Mechanisms supporting cooperation for Prisoner's Dilemma games

György Szabó

Research Institute for Technical Physics and Materials Science
H-1525 Budapest, POB. 49., Hungary

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Outline

- Motivations, basic concepts
- Evolutionary Prisoner's Dilemma games on a lattice
- Effect of topology of connectivity and of noise on cooperation
- PD games with different strategy adoption rates

Conclusions

Evolutionary Prisoner's Dilemma games on a lattice

N players are located on the site *x* of a lattice (periodic boundary conditions)

Each player *x* follows one of the two possible pure (unconditional) strategies,

$$s_x = D = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{(defector)} \quad \text{or} \quad C = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{(cooperator)}$$

Players' payoff comes from games with their neighbors at sites $x+\delta$

$$U_x = \sum_{\delta} s_x^{+} A s_{x+\delta}$$

The game is uniform and symmetric

$$s_x^{+} A s_y = \begin{pmatrix} s_{x1} & s_{x2} \end{pmatrix} \begin{pmatrix} P & T \\ S & R \end{pmatrix} \begin{pmatrix} s_{y1} \\ s_{y2} \end{pmatrix} \quad \text{with rank } T > R > P > S \quad \text{and} \quad T + S < 2R$$

Nash equilibrium for Prisoner's Dilemma: *DD*

social dilemma (applications)

Stochastic evolutionary rule (local Darwinian selection with noise)

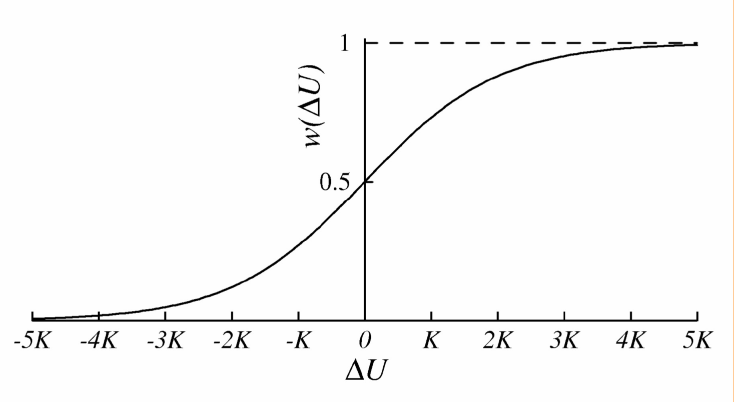
Comparison of pairs:

1. choose a neighboring pair (*x*,*y*) at random
2. determine their payoff (U_x and U_y) dependent on surrounding
3. x adopts the neighboring strategy s_y with a probability

$$w(s_x \to s_y) = \frac{1}{1 + \exp[(U_x - U_y)/K]}$$

K: average amplitude of noise (temperature)

irrational choice is allowed



Steps 1-3 are repeated,

start from a random initial state

Stationary state is investigated for a rescaled payoff matrix:

R=1; T=b; P=0; S=0; $1 < b < 2$

Mean-field approximation

The average payoff for *C* és *D* strategies:

$$U_C = z\rho, \quad U_D = z\rho b$$

where ρ is the concentration of C, and z is the number of neighbors.

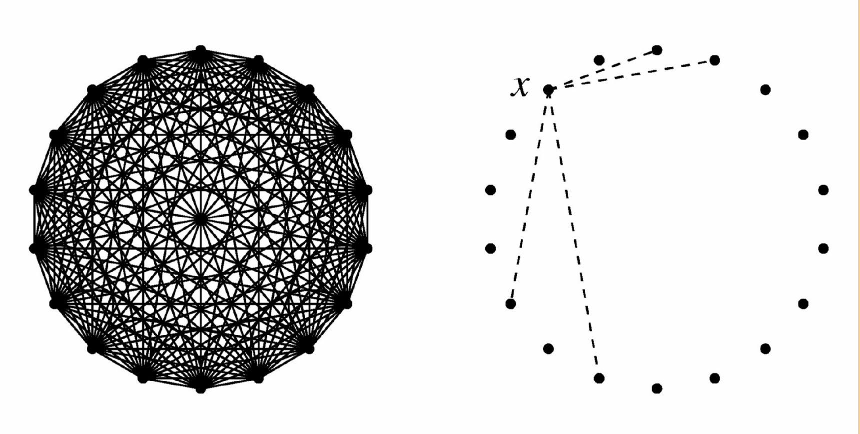
The variation of the *C* strategy concentration:

$$\frac{\partial \rho}{\partial t} = \rho(1-\rho)[w(s_D \to s_C) - w(s_C \to s_D)] = -\rho(1-\rho)\tanh\left(\frac{U_D - U_C}{2T}\right)$$

Notice: $\rho(t) \to 0 \,, \quad as \quad U_D > U_C \,.$

C strategy dies out!!!

Structures satisfying MF conditions



Cooperators become extinct on the one-dimensional lattice (z=2) too.

Mechanisms supporting cooperation

-Kin selection (Hamilton)

advance if relatives help each other

- Direct reciprocity (Axelrod)

application of tit-for-tat strategy

- Indirect reciprocity (Fehr, experiments)

altruistic punishment

- Group selection (Traulsen)

green-beard effect and/or separated groups

- Formation of C colonies in spatial systems

- Influential players

positioned on hubs of scale-free connectivity structure

enhanced teaching activities (in strategy adoption)

Cellular automaton model (Nowak and May 1992)

players on square lattice collect income from neighbors (z=8, 9)

in discrete time steps (t=0,1,2, …) players adopt the best of neighboring strategies

Simulation for b=1.56

Phenomena

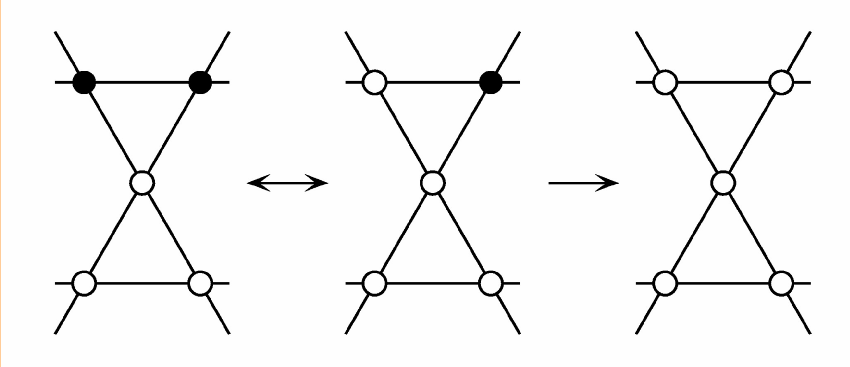
- Cs can survive if they form rectangular colonies
- C invasions along the horizontal and vertical interfaces
- Growth of a C colony is stopped by other growing C colonies
- Solitary defectors have the highest score (zb)

the increase of b is beneficial for D

Spreading of cooperators on overlapping triangles

Cooperators on a triangle receive 2

Neighboring *D* receives *b*



Subsequent invasions from a C triangle result in additional C-occupied triangles in the low noise limit if $b<3/2$

Growth from a *C* seed is blocked by *D*s separating two branches of *C* domain

this process is excluded on tree-like structures (RRG2)

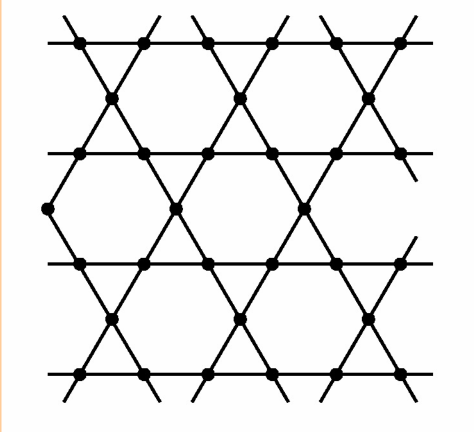
possible on the Kagomé lattice

Consequently, ρ (and b_{c2}) is larger on RRG2.

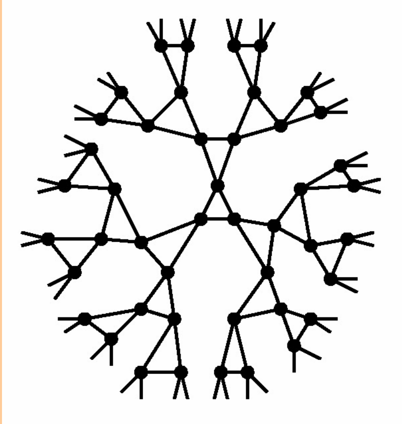
The finite value of ρ is controlled by the collision of *C* branches growing from different *C* seeds (and by other stochastic events).

Structures with (one-site) overlapping triangles

kagomé lattice



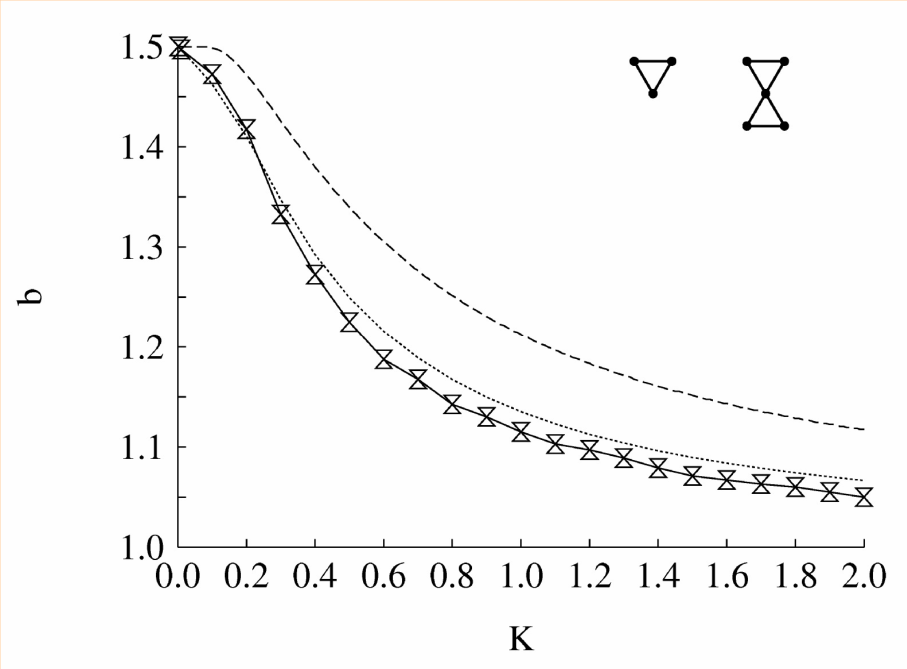
RRG2



Two structures

with locally similar features

The 3- and 5-site approximations are quivalent



MC data on RRG2 are well predicted by the 5-site approximation

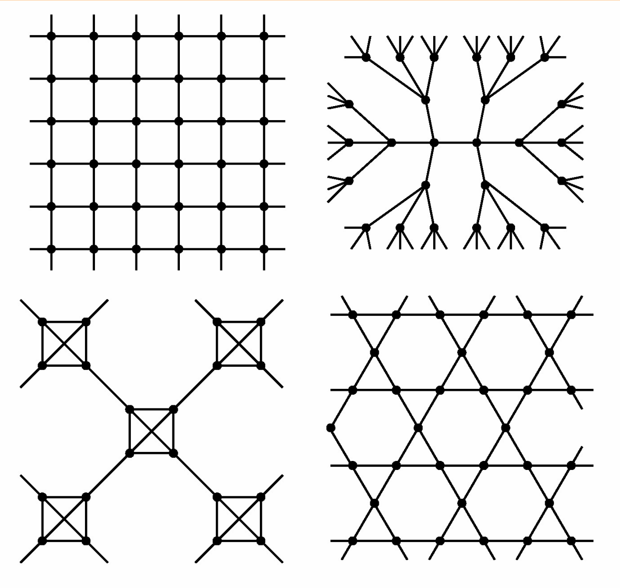
In comparison to Kagomé lattice

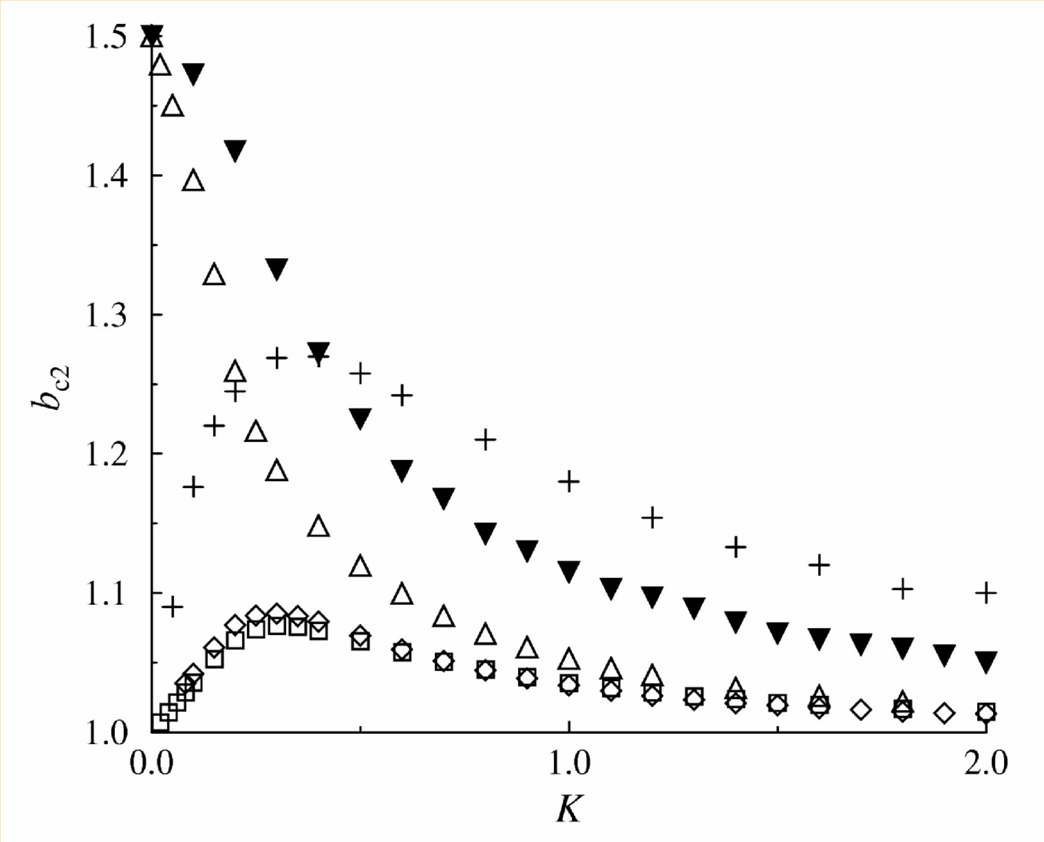
RRG2 has higher b_{c2}

(this feature is related to the tree-like structure)

Comparison of phase diagrams (z=4)

Simulations: □ : square lattice
Δ : kagome lattice
+ : random regular graph (or Bethe?)
▼: RRG2
◊ : lattice of 4-site cliques





For large K the highest cooperation is provided by RRG (the fixed random partnership is better than the spatial structure).

For low noise ($K\rightarrow 0$) some spatial structures sustain cooperation

Importance of overlapping triangles!

Confirmed on some 3d lattices

Effect of inhomogeneous degree distribution Santos et al., PRL (2005)

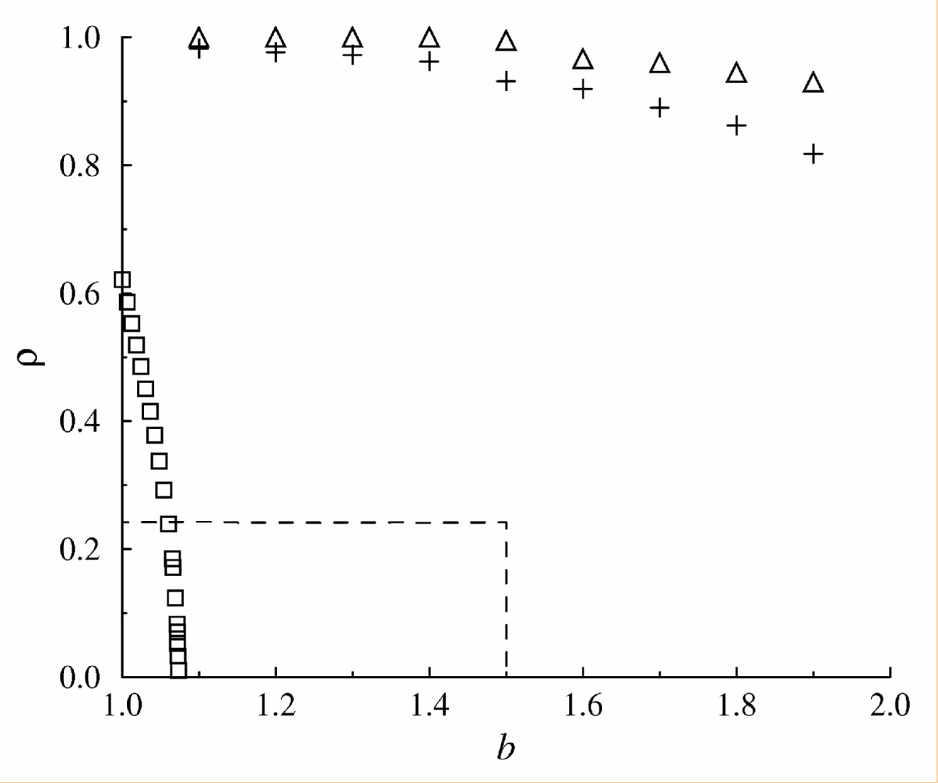
number of neighbors varies for real social networks

scale-free networks [$f(z) \sim z^{-3}$] (e.g., BA and DMS models)

strategy adoption probability is controlled by the difference of total payoffs

favors sites with many neighbors (hubs)

Comparison of MC results on different networks for low noise:



DMS model: Δ

Barabási-Albert model: +

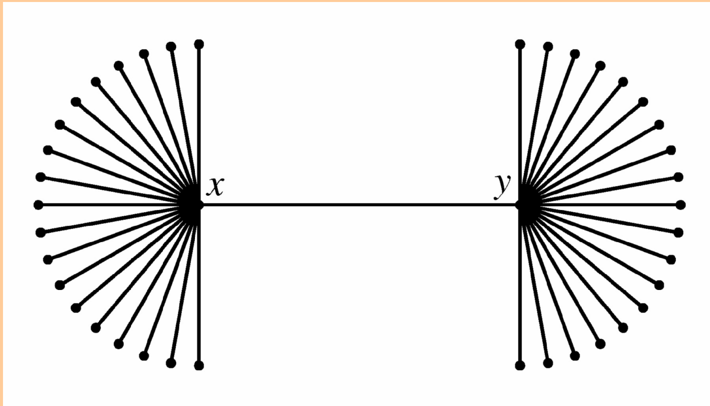
kagome lattice (K=0): - - - - -

Square lattice: □ (K=0.4 optimum)

The mechanism supporting cooperation

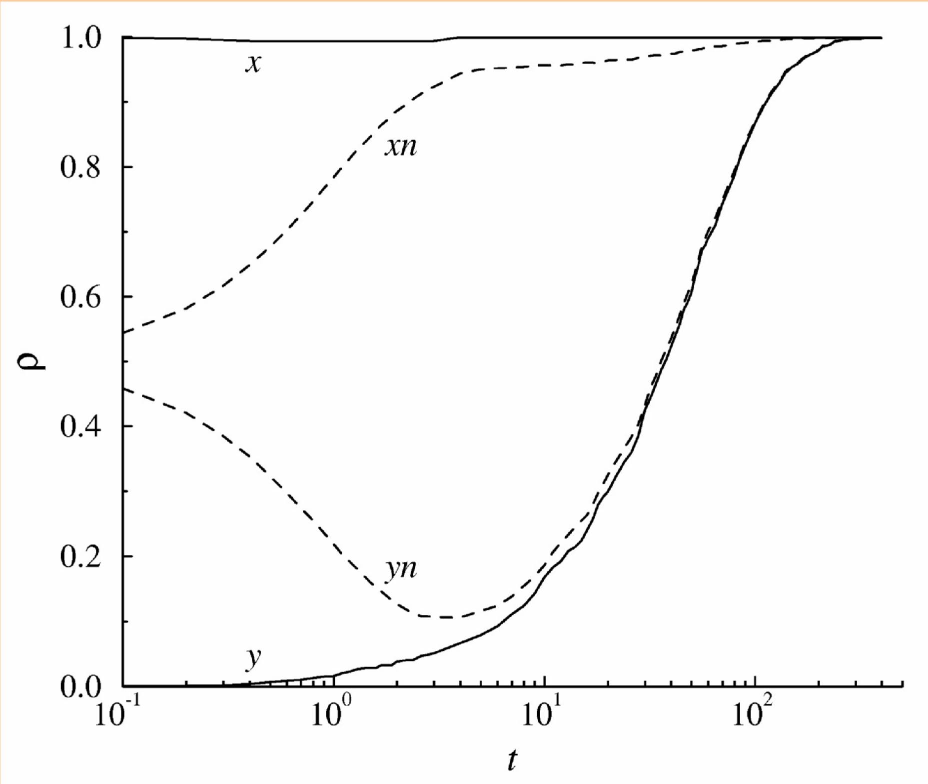
Consider a part of scale-free network with two linked hubs (with large degree)

$$s_x = C \quad \text{and} \quad s_y = D \quad \text{at} \quad t = 0$$



Their neighbors follow *C* or *D* with the same probs.

The effect of surrounding is modelled by strategy adoption from a random site with probability *R*=0.5.



Results averaged over 10,000 runs

At the beginning $U_x > U_{xn}$ and $U_y > U_{yn}$

This yields $s_{xn} \rightarrow C$ and $s_{yn} \rightarrow D$

After some time $s_x = C$ becomes the most successful player to be followed

Inhomogeneity in the strategy adoption rate promotes cooperation

The same evolutionary PD game as before

Szolnoki, EPL (2007)

New features: two types of players (good teachers and bad teachers)

$$n_x = A \quad or \quad B$$

probability of strategy adoption from y to x depends on n_y

$$w(s_x \to s_y) = W_y \frac{1}{1 + \exp[(U_x - U_y)/K]}$$

$$\text{where} \quad W_y = \begin{cases} 1, & \text{if} \quad n_y = A \\ w, & \text{if} \quad n_y = B \end{cases} \quad \text{and} \quad w < 1$$

ν portion of players is A
the initial random distribution of players A and B are quenched
the initial strategy distribution is random

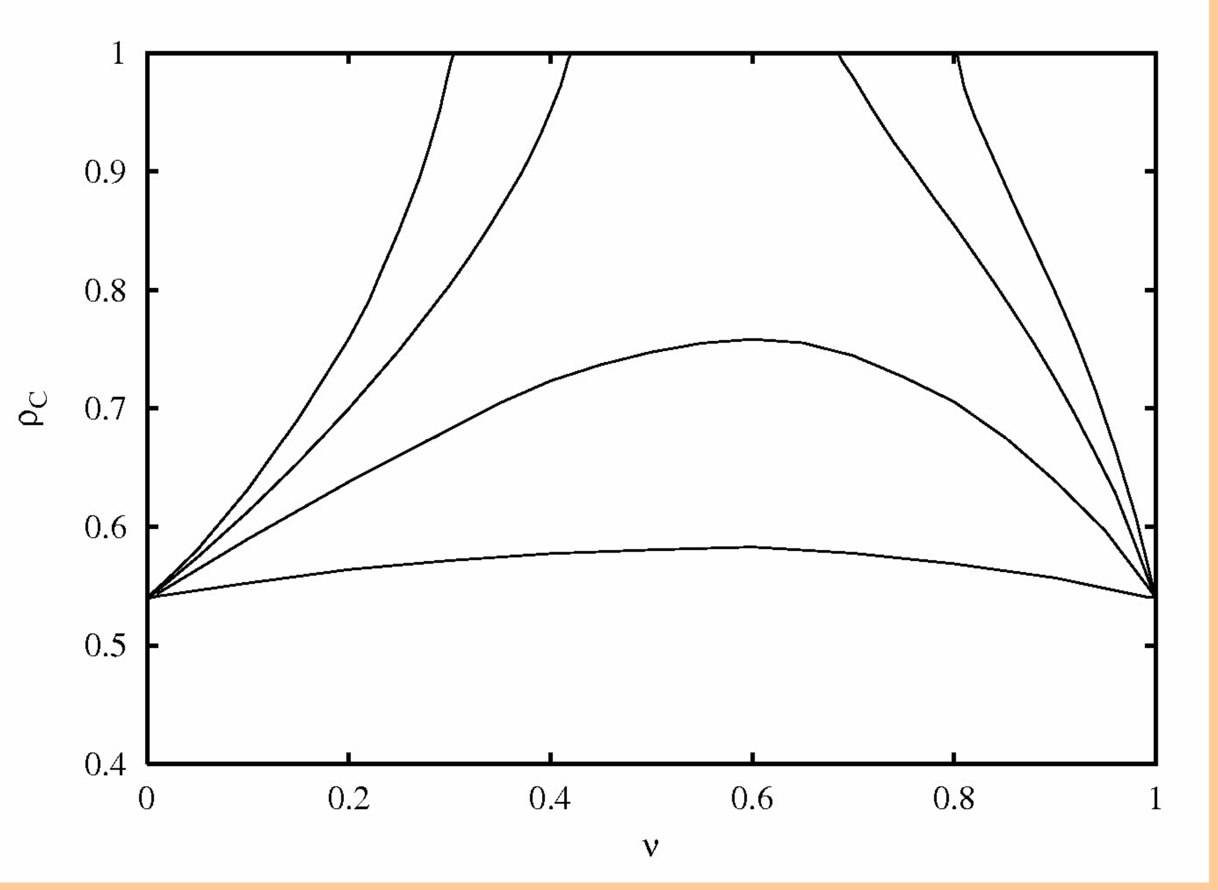
The effect of good teachers is resembling to hubs on scale-free networks
(both are influential players)

Monte Carlo results on kagome lattice

simulations for b=1.03, K=0.5,

w=0.5, 0.2, 0.1, and 0.05

Density of cooperators versus ν



Stationary states

are equivalent

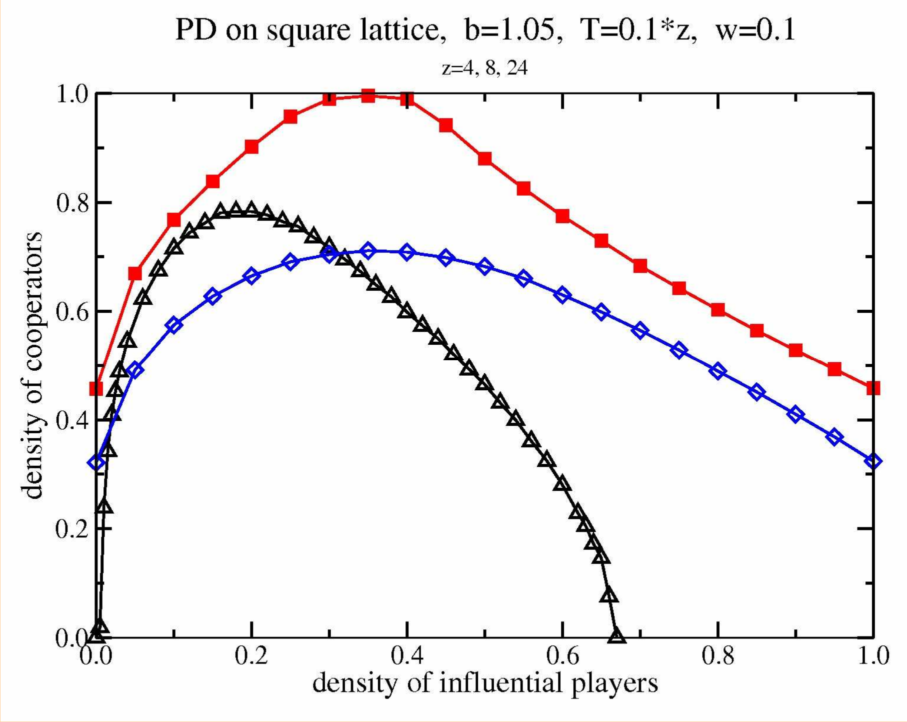
for ν=0 and 1.

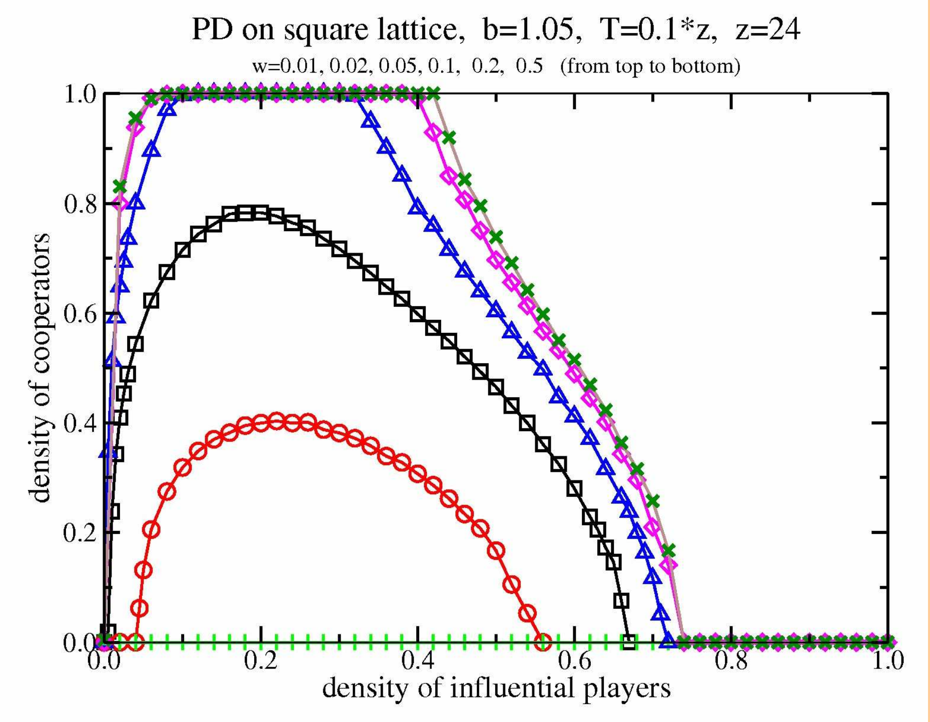
Increase of neighborhood (z)

Two opposite effects: - mean-field type behavior if z $\rightarrow \infty$

- supports the effects of influential players

Preliminary MC results on square lattice for z=4, 8, and 24



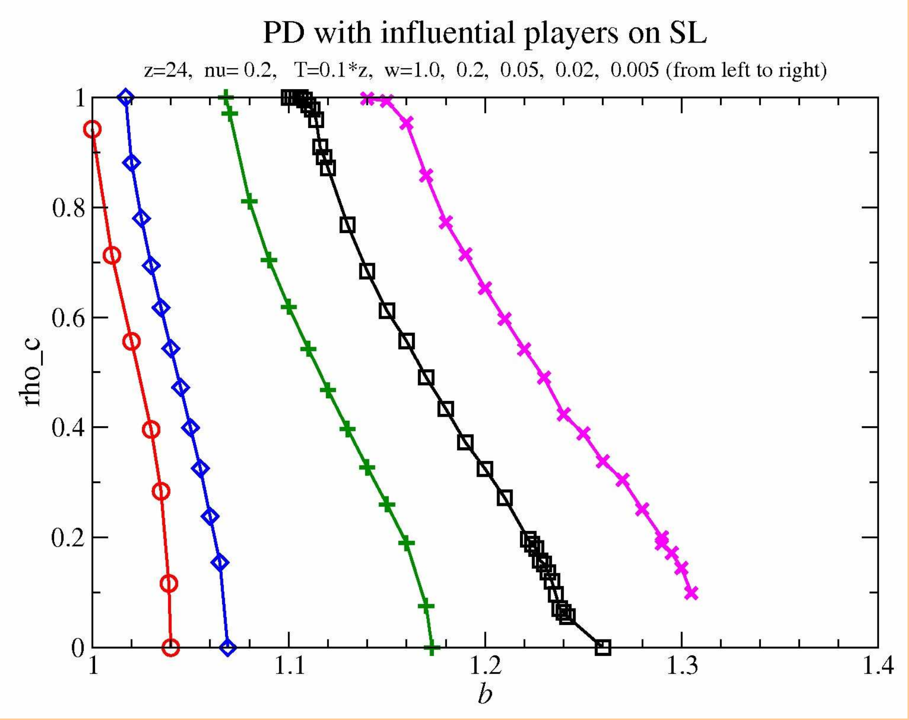


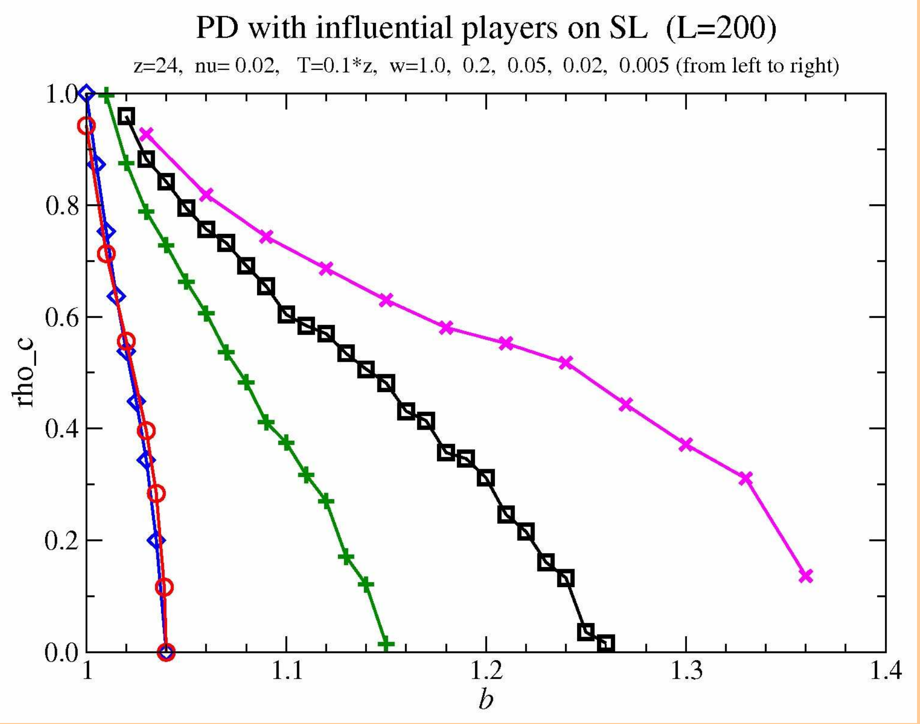
Moving influential players

the same model as before but players A can move to one of the nn. sites.

Preliminary results (in collaboration with M. Droz, J. Szwabinski, and A. Szolnoki)

Comparison of MC data for high (ν=0.2) and low (ν=0.02) densities of standing As





Simulations with standing As

Simulations with moving As

Conclusions

The spatial evolutionary games provide a mathematical basis to explore those mechanisms, structures and evolutionary rules which support the maintenance of cooperation in societies of selfish individuals.

Thank you for your attention