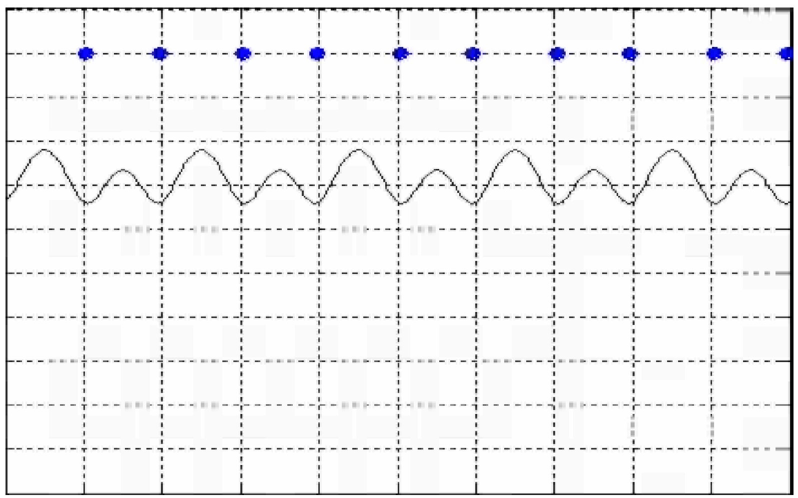
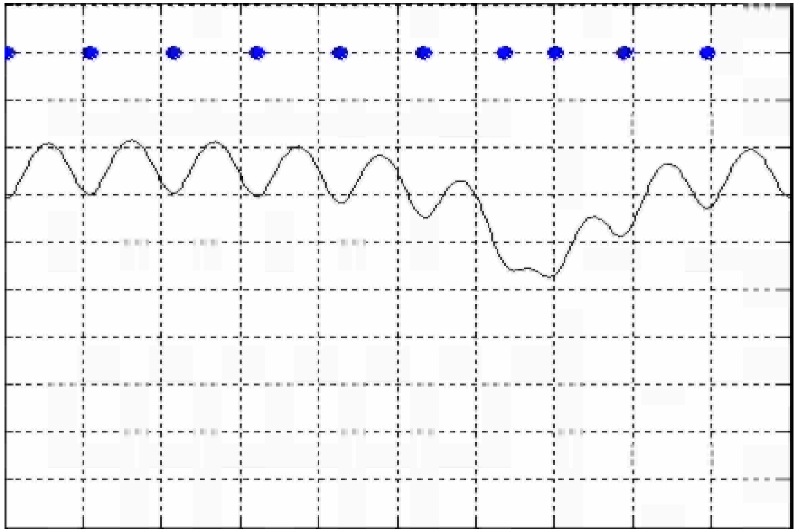


Fig. 7. Projections of the two limit cycles for $m:n = 3$ to the $\{x_1, x_2, v_2\}$-plane obtained from simulations.

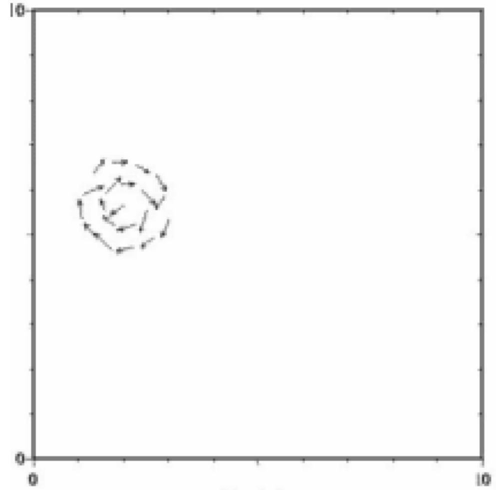
Nichtlineare Kette mit Antrieb



Angetriebene laufende Wellen: stabile dissipative Solitonen



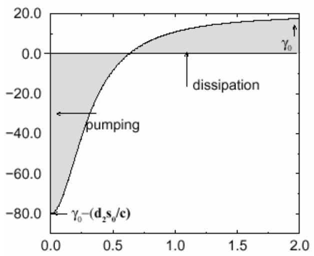
rotating cluster of Morse particles: bistability of L



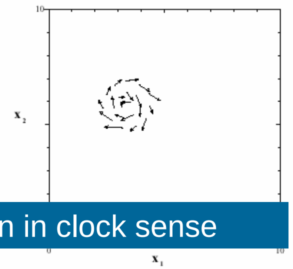
7. Schwärme aktiver Teilchen mit Wechselwirkung

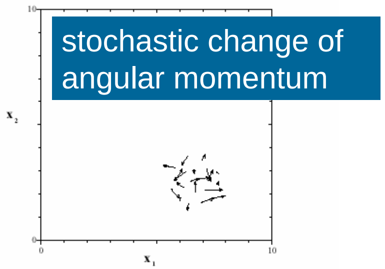
$$\frac{d}{dt}\boldsymbol{r}_i = \boldsymbol{v}_i\,;\;\; \frac{d}{dt}\boldsymbol{v}_i = -\gamma(v^2)\boldsymbol{v}_i - a\boldsymbol{r}_i - \sum_j \frac{\boldsymbol{r}_{ij}}{r_{ij}}\Phi'(r_{ij}) + \sqrt{2D_0}\,\boldsymbol{\xi}_i(t)$$

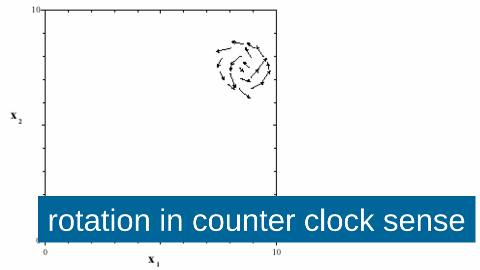
negative friction at small velocities



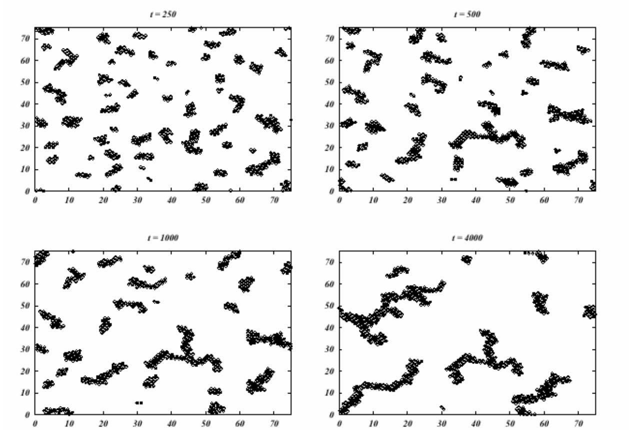
Cluster of ABT with Morse-inter.



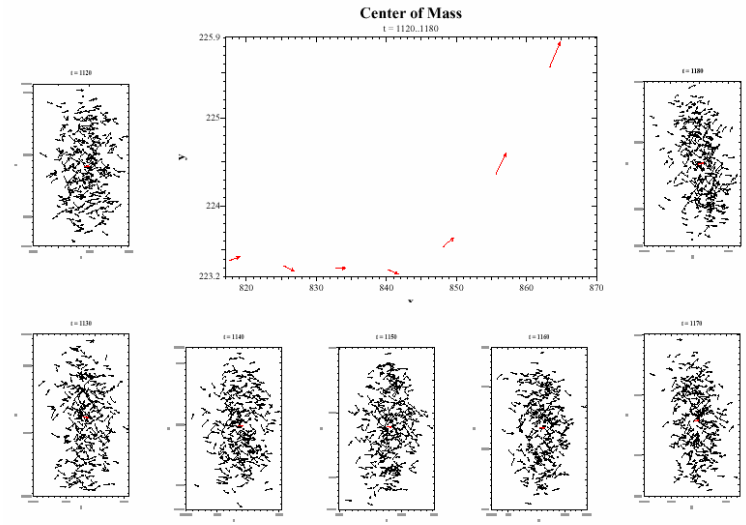




Cluster formation of ABT with Morse-int.

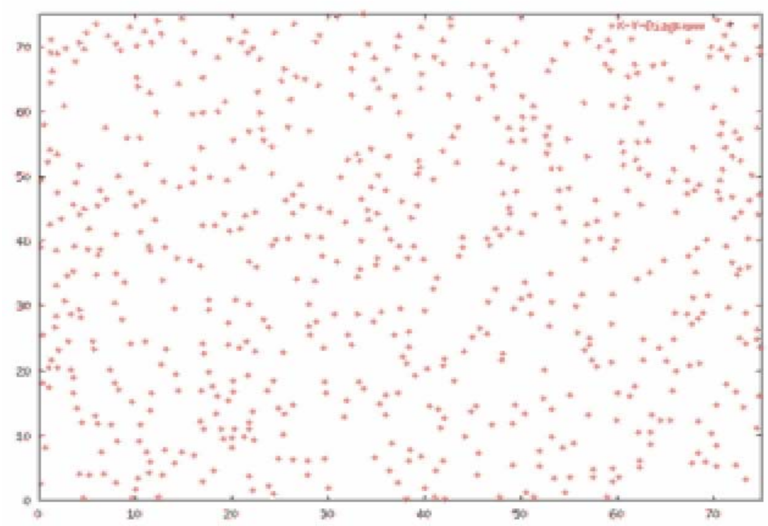


structures of amoeba kind show translations, rotations and stochastic change of forms

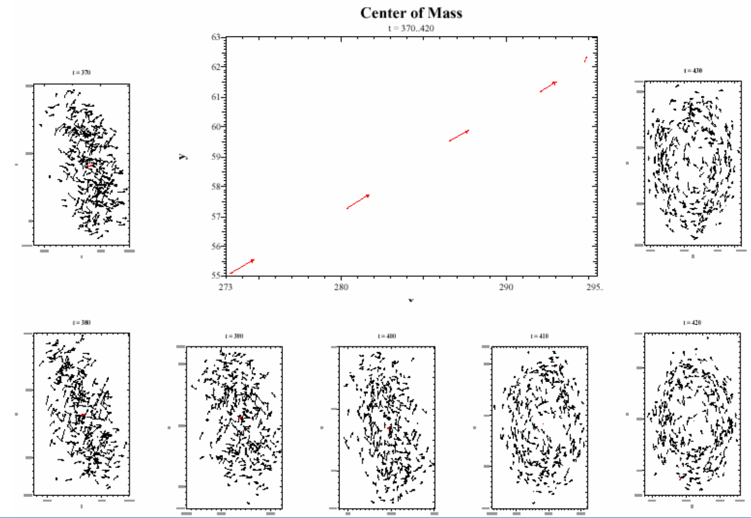


With Erdmann/Mikhailov (PRE 2005): translational mode

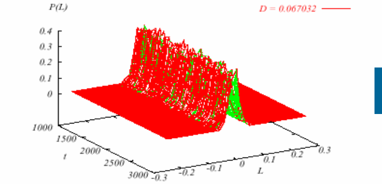
Cluster aktiver Partikel mit WW



With Erdmann/Mikhailov: noise induced phase transition:

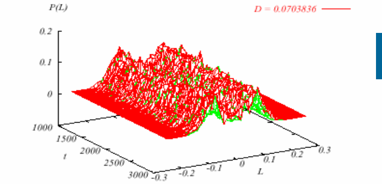


translation comes to stop --> rotational mode with increasing noise occurs a transition from translation (no angular momentum) to rotation (bistable angular momentum)



small noise

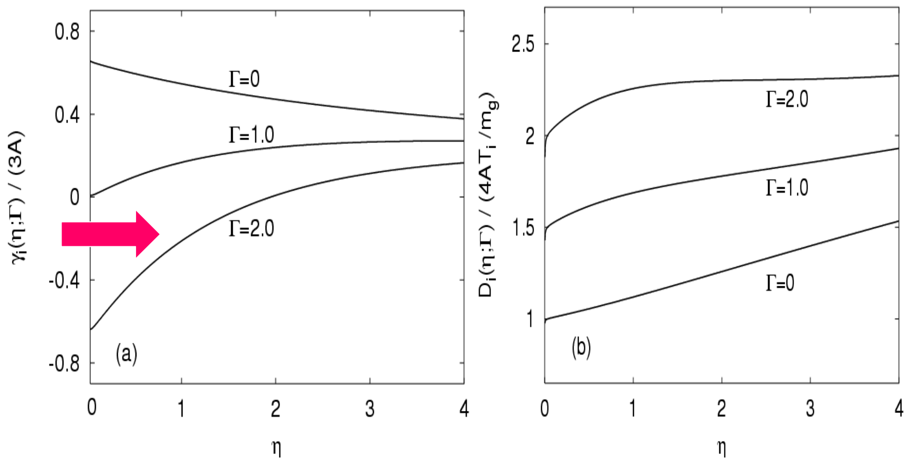
FIG. 7: Time evolution of the angular momentum distribution with a strength of the fluctuations before the critical one.



big noise

FIG. 8: Time evolution of the angular momentum distribution beyond the critical noise strength

Fkt aktive Reibung + Diffusion für Staubplasmen (Trigger/Zagorodny 2003)



8. Dynamik geladener Teilchen

Two charged Brownian particles in parabolic confinement

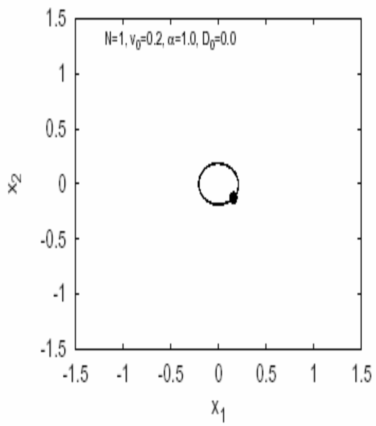
$$H = \frac{m}{2}\boldsymbol{v}_1^2 + \frac{m}{2}\boldsymbol{v}_2^2 \frac{m\omega_0^2}{2}\boldsymbol{r}_1^2 + \frac{m\omega_0^2}{2}\boldsymbol{r}_2^2 + \frac{e^2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|}$$

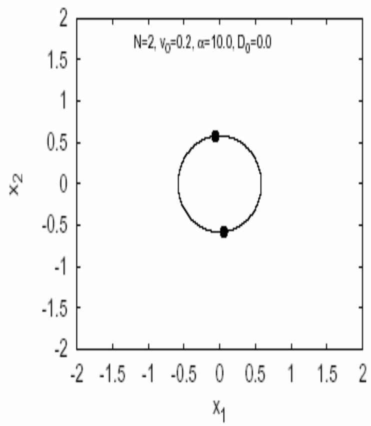
Equilibrium (no driving):

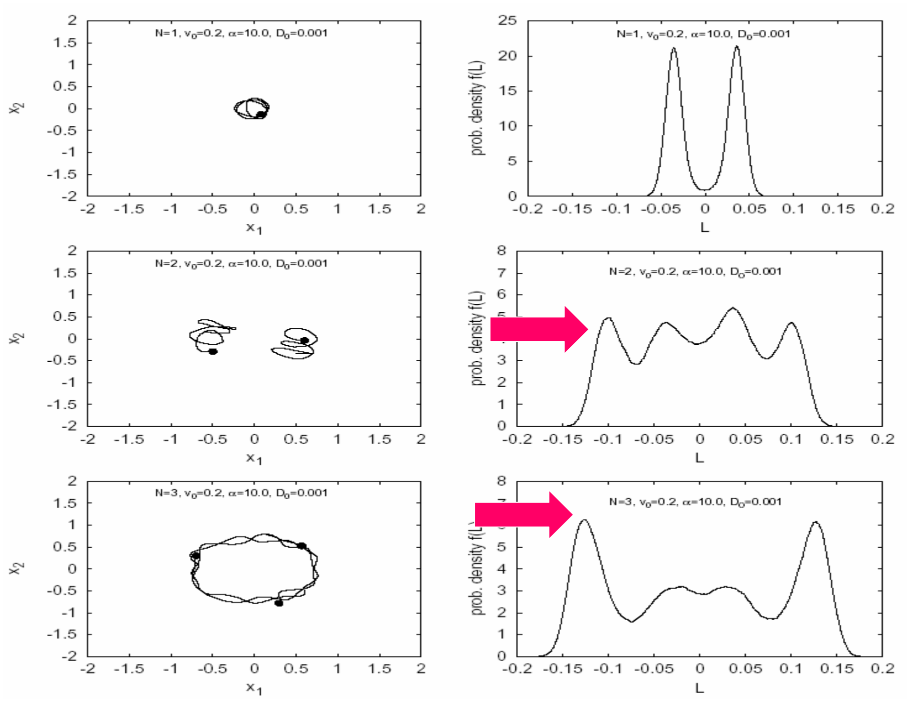
$$r_0 = \left[\frac{e^2}{2m\omega_0^2}\right]^{1/3}$$

$r$otations: no threshold, require only kinetic energy!
$o$scillations: require kinetic + potential energy!

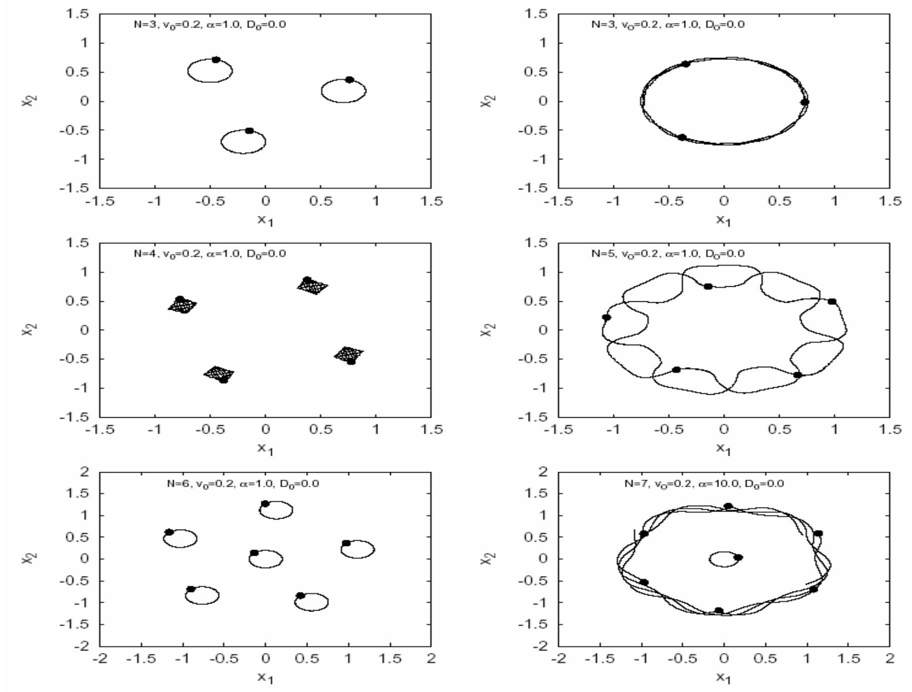
Study 1 or 2 charged particles with negative friction: limit cycles



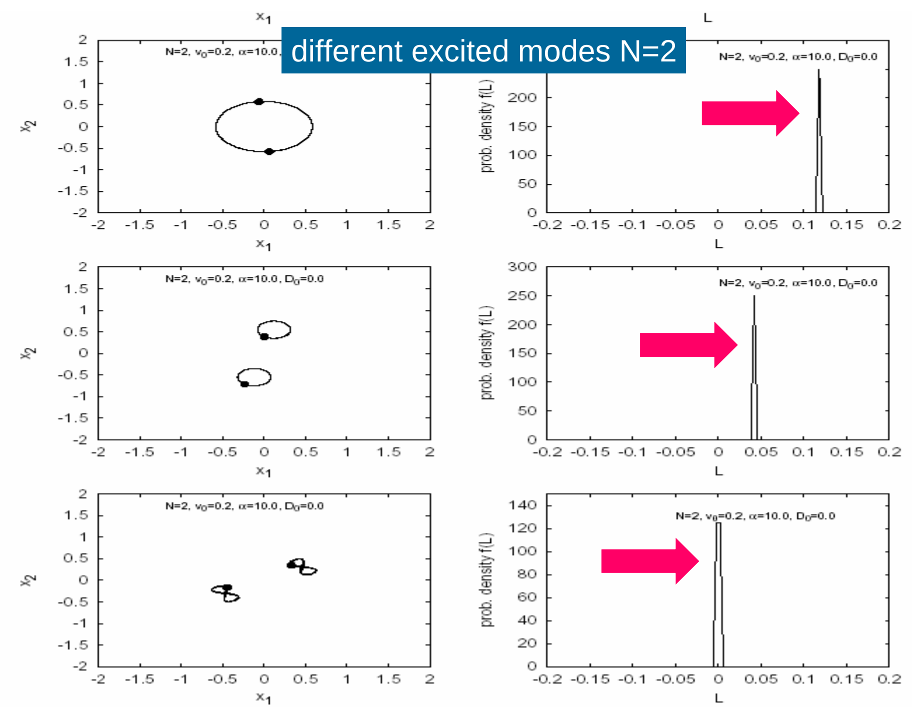




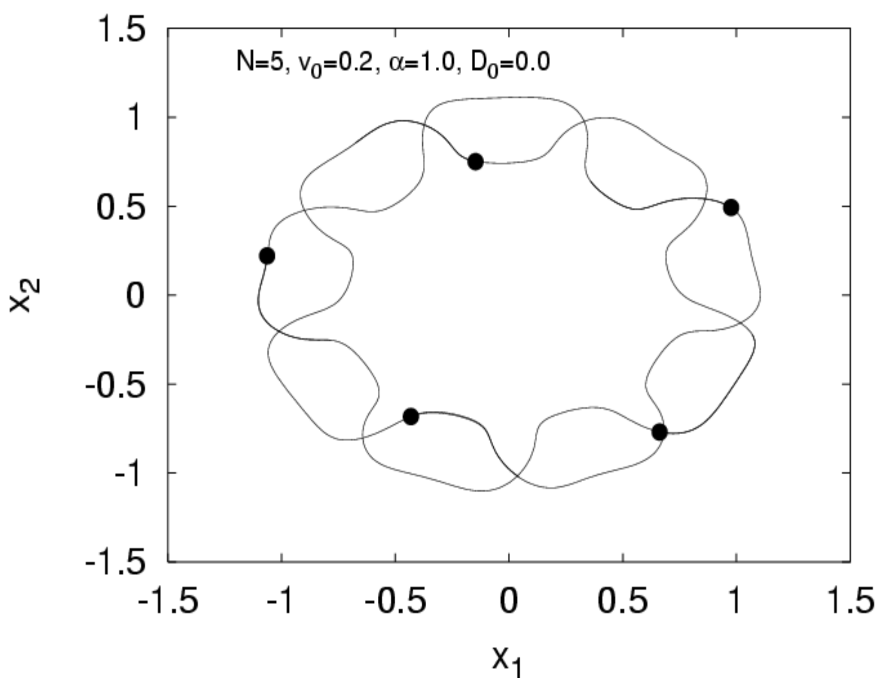
N=1, v0=0.2, α=10.0, D0=0.001
N=2, v0=0.2, α=10.0, D0=0.001
N=3, v0=0.2, α=10.0, D0=0.001
x1
x2
L
prob. density f(L)



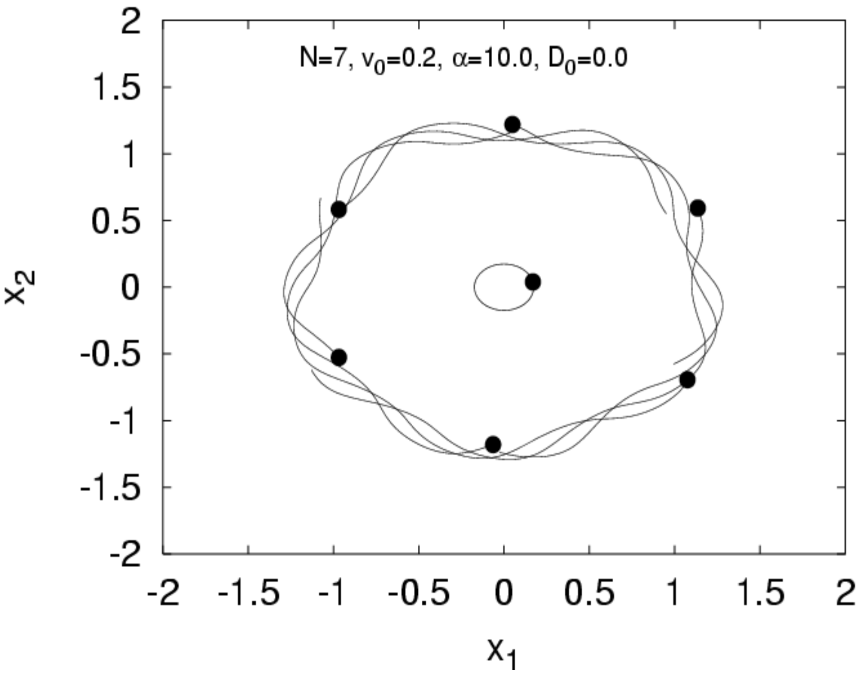
N=3, v0=0.2, α=1.0, D0=0.0
N=4, v0=0.2, α=1.0, D0=0.0
N=5, v0=0.2, α=1.0, D0=0.0
N=6, v0=0.2, α=1.0, D0=0.0
N=7, v0=0.2, α=10.0, D0=0.0
x1
x2



different excited modes N=2
N=2, v0=0.2, α=10.0, D0=0.0
x1
x2
L
prob. density f(L)



N=5, v0=0.2, α=1.0, D0=0.0
x1
x2



Bewegungsgl. Für die dissipativ getriebenen Ionen (oben) und "Elektronen" (unten)

$$\frac{d}{dt}x_k = v_k, \qquad (2.a)$$

$$m\frac{dv_k}{dt} + \frac{\partial U}{\partial x_k} = e_k E + F(v_k) + \sqrt{2D}\,\xi_k(t), \qquad (2.b)$$

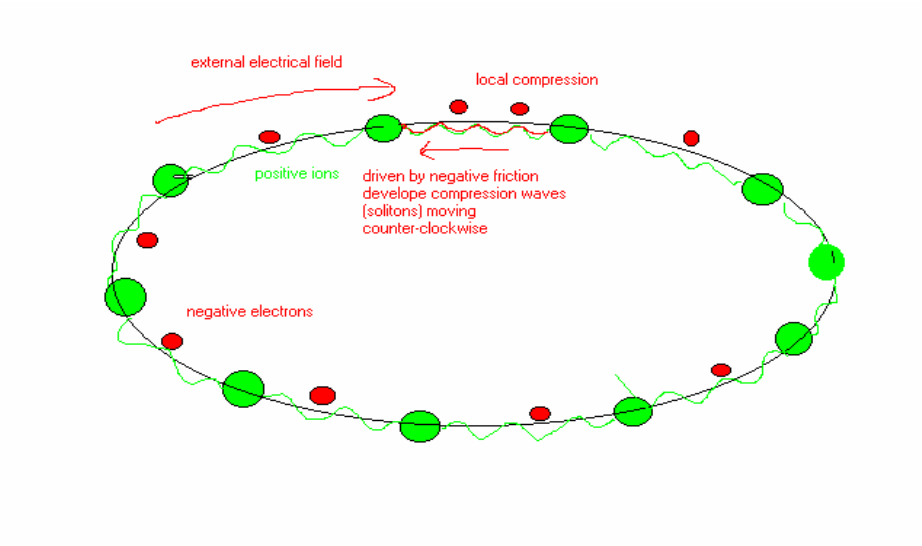
$$U_e(y_j, x_k) = \frac{(-e)e_k}{\sqrt{(y_j - x_k)^2 + h^2}}, \qquad (6)$$

$$\frac{d}{dt}y_j = v_j, \qquad (7.a)$$

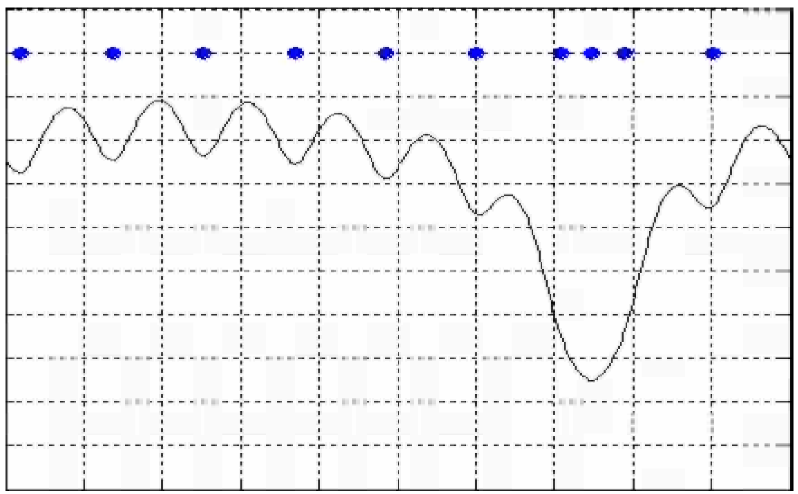
$$m_e\frac{d^2}{dt^2}y_j + \frac{\partial U_e}{\partial y_j} = -eE - m_e\gamma_{e0}v_j + \sqrt{2D_e}\,\xi_j(t), \qquad (7.b)$$

9. Geladene Teilchen in WW mit getriebenen nichtlinearen Gitterschwingungen

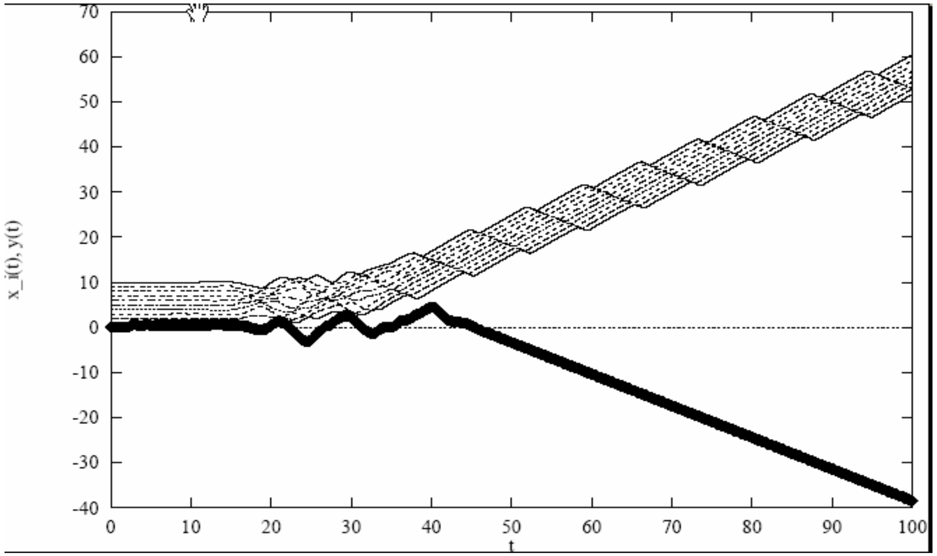
(mit Velarde/Chetverikov/Makarov UC Madrid)



Elektrisches Potential erzeugt durch solitäre Anregungen



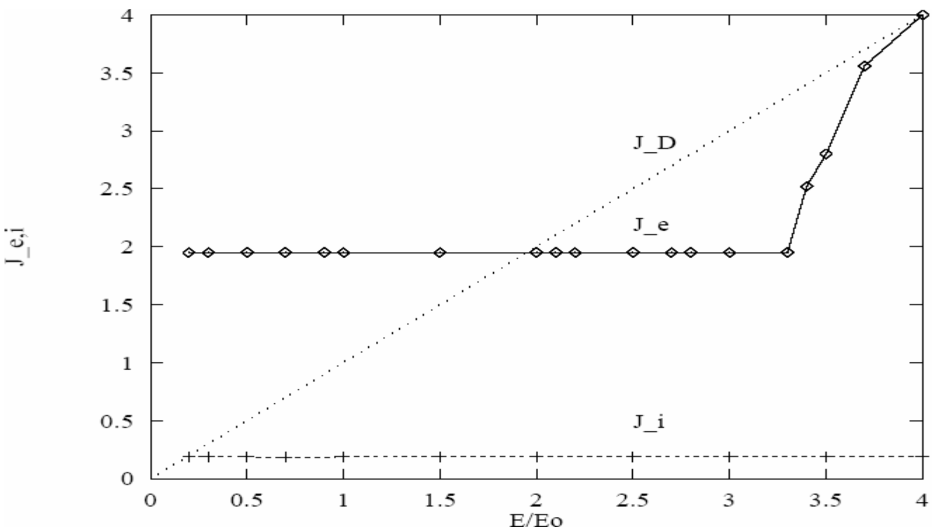
Die “Elektronen” können durch Solitonen (lokale Kompress -> Pottopf) eingefangen werden



Einige Referenzen

- Phys. Rev. Lett. **80**, 5044-5047 (1998) w S&T
- BioSystems **49**, 5044-5047 (1999) “
- Physica A **273,** 294 (1999) “
- Eur. Phys. Journal B **15**,105-113 (2000)
- Phys. Rev. E **64**, 021110 (2001); **65**, 061106 (2002); **67**, 046403 (2003)
- Schweitzer: Brownian agents ... (2003)
- Phys.Rev.E (2004) with Dunkel/Trigger
- Eur. Phys. J. (2005) with Chet&Velarde
- Eb/Sokolov: Stat.TD+Stoch.,Singapore2005

Electron. Strom (soliton-driven) und ion. Strom vgl. mit Drude Strom (nichtlin Char)



10. Zusammenfassung

- Neue Modelle der aktiven Brownschen Bewegung zeigen sehr komplexes dynamisches Verhalten
- Typische Phänom der indiv Bew: Schwing., Rotationen, ...
- Typische kollektive Bew: Schwarmbildung, Clustering, kollektive Translationen, Rotationen, ...
- Neue Anw.: Physik, Biologie, sozio-ökon Systeme ?